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AMALGAMATION OF EVEN HARMONIOUS GRAPHS WITH STAR GRAPHS

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ABSTRACT

***A** graph G (V, E) with 'n' vertices and 'm' edges is said to be even harmonious graph if f is an injection from the vertices of G to the integers from 0 to 2q such that the induced mapping f* from the edges of G to {0, 2, 4,…2(q-1)} defined by f*(uv) = [f (u) + f (v)](mod 2q) is bijective.*

Key words: *Cyclic graphs, Wheel graphs, Fan graphs, Web graph.*

1. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Graph labeling was introduced in the late 1960's. For detailed survey on graph labeling we refer to Gallian [1]. To begin with simple graph with 'p' vertices and 'q' edges. The definitions and other information which are used for the present investigation are given.

2. DEFINITIONS

Definition 2.1: Even harmonious graph: A function f is said to be an even harmonious graph with q edges if f is an injection from the vertices of G to the integers of 0 to 2q and the induced function f* from the edges of G to {0, 2, …2(q-1)} defined by f*(uv) = [f (u) + f (v)] (mod 2q) is bijective.

Definition 2.2: Wheel graph: The wheel graph W_n is defined to be the join of K_1 and C_n ie the wheel graph consists of edges which joins a vertex of K_1 to every vertex of C_n .

Definition 2.3: Fan graph: Fan f_n ($n \geq 2$) is obtained by joining all vertices of P_n to K_1 and K_1 is called as the center of f_n and contains n+1 vertices and 2n-1 edges ie $f_n = P_n + K_1$.

3. RESULTS

Theorem 3.1: G is a C_{2n+1} graph where $n \geq 0$ of order n and size m. The amalgamation of G with Star graph S_n order 'n' and size (n-1) is even harmonious graph.

Proof: G_{amal} = {V, E, f*} is a graph. Let the vertex set of G_{amal} (V) = {v_1, v_2, v_3, v_4, v_5 ….v_n} and edges set of G_{amal} (E) = {e_1, e_2, e_3, e_4,e_5,…..e_n} label the vertices from 0 to 2q and edge 0 to 2(q-1) such that $G_{(amal)} = V \times V \rightarrow E$ is bijective and is defined as f*(uv) =[f(u) + f(v)] (mod 2q) u, v ϵ G_{amal} (V) and edge labels{0, 2, 4, …. 2 (q-1)} are distinct .Hence G is even harmonious graph.

Example: The graph in fig (1) is $G_{(amal)}$ of C_n with S_2 which is of order 5 and size 5. So G_{amal} (V)= {0, 2, 4, 6, 8, 10} then f* (uv) =[f(u) + f(v)] (mod 2q) such that edge labels are {0, 2, 4, 6, 8} which admits even harmonious labeling similarly, G_{amal} (C_3 ,S_3) in fig (2) is also even harmonious graph. The graph on fig (3) is general case of the above graphs.

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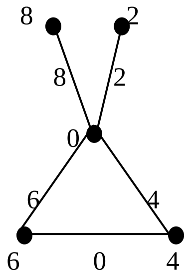
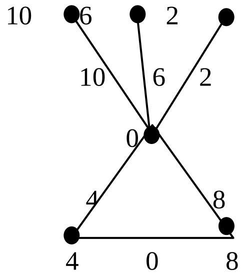




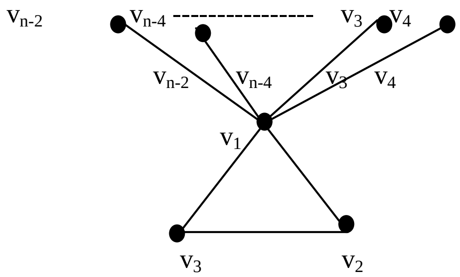


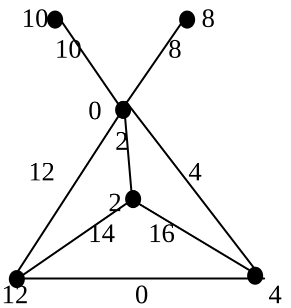
Fig (1) G_{amal} (C_3, S_2)   fig (2) G_{amal}(C_3 , S_3)   fig (3) G_{amal} (C_3 , S_n)

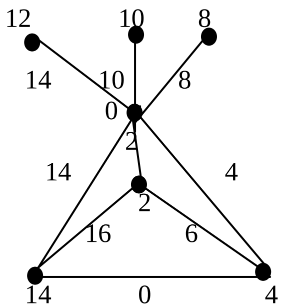
Note: The same is also denoted as amalgamation of (K_3 ● S_n) where $n \geq 2$

Theorem 3.2: G is W_{2n+1} graph where $n \geq 1$ of order n and size m .The amalgamation of G with star graph S_n order 'n' and size (n-1) is even harmonious graphs.

Proof: G_{amal} = {V, E, f*} is a graph. Let the vertex set of G_{amal} (V) = {v_1, v_2, v_3,v_4 v_5 …..v_n} and edges set G_{amal} (E) = {e_1, e_2, e_3, e_4,e_5, ….e_n} label the vertices from 0 to 2q and edge 0 to 2(q-1), label the central vertex as 2 is fixed such that $G_{(amal)}$ = V x V → E is bijective and is defined by. f* (uv) = [f (u) + f (v)] (mod 2q) u, v є G_{amal} (V) and edge labels{0, 2, 4, …. 2 (q-1)} are distinct .Hence G is even harmonious graph.

Example: The graph in fig (1) is $G_{(amal)}$ of W_n with S_2 which is of order 6 and size 8. So G_{amal} (V) = {0, 2, 4, 6, 8, 10, 12, 14, 16} then f*(uv) = [f (u) + f (v)] (mod 2q) such that edge labels are {0, 2, 4, 6, 8,10, 12, 14} which admits even harmonious labeling similarly, $G_{(amal)}$ (W_3, S_3) in fig (2) is also even harmonious graph. The graph on fig (3) is general case of the above graphs.





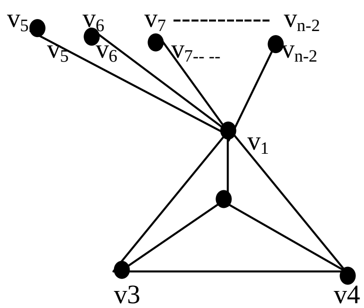


fig (1) G_{amal} (W_3, S_2)   fig (2) G_{amal}(W_3, S_3)   fig (3) G_{amal} (W_3, S_n)

Note: The same is also denoted as amalgamation of (K_4 ●S_n) where $n \geq 2$

Theorem 3.3: G is a fan graph. The amalgamation of G with star graph S_n order 'n' and size (n -1) is even harmonious graphs.

Proof: G_{amal} = {V, E, f*} is a graph. Let the vertex set of G_{amal} (V) = {v_1, v_2, v_3, v_4 v_5 ….v_n} and edges set G_{amal} (E) = {e_1, e_2, e_3, e_4,e_5,…..e_n} label the vertices from 0 to 2q and edge label 0 to 2(q-1), label one duplicate vertex in star graph [5] such that $G_{(amal)}$ = VxV→ E is bijective is defined as. f*(uv) = [f (u) + f (v)] (mod 2q) u, v є G_{amal} (V) and edge labels {0, 2, 4, ---- 2 (q-1)} are distinct .Hence G is even harmonious graph.

Example: The graph in fig (1) is G_{amal} of f_n with S_2 which is of order 6 and size7. So G_{amal} (V) = {0, 2, 4, 6, 8, 10,12, 14} then f* (uv) = [f(u) + f(v)] (mod 2q) such that edge labels are {0, 2, 4, 6, 8,10, 12,} which admits even harmonious labeling similarly, $G_{(amal)}$ (f_3 ,S_3) in fig (2) is also even harmonious graph. The graph on fig (3) is general case of the above graphs.

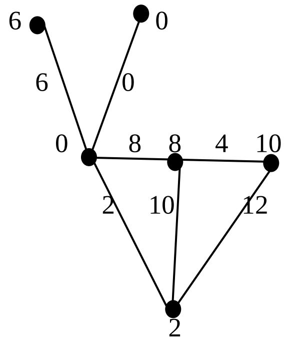


Fig. (1)

$G_{(amal)}$ (f_2 ,S_2)

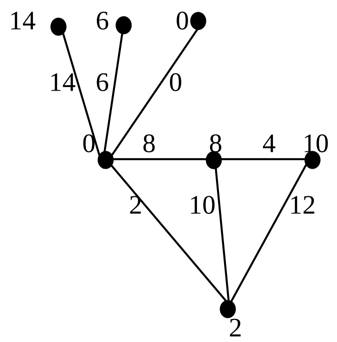


Fig. (2)

$G_{(amal)}$ (f_2 ,S_3)

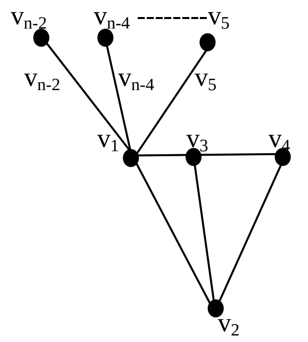


Fig. (3)

$G_{(amal)}$ (f_2 ,S_n)

Theorem 3.4: G is a $\{(C_3 \times K_2) \bullet S_n\}$also known as closed web graph denoted as $(\mathcal{W}_3)$where $n \geq 2$ which is of order 6 and size 9 The amalgamation of G with Star graph S_n order n and size (n-1) is even harmonious graph.

Proof: G_{amal} = {V, E, f*} is a graph. Let the vertex set of G_{amal} (V) = $\{v_1, v_2, v_3, v_4\ v_{5,\ldots\ldots}v_n\}$ and edge set G_{amal} (E) = $\{e_1, e_2, e_3, e_4, e_5, \ldots e_n\}$ label the vertices from 0 to 2q and edge label 0 to 2(q-1), label one duplicate vertex in star graph [5] such that $G_{(amal)}$ = VxV→E is bijective is defined as f* (uv) = [f (u) + f (v)] (mod 2q) u, v є G_{amal} (V) and edge labels {0, 2, 4, …..2(q-1)} are distinct. Hence G is even harmonious graph.

Example: The graph in fig (1) is G_{amal} of (C_3 x K_2) with S_2 which is of order 6 and size 9.So G_{amal} (V) = {0, 2, 4, 6, 8, 10,12,14, 16, 18} then f*(uv) = [f(u) + f(v)] (mod 2q) such that edge labels are {0, 2, 4, 6, 8,10,12,14,16} which admits even harmonious labeling similarly, G_{amal} ($\mathcal{W}_3$,S_2) in fig (2) is also even harmonious graph. The graph on fig (3) is general case of the above graphs.

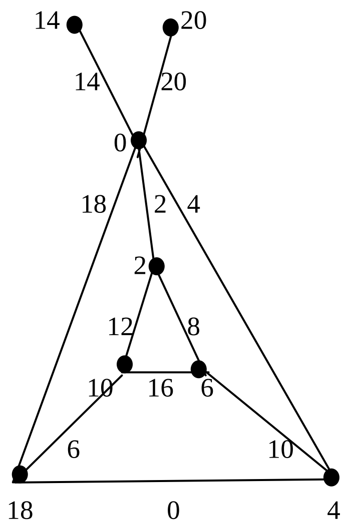


fig (1)

G_{amal} ($\mathcal{W}_3$, S_2)

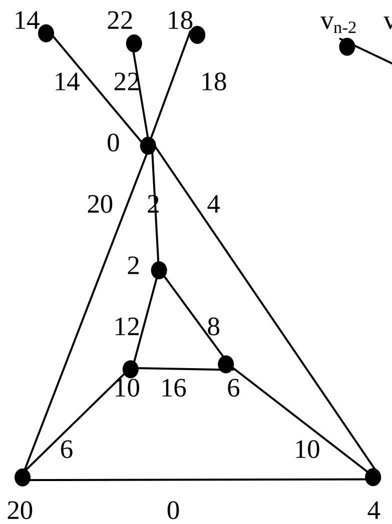


fig (2)

G_{amal} ($\mathcal{W}_3$, S_3)

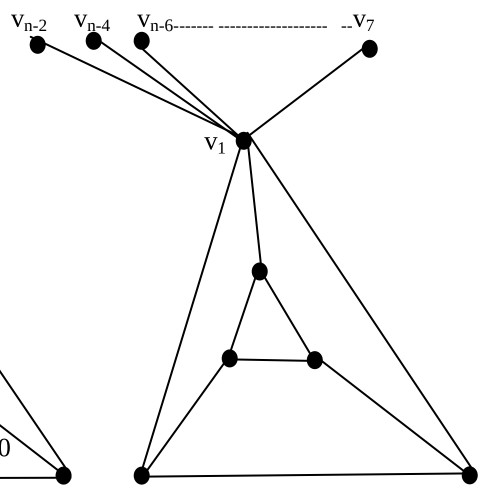


fig (3)

G_{amal} ($\mathcal{W}_3$, S_n)

CONCLUSION

In this paper we have observed that Even harmonious graphs most of the graph obtained by amalgamation are even harmonious in future the same process will be analyzed for some graphs.

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