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A NEW CLASS OF CONTRA MAPPINGS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce a new class of sets called soft regular generalized β closed sets (briefly soft rg β – closed set). Further some properties of soft contra regular generalized β continuous mapping and soft contra regular generalized β irresolute mappings are discussed in the soft topological spaces which are defined over an initial universe with a fixed set of parameters.

Keywords: *Soft sets, soft regular generalized β closed set, soft contra regular generalized β continuous mapping, soft contra regular generalized β irresolute mapping.*

1. INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. Molodtsov's [10] soft set theory was originally proposed as general mathematical tool for dealing with uncertainty problems. He proposed soft set theory, which contains sufficient parameters such that it is free from the corresponding difficulties, and a series of interesting applications of the theory instability and regularization, Game Theory, Operations Research, Probability and Statistics. Topological structure of soft sets was initiated by Shabir and Naz [12] and studied the concepts of soft open set, soft interior point, soft neighborhood of a point, soft separation axioms and subspace of a soft topological space. Many researchers extended the results of generalization of various soft closed sets in many directions. Athar Kharal and B. Ahmad [3] defined the notion of a mapping on soft classes and studied several properties of images and inverse images of soft sets.

In this paper we introduce the concept of soft regular generalized β closed sets (briefly soft rg β – closed set) ,soft contra regular generalized β continuous mappings and soft contra regular generalized β irresolute mappings on the topological space. Also the relation between the existing space and newly defined space are discussed.

2. PRELIMINARIES

Definition 2.1[10]: Let U be an initial universe and E be a set of parameters. Let P(U) denote the power Set U of and A be a non empty subset of E.A pair (F,A) is called a soft set over F, where F is a mapping given by F:A⟶P(U).

Definition 2.2[8]: A subset (A, E) of a topological space X is called Soft generalized closed (soft g –closed) is cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U,E) and (U, E) is open in X.

Definition 2.3[1]: A subset (A, E) of a topological space X is called Soft semi generalized closed (soft sg –closed) is scl(A, E) $\tilde{\subseteq}$ (U,E) and (U, E) is soft semi-open in X.

Definition 2.4[1]: A subset (A, E) of a topological space X is called Soft generalized- semi closed (soft gs –closed) is scl(A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft open in X.

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Definition 2.5[1]: A subset (A, E) of a topological space X is called Soft β closed if int(cl(int(A, E))) $\tilde{\subseteq}$ (A, E)

Definition 2.6[1]: A subset (A, E) of a topological space X is called Soft α generalized closed(soft α g –closed) is α cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft open in X.

Definition 2.7[6]: A subset (A, E) of a topological space X is called Soft generalized α -closed (soft g α –closed) is α cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft α -open in X.

Definition 2.8[1]: A subset (A, E) of a topological space X is called Soft generalized- β closed (soft g β –closed) is β cl(A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft open in X.

Definition 2.9[8]: A subset (A, E) of a topological space X is called generalized- pre closed (soft gp –closed) is pcl(A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft open in X.

Definition 2.10[6]: Let (X, τ, E) and (Y, τ′, E) be a two soft topological spaces. A function f: (X, τ, E)⟶ (Y, τ′, E) is said to be soft contra semi-continuous if f^{-1}(G, E) is soft semi- closed (open) in (X, τ, E) for every soft open (closed) set (G, E) of (Y, τ′, E).

Definition 2.11[6]: Let (X, τ, E) and (Y, τ′, E) be a two soft topological spaces. A function f: (X, τ, E)⟶(Y, τ′, E) is said to be soft contra α -continuous if f^{-1}(G, E) is soft α - closed (open) in (X, τ, E) for every soft open (closed) set (G, E) of (Y, τ′, E).

Definition 2.12[6]: Let (X, τ, E) and (Y, τ′, E) be a two soft topological spaces. A function f: (X, τ, E) ⟶(Y, τ′, E) is said to be soft contra sg-continuous if f^{-1}(G,E) is soft sg- closed (open) in (X, τ, E) for every soft open (closed) set (G, E) of (Y, τ′, E).

Definition 2.13[6]: Let (X, τ, E) and (Y, τ′, E) be a two soft topological spaces. A function f: (X, τ, E) ⟶ (Y, τ′, E) is said to be soft contra gs-continuous if f^{-1}(G, E) is soft gs- closed (open) in (X, τ, E) for every soft open (closed) set (G, E) of (Y, τ′, E).

Definition 2.14[6]: Let (X, τ, E) and (Y, τ′, E) be a two soft topological spaces. A function f: (X, τ, E) ⟶ (Y, τ′, E) is said to be soft contra g β -continuous if f^{-1}(G,E) is soft g β - closed (open) in (X, τ, E) for every soft open (closed) set (G, E) of (Y, τ′, E).

Definition 2.15[6]: Let (X, τ, E) and (Y, τ′, E) be a two soft topological spaces. A function f: (X, τ, E) ⟶(Y, τ′, E) is said to be soft contra gs β -continuous if f^{-1}(G, E) is soft gs β - closed (open) in (X, τ, E) for every soft open (closed) set (G, E) of (Y, τ′, E).

3. SOFT REGULAR GENERALIZED β CLOSED SETS

Definition 3.1: A subset (A, E) of a topological space X is said to be soft regular generalized β closed (soft rg β –closed) in a soft topological space (X, τ , E), if β cl(A, E) $\tilde{\subseteq}$ (U, E), whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft regular open in X.

Theorem 3.2:
1. Every soft closed set is soft rg β – closed.
2. Every soft semi-closed set is soft rg β – closed.
3. Every soft pre-closed set is soft rg β – closed.
4. Every soft g-closed set is soft rg β – closed.
5. Every soft sg-closed set is soft rg β – closed.
6. Every soft gs-closed set is soft rg β – closed.
7. Every soft β -closed set is soft rg β – closed.
8. Every soft α g-closed set is soft rg β – closed.

9. Every soft g α -closed set is soft rg β – closed.
10. Every soft rg-closed set is soft rg β – closed.
11. Every soft g β -closed set is soft rg β – closed.
12. Every soft gp-closed set is soft rg β – closed.

Proof:

1. Let (A, E) be any soft closed set in X such that (A, E) $\tilde{\subseteq}$(U, E), where (U, E) is soft regular open. Since β cl(A, E) $\tilde{\subseteq}$ cl(A, E)=(A, E).Therefore β cl(A, E) $\tilde{\subseteq}$ (U, E). Hence (A, E) is soft rg β – closed set in X.
2. Let (A, E) be any soft semi-closed set in X such that (A, E) $\tilde{\subseteq}$(U, E), where (U, E)is soft regular open. Since β cl(A, E) $\tilde{\subseteq}$scl(A, E) $\tilde{\subseteq}$ (U, E).Therefore β cl(A, E) $\tilde{\subseteq}$ (U, E).Hence (A, E) is soft rg β – closed set in X.

The remaining proofs are straight forward.

The Converse of the above implications need not be true which are proved by the below examples.

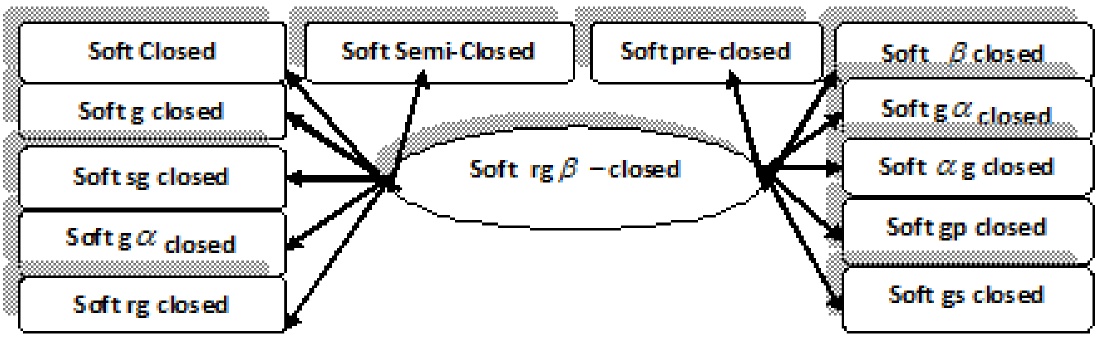
Example 3.3: Let X= {a, b, c}, E= {e_1, e_2} and τ = { ϕ , X, (F_2, E), (F_3, E), (F_6, E)},
Where
(F_1, E), (F_2, E), (F_3, E),……(F_{13}, E) are soft sets over X defined by
(F_1, E) = {(e_1,{a}), (e_2, ϕ), (F_2, E) = {(e_1,{a}), (e_2, {a})}, (F_3, E) = {(e_1, {b}), (e_2,{b})}
(F_4, E) = {(e_1, ϕ), (e_2,{b})}, (F_5, E) = {(e_1,{b, c}), (e_2,{b, c})}, (F_6, E) = {(e_1,{a, b}), (e_2,{a, b})}
(F_7, E) = {(e_1,{a, c}),(e_2,{a, c})}, (F_8, E) = {(e_1,{c}), (e_2,{c})}, (F_9, E) = {(e_1,{a}), (e_2, {a, b})}
(F_{10}, E) = {(e_1,{a, b}), (e_2,{a})}, (F_{11}, E) = {(e_1,{a, b, c}), (e_2,{a, c})}, (F_{12}, E) = {(e_1,{b}), (e_2, ϕ)}
(F_{13}, E) = {(e_1, {b}), (e_2,{a, c})} and (X, τ , E) be soft topological spaces over X. Then

(i) (F_3, E) is soft rg β – closed but not soft closed.
(ii) (F_6, E) is soft rg β – closed but not soft semi-closed.
(iii) (F_{12}, E) is soft rg β – closed but not soft g-closed.
(iv) (F_9, E) is soft rg β – closed but not soft gp-closed.
(v) (F_{10}, E) is soft rg β – closed but not soft β -closed.
(vi) (F_{11}, E) is soft rg β – closed but not soft g β -closed.
(vii) (F_6, E) is soft rg β – closed but not soft pre-closed.
(viii) (F_8, E) is soft rg β – closed but not soft rg-closed.
(ix) (F_4, E) is soft rg β – closed but not soft α g-closed.

Remark:

1. The intersection of two subsets of a soft rg β – closed set in X need not be soft rg β – closed set in X.
2. If A and B are soft rg β – closed set in X, then A $\cup$ B need not be soft rg β – closed set in X.

From the above results and examples the following implications are made:



Theorem 3.4: If (A, E) $\tilde{\subseteq}$ Y $\tilde{\subseteq}$ X and suppose that (A, E) is soft rg β – closed sets in X then (A, E) is soft rg β – closed set relative to Y.

Proof: (A, E) is a is soft rg β – closed sets relative to Y. Then (A, E) Y $\cap$ (U, E),where (U, E) is regular open. Since (A, E) is soft rg β – closed (A, E) $\tilde{\subseteq}$(U, E).that is β cl(A, E) $\tilde{\subseteq}$ (U, E). Such that (A, E) is soft rg β – closed set relative to Y.

Theorem 3.5: If (A, E) is both soft regular open and soft rg β – closed set in X then (A, E) is soft regular closed set.

Proof: Since (A, E) is soft regular open and soft rg β – closed set in X, β cl(A,E) $\tilde{\subseteq}$ (U, E).But (A, E) $\tilde{\subseteq}$ β cl(A,E). Therefore A= β cl(A, E).Hence (A,E) is soft regular closed set.

Theorem 3.6: For x $\in$ X, then the set x-{X} is a soft rg β – closed set or soft regular open.

Proof: Suppose that X-{x} is not soft regular open, then X is the only soft regular open set containing X-{x}.

(i.e) β cl(X-{x}) $\tilde{\subseteq}$ X. Then x-{X} is a soft rg β – closed in X.

4. SOFT CONTRA rg β –CONTINUOUS AND SOFT CONTRA rg β –IRRESOLUTE FUNCTIONS IN TOPOLOGICAL SPACES

Definition 4.1: Let (X, τ, E) ⟶(Y, τ′, E) be two soft topological spaces. A function f: (X, τ, E)↦ (Y, τ′, E) is said to be Soft contra rg β -continuous, if f^{-1}(G, E) is soft rg β -closed (open) in (X, τ, E), for every soft open (closed) set (G, E) of (Y, τ′, E)

Definition 4.2: Let (X, τ, E) ⟶(Y, τ′, E) be two soft topological spaces. A function f: (X, τ, E)↦ (Y, τ′, E) is said to be soft contra rg β -irresolute, if f^{-1} (G, E) is soft rg β -closed (open) in (X, τ, E), for every soft rg β - open (closed) set (G, E) of (Y, τ′, E).

Theorem 4.3: If a function f: (X, τ, E) ⟶ (Y, τ′, E) is soft contra rg β -continuous and (U, E) is soft open in (X, τ, E). Then (f/U): (U, τ, E) ⟶(Y, τ′, E) is soft contra rg β -continuous.

Proof: Let (V, E) be any soft closed in (Y,′,τE). Since f: (X, τ, E) → (Y, τ′, E) is soft contra rg β -continuous, f^{-1}(V, E) is rg β -open in (X, τ, E), $(f/U)^{-1}$(V,E) $\tilde{\cap}$ (U,E) is soft contra rg β -open in (X, τ, E). Hence f($(f/U)^{-1}$(V)) is soft rg β -open in (U, τ, E).

Definition 4.4: A soft topological space (X, τ, E) is called
- (i) rg β -locally indiscrete if every rg β -open set is closed
- (ii) Tβ-space if every rg β -closed set is β-closed.

Theorem 4.4: Let (X, τ, E) and (Y, τ′, E) be two soft topological spaces and f : (X, τ, E)⟶ (Y, τ′, E) be a soft function, Then
- (1) If f is soft rg β -continuous and the space (X, τ, E) is rg β -locally indiscrete then f is soft contra continuous.
- (2) If f is soft rg β -irresolute and the space (X, τ, E) is rg β -locally indiscrete then f is soft contra continuous.
- (3) If f is soft contra rg β -continuous and the space (X, τ, E) is rg β -space then f is soft contra continuous.
- (4) If f is soft contra rg β -continuous and the space (X, τ, E) is $T_{rg}\beta$ - space then f is soft contra β continuous.

Proof:
(1) Let (V, E) be a soft open in (Y, τ′, E). Since f is soft rg β -continuous. f^{-1}(V, E) is soft rg β -open in (X, τ, E). Since (X, τ, E) is soft locally rg β -indiscrete, f^{-1}(V, E) is closed in (X, τ, E). Hence f is contra continuous.
The remaining proofs are straight forward.

Theorem 4.6: If a function f : (X, τ, E) ⟶(Y, τ′, E), then the following conditions are equivalent

1) f is soft contra rg β - continuous.
2) The inverse image of every soft closed set of (Y, τ′, E) is soft rg β -open.

Proof: proof is obvious.

Theorem 4.7: Let function f: (X, τ, E) ⟶(Y, τ′, E), Then

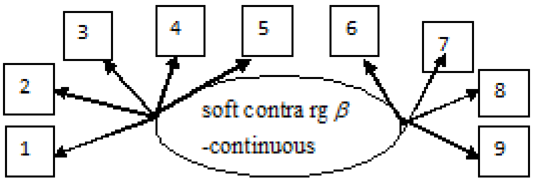
1) If f is soft contra continuous, then f is soft contra rg β -continuous.
2) If f is soft contra gs- continuous, then f is soft contra rg β -continuous.
3) If f is soft contra pre- continuous, then f is soft contra rg β -continuous.
4) If f is soft contra g- continuous, then f is soft contra rg β -continuous.
5) If f is soft contra sg- continuous, then f is soft contra rg β -continuous.
6) If f is soft contra semi- continuous, then f is soft contra rg β -continuous.
7) If f is soft contra α g- continuous, then f is soft contra rg β -continuous.
8) If f is soft contra g α - continuous, then f is soft contra rg β -continuous.
9) If f is soft contra α - continuous, then f is soft contra rg β -continuous.

Proof:

1) Let (F, E) be a soft open set in (Y,′,τE), Since f is soft contra continuous mapping, then f^{-1}(F, E) is soft closed in (X, τ, E). As every soft closed set is soft rg β -closed, then f^{-1} (F, E) is soft rg β -closed in (X, τ, E). Hence f is soft contra rg β -continuous.
2) Let (F, E) be a soft open set in (Y, τ′, E), Since f is soft contra gs-continuous mapping, then f^{-1}(F, E) is soft gs-closed in (X, τ, E). As every soft gs-closed set is soft rg β -closed, then f^{-1}(F, E) is soft rg β -closed in (X, τ, E). Hence f is soft contra rg β -continuous.

The remaining proofs are straight forward.

From the above results the following implications are made:



1. Soft contra continuous 2. Soft contra gs- continuous 3. Soft contra pre- continuous
4. Soft contra g- continuous 5. Soft contra sg- continuous 6. Soft contra semi- continuous
7. Soft contra α g- continuous 8**.** Soft contra g α - continuous 9. Soft contra α - continuous

5. COMPOSITIONS OF SOFT MAPPINGS

Theorem 5.1:

1) If f: (X, τ, E) ⟶ (Y, τ′, E) is soft rg β -irresolute and g: (Y, τ′, E) ⟶ (Z, σ , E) is soft continuous then gof: (X, τ, E)⟶ (Z, σ , E) is soft rg β -continuous.
2) If f: (X, τ, E)⟶(Y, τ′, E) is soft contra rg β -irresolute and g: (Y, τ′, E) ⟶(Z, σ , E) is soft contra rg β -continuous then gof: (X, τ, E)⟶(Z, σ , E) is soft rg β - continuous.
3) If f : (X, τ, E)⟶ (Y, τ′, E) is soft contra continuous and g: (Y, τ′, E)⟶(Z, σ , E) is soft contra continuous then gof: (X, τ, E)⟶(Z, σ , E) is soft continuous.
4) If f: (X, τ, E) ⟶(Y, τ′, E) is soft rg β -irresolute and g: (Y, τ′, E) ⟶ (Z, σ , E) is soft contra rg β continuous then gof: (X, τ, E)⟶ (Z, σ , E) is soft contra rg β continuous.
5) If f: (X, τ, E) ⟶ (Y, τ′, E) is soft contra rg β -continuous and g: (Y, τ′, E)⟶ (Z, σ , E) is soft continuous then gof : (X, τ, E)⟶ (Z, σ , E) is soft contra rg β -continuous.

6) If f: (X, τ, E) ⟶ (Y, τ′, E) is soft contra rg β - irresolute and g: (Y, τ′, E) ⟶ (Z, σ , E) is soft rg β - irresolute then gof: (X, τ, E)⟶ (Z, σ , E) is soft contra rg β - irresolute.
7) If f : (X, τ, E) ⟶ (Y, τ′, E) is soft contra rg β -irresolute and g: (Y, τ′, E) ⟶(Z, σ , E) is soft contra rg β - irresolute then gof : (X, τ, E)⟶ (Z, σ , E) is soft rg β - irresolute
8) If f: (X, τ, E)⟶ (Y, τ′, E) is soft rg β -irresolute and g: (Y, τ′, E) ⟶(Z, σ , E) is soft rg β - continuous then gof: (X, τ, E)⟶ (Z, σ , E) is soft rg β - continuous.
9) If f: (X, τ, E) ⟶(Y, τ′, E) is soft rg β -irresolute and g: (Y, τ′, E) ⟶ (Z, σ , E) is soft contra rg β -irresolute then gof: (X, τ, E)⟶ (Z, σ , E) is soft contra rg β -irresolute.

Proof:

1) Let (B,E) be a soft closed set in (Z, σ , E), As g is soft continuous then g^{-1}(B, E) is soft closed in (Y, τ′, E) .As f is soft rg β - irresolute then $f^{-1}(g^{-1}$(B,E)) is soft rg β -closed in (X, τ, E).

Thus gof is soft contra rg β -continuous.

2) Let (B,E) be a soft open set in (Z, σ , E), As g is soft contra rg β - continuous then g^{-1}(B,E) is soft rg β -closed in (Y, τ′, E) .As f is soft contra rg β - irresolute then f^{-1} (g^{-1}(B, E)) is soft rg β -open in (X, τ, E).

But f^{-1} (g^{-1}(B,E)) = $(gof)^{-1}$(B,E) soft rg β -open in (X, τ, E).

Thus gof is soft rg β -continuous.

The remaining proofs are straight forward.

5. CONCLUSION

In this paper soft regular generalized β closed sets, soft contra regular generalized β continuous mappings, and soft contra regular generalized β irresolute mappings were studied and their relationship with the already existing sets in soft topological spaces were discussed. The Scope for further research can be focused on the application of soft topological spaces.

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