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SELF-GRAVITATING INSTABILITY OF ROTATING VARIABLE STREAMS

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ABSTRACT

The instability of a rotating self-gravitating non-viscous infinite medium with variable streams has been developed for general wave propagation. Jeans instability restriction is influenced by the streaming whether it is non-uniform or not. This is because the streaming has originally strong destabilizing influence. As the rotating force is being with two dimensional angular velocity, the model is gravitational unstable under Jeans gravitational instability condition. We predict that the rotating forces increase the pure self-gravitational instability states. However, the latter could be completely suppressed as the rotation is high and perpendicular to the direction of the perturbed wave propagations.

Keywords: *Instability, rotation, Non-viscous fluid.*

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1. INTRODUCTION:

The self-gravitation instability is studies for first time by jeans (1902). Such studies are considered as an extension of the work which is carried out by Darwin (1888). The modifications of Jeans criterion were made by several researchers, e.g. Chandrasekhar (1981) and Fermi (1953), Fricke (1954), Chandrasekhar (1981) and Radwan (1988) et.al., upon considering effects of different parameters on the self- gravitating force. Recently the self-gravitating instability of variable streams has been developed by Senger (1981).

In the present work, the influence of rotating forces separately or simultaneously together with the streaming effects on aself-gravitating homogenous medium of Jeans has been developed.

2. BASIC EQUATION:

We consider an infinite homogenous self-gravitational fluid medium. The fluid is assumed to rotating with a general angular velocity.

$$\vec{\mathbf{\Omega}} = \left(\Omega_x\,,\,\Omega_y\,,\Omega_z\right) \tag{1}$$

Where the components $\vec{\mathbf{\Omega}}$ of considered along the utilizing Cartesian coordinates $(x,\ y,\ z)$. The fluid is also assumed to be streaming in the initial state with the variable velocity

$$\vec{\mathbf{u}}_{\mathbf{0}} = \left(u_o(z),\ 0,0\right) \tag{2}$$

Here the speed $u_o(z)$ is in the- direction but varying along the –direction of the coordinates. The model is acted upon by the pressure gradient, self-gravitating and rotating forces. Under the present circumstances the basic equations are given as follows:

THE VECTOR EQUATION OF MOTION:

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}\right) = -\nabla\, p + \rho\nabla V + 2\rho(\vec{u}\wedge\vec{\Omega}) - \frac{1}{2}\rho(\vec{\Omega}\wedge\vec{r})^2 \tag{3}$$

shows the acting forces on the fluid in th right-hand side.

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The equation of continuity:

$$\frac{\partial \rho}{\partial t}+\left(\vec{u}.\nabla\right)\rho=-\rho(\nabla.\vec{u}) \tag{4}$$

The self-gravitating Poissons equation:

$$\nabla^2 V=-4\pi G\rho \tag{5}$$

The polytropic equation of state:

$$p=K\rho^{\Gamma} \tag{6}$$

Relates the pressure p and density ρ of the fluid, where K and Γ are constants.

Here $\rho, \vec{u}$ and p are the fluid mass density, velocity vector and kinetic pressure, V and G are the gravitational potential and gravitational constant respectively.

For a small departure from the unperturbed initial state, every variable quantity $Q(x, y, z)$ may be expressed a

$$Q=Q_o+Q_1+\cdots, \qquad \left|Q_1\right|<<Q_o \tag{7}$$

By the use the expansion for the basic equations the relevant perturbation equations are being:

$$\rho_o\left(\frac{\partial \vec{u}_1}{\partial t}+(\vec{u}_o.\nabla)\vec{u}_1+(\vec{u}_1.\nabla)\vec{u}_o\right)=-\nabla p_1+\rho_o\nabla V_1+2\rho_o(\vec{u}\wedge\vec{\Omega}) \tag{8}$$

$$\frac{\partial \rho_1}{\partial t}+\left(\vec{u}_o.\nabla\right)\rho_1=-\rho_o\left(\nabla.u_1\right) \tag{9}$$

$$\nabla^2 V_1=-4\pi G\rho_1 \tag{10}$$

$$\frac{\partial p_1}{\partial t}+\left(\vec{u}_o.\nabla\right)p_1=c^2\left(\frac{\partial \rho_1}{\partial t}+\left(\vec{u}_o.\nabla\right)\rho_1\right) \tag{11}$$

From the view point of equations (1) and (2) and expressing the velocity $\vec{u}_1$ in terms of its components

$$\vec{u}_1=(u_{1x}, u_{1y} u_{1z}) \tag{12}$$

The linearized system (8)- (11) could be expressed a

$$\rho_o\frac{\partial u_{1x}}{\partial t}+u_o\frac{\partial u_{1x}}{\partial x}{}_1+u_{1z}\frac{\partial u_o}{\partial z}=-c^2\frac{\partial p_1}{\partial x} +\rho_o\frac{\partial V_1}{\partial x}+2\rho_o(\Omega_z u_{1y}-\Omega_y u_{1z}) \tag{13}$$

$$\rho_o\frac{\partial u_{1y}}{\partial t}+u_o\frac{\partial u_{1y}}{\partial x}=-c^2\frac{\partial \rho_1}{\partial y}+\rho_o\frac{\partial V_1}{\partial x}+2\rho_o(\Omega_x u_{1z}-\Omega_z u_{1x}) \tag{14}$$

$$\rho_o \frac{\partial u_{1z}}{\partial t} + u_o \frac{\partial u_{1z}}{\partial x} = -c^2 \frac{\partial \rho_1}{\partial z} + \rho_o \frac{\partial V_1}{\partial z} + 2\rho_o (\Omega_y u_{1x} - \Omega_x u_{1y}) \tag{15}$$

$$\frac{\partial \rho_1}{\partial t} + u_o \frac{\partial \rho_1}{\partial x} = -\rho_o \left(\frac{\partial u_{1x}}{\partial x} + \frac{\partial u_{1y}}{\partial y} + \frac{\partial u_{1z}}{\partial z} \right) \tag{16}$$

$$\frac{\partial^2 V_1}{\partial x^2} + \frac{\partial^2 V_1}{\partial y^2} + \frac{\partial^2 V_1}{\partial z^2} = -4\pi G \rho_1 \tag{17}$$

Where use has been made of equation (11) for equation (13)-(15).

3. EIGENVALUE RELATION:

We consider a sinusoidal propagation wave in the fluid medium. Then, based on the linearized theory of stability and in considering general propagation, time dependence of the perturbed variables could be

$$Q_1 \approx \exp[i(\sigma t + k_x x + k_y y + k_z z)] \tag{18}$$

Here σ is the oscillation frequency, ($i = \sqrt{-1}$) while $k_x, k_{y,} k_z$ are the wave numbers along x, y and z directions respectively. Consequently the linearized equation (13)-(17) degenerate to:

$$i\rho_o n u_{1x} - \rho_o \Omega_z + (2\rho_o \Omega_y + \rho_o D u_o) u_{1z} + \quad + i c^2 k_x - i\rho_o k_x V_1 = o \tag{19}$$

$$2\rho_o \Omega_z u_{1x} - i\rho_o n u_{1y} - 2\rho_o \Omega_x u_{1z} + i k_y c^2 \rho_1 - i\rho_o k_y V_1 = 0 \tag{20}$$

$$-\rho_o \Omega_y u_{1x} + 2\rho_o \Omega_x u_{1y} + i\rho_o n u_{1z} + i k_z c^2 \rho_1 - i\rho_o k_z V_1 = 0 \tag{21}$$

$$i\rho_o k_x u_{1x} + i\rho_o k_y u_{1y} + i\rho_o k_z u_{1z} - i n \rho_1 = 0 \tag{22}$$

$$4\pi G \rho_1 - k^2 V_1 = 0 \tag{23}$$

$$4\pi G \rho_1 - k^2 V_1 = 0$$

Where

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} \tag{24}$$

$$n = \sigma + k_x u_o \tag{25}$$

The algebraic equations (19) – (23) could be expressed in the matrix form

$$A_{ij} x_j = 0 \tag{26}$$

Where the elements a_{ij} ($i = 1,2,3,4,5$ and $j = 1,2,3,4,5$) of the matrix A_{ij} are given in an appendix while x_j is a column whose elements, respectively, are $u_{1x}, u_{1y}, u_{1z}, \rho$ and V_1.

For non-trivial solutions of equation (19) -(23), we must have -

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$$|a_{ij}| = 0$$

This gives the required eigenvalue relation of fifth order equation in n as:

$$4\pi G\begin{vmatrix} i\rho n & -\rho_o\Omega_z & (2\rho_y+\rho_o Du_o) & -i\rho_o k_x \\ \rho_o\Omega_z & i\rho n & \rho_o\Omega_z & -i\rho_o k_y \\ -2\rho_o\Omega_y & 2\rho_o\Omega_x & i\rho n & -i\rho_o k_z \\ i\rho_o k_x & i\rho_o k_y & i\rho_o k_z & 0 \end{vmatrix}$$
$$+k^2\begin{vmatrix} i\rho n & -2\rho_o\Omega_z & (2\rho_o\Omega_x+\rho_o Du_o) & ic^2k_x \\ 2\rho_o\Omega_z & i\rho_o n & -2\rho_o\Omega_x & ic^2k_y \\ 2\rho_o\Omega_y & 2\rho_o\Omega_x & i\rho_o n & ic^2k_y \\ i\rho_o k_x & i\rho_o k_y & i\rho_o k_z & in \end{vmatrix} \tag{27}$$
$$=0$$

In non-rotating frame i.e. $\vec{\Omega}=0$, the general eigenvalue relation (27) degenerates to

$$k^2 N^3 + k^2(k^2c^2 - 4\pi G\rho_o)N - k_x k_z(k^2c^2 - 4\pi G\rho_o)Du_o = 0 \tag{28}$$

Where

$$N = \omega + i k_x u_o \tag{29}$$

With

$$\omega = i\rho_o \tag{30}$$

is the growth rate while is the wave oscillation frequency. The relation (28) coincides with that derived by Sengar (1981). It is found the relation (28) gives at least one positive real root. So that N is real positive showing that the medium is unstable. Therefore, the streaming has strong stabilizing influence.

As a result of this, the streaming increases the self-gravitating unstable domains and simultaneously decreases those of stability.

One has mention here that if the fluid is at rest or with uniform streaming in the initial unperturbed state, then equation (28) reduces to of Jeans

$$N^2 = 4\pi G\rho_o - k^2c^2 \qquad as\ u_o\ uniform$$
$$\omega^2 = 4\pi G\rho_o - k^2c^2 \qquad as\ u_o\ uniform \tag{31}$$

The model is unstable if

$$\omega^2 > 0 \tag{32}$$

i.e.

$$k_j < \sqrt{4\pi G\rho_o}\ /c \qquad , \qquad k^* < \sqrt{4\pi G\rho_o + k^2u_o^2}\ /c \tag{33}$$

where is the Jeans critical wave number while is the critical wave number as the fluid streams with

we conclude that Jeans criterion for self-gravitating instability of homogeneous medium is influenced by the fluid steaming whether the stearms are uniform or non-uniform.

Let us consider the fluid is rotating with two dimensional angular velocity $\vec{\Omega}$ with $\Omega_x = 0$ and wave propagation in the z-direction only i.e. $k_x = 0$, $k_y = 0$ .Then the general eigenvalue relation in such a case is given by

$$\sigma^4 - A\sigma^2 + B = 0 \tag{34}$$

Where

$$A = c^2 k^2 - 4\pi G + 4\Omega^2 \tag{35}$$

$$B = 4\Omega_z^2 (c^2 k^2 - 4\pi G \rho_o) \tag{36}$$

$$\Omega = \sqrt{\Omega^2 + \Omega^2} \tag{37}$$

Equation (34) is a quadratic equation in σ^2, therefore in generally there will be two modes in which the proposed wave equation (18) may be propagated. These modes are given such that

$$\sigma_1^2 + \sigma_2^2 = c^2 k^2 - 4\pi G \rho_o + 4\Omega^2 \tag{38}$$

$$\sigma_1^2 \ \sigma_2^2 = 4\Omega_z^2 (c^2 k^2 - 4\pi G \rho_o) \tag{39}$$

Where σ_1^2 and σ_2^2 are the square of the oscillation frequencies σ_1 and σ_2 of two modes. By an appeal to the properties of the quadratic equation (34), on utilizing the theory of non-linear equations, we see that each of the two roots σ_1^2 and σ_2^2 must be real.

Assume that the Jeans instability condition

$$c^2 k^2 - 4\pi G \rho_o < 0 \tag{40}$$

Is valid here for the present rotating model. \then on using the condition (39) for equation (38) we find

$$\sigma_1^2 < 0 \qquad \text{or} \qquad \sigma_2^2 < 0 \tag{41}$$

Consequently, one of the two modes of the propagating wave must be unstable. This means that the rotation has a destabilizing influence as Jeans condition (39) is satisfied.

we conclude that Jeans self-gravitating instability condition is not influenced by the rotating forces as the rotation velocity has two components. However, if the propagation is in the z-direction and , the stability equation (33) yields

$$\sigma^2 = 4\Omega^2 - 4\pi G \rho_o + c^2 k^2 \tag{42}$$

If the rotation is so strong such that

$$\Omega > \sqrt{\pi G \rho_o} \tag{43}$$

Then the model will be never unstable.

This shows that the self-gravitating instability of the fluid medium will be completely suppressed if the propagation of the perturbed wave is perpendicular to the direction of $\vec{\Omega}$.

APPENDIX:

The elements a_{ij} (I = 1, 2, …,5 and j = 1, 2, …,5) of the matrix equation (26) are given by

$a_{11} = i\rho_o n,\ a_{12} = -2\rho_o \Omega_z,\ a_{13} = 2\rho_o \Omega_y + \rho_o D u_o\ \ a_{14} = i c^2 k_x,\ a_{15} = -i\rho_o k_x$,

$a_{21} = 2\rho_o \Omega_z,\ a_{22} = i\rho_o n,\ a_{23} = -2\rho_o \Omega_x,\ a_{24} = i c^2 k_y,\ a_{25} = -i\rho_o k_y$

$a_{31} = -2\rho_o \Omega_y,\ a_{32} = 2\rho_o \Omega_x,\ a_{33} = i\rho_o n\ \ a_{34} = i c^2 k_x,\ a_{35} = -i\rho_o k_z$,

$a_{41} = i\rho_o k_x,\ a_{42} = i_o \rho_o k_y,\ a_{43} = i\rho_o k_z,\ a_{44} = i n\ ,\ a_{45} = 0$,

$a_{51} = 0,\ a_{52} = 0_z,\ a_{53} = 0,\ a_{54} = 4\pi G,\ a_{55} = -k^2$

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