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ALGEBRAIC RELATIONS CONNECTING 3- STRUCTURE METRIC MANIFOLDS

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ABSTRACT

The aim of this paper is to connect 3- structure metric, 3-structure almost Sasakian or 3-structure contact Riemannian, K- contact 3- structure metric, Sasakian 3- structure metric,3- structure co-symplectic and 3 – structure nearly co-symplectic manifolds by an algebraic relations.

Index Terms*- 3- structure metric, Sasakian, Co-Symplectic, Contact Riemannian, K- contact.*

1. INTRODUCTION

Let us consider an n-dimensional manifold V_n with three vector fields $\underset{x}{U}$, three 1-forms $\overset{x}{u}$ and three tensor fields $\underset{x}{F}$ of the type (1,1), such that

$$\varepsilon_{xyz}\underset{z}{F} = \underset{x}{F}\,\underset{x}{F} - \overset{y}{u}\otimes\underset{x}{U} + \underset{xy}{\delta} I_n, \tag{1.1)a}$$

$$\underset{x}{F}\,\underset{y}{U} = \varepsilon_{xyz}\underset{z}{U}, \tag{1.1)b}$$

$$\overset{x}{u}\circ\underset{y}{F} = \varepsilon_{xyz}\overset{z}{u}, \tag{1.1)c}$$

$$\overset{x}{u}\left(\underset{y}{U}\right) = \overset{x}{\underset{y}{\delta}}. \tag{1.1)d}$$

Where ε_{xyz} = 1 or – 1 according as xyz is an even or odd permutation of 123 and 0 otherwise. Then $\left\{\underset{x}{F},\underset{x}{U},\overset{x}{u}\right\}$, where x = 1,2,3 are said to define an almost contact 3 – structure on V_n or almost co- quaternian Riemannian structure on V_n and the manifold is called an almost contact 3 – structure manifold.

Let a metric tensor g be defined on an almost contact 3 – structure V_n, satisfying

$$g\left(\underset{x}{F}X,\underset{x}{F}Y\right) = \varepsilon_{xyz}\,g\left(\underset{z}{F}X,Y\right) - \overset{x}{u}(X)\overset{y}{u}(Y) + \underset{xy}{\delta}\,g(X,Y), \tag{1.2)a}$$

Where

$$\overset{x}{u}(X) = g\left(X,\underset{x}{U}\right). \tag{1.2)b}$$

Then the system $\left\{\underset{x}{F},\underset{x}{U},\overset{x}{u},g\right\}$ is said to give to V_n a metric 3- structure and the manifold V_n is called 3- structure metric manifold.

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If on the three structure metric manifold

$$2\,'\underset{x}{F}\left(X,Y\right) = \mathrm{d}\overset{x}{u}\left(X,Y\right) \tag*{(1.3)}$$

Then V_n is called 3- structure almost Sasakian manifold or 3- structure contact Riemannian manifold.

On this manifold, we have

$$\mathrm{d}\,'\underset{x}{F} = 0. \tag*{(1.4)}$$

If on a 3- structure almost Sasakian manifold, $\underset{x}{U}$ are a set of mutually orthogonal unit killing vectors:

$$g\left(\underset{x}{U},\underset{y}{U}\right) = \underset{xy}{\delta}, \tag*{(1.5)a}$$

$$\left(D_X\,\overset{x}{u}\right)X = 0. \tag*{(1.5)b}$$

Satisfying

$$\left[\underset{x}{U},\underset{y}{U}\right] = 2\varepsilon_{xyz}\,\underset{z}{U}, \tag*{(1.6)a}$$

$$4\,\underset{z}{U} = \varepsilon_{xyz}\left[\underset{x}{U},\underset{y}{U}\right]. \tag*{(1.6)b}$$

Then V_n is called a K- contact 3- structure metric manifold.

On such a manifold

$$\left(D_X\,\underset{x}{U}\right) = \underset{x}{F}\,X, \tag*{(1.7)a}$$

$$\left(D_X\,\underset{x}{F}\right)Y = K\left(X,\underset{x}{U},Y\right), \tag*{(1.7)b}$$

Where K is Riemannian-Christoffel curvature tensor.

If on a K- contact 3- structure metric manifold

$$\left[\underset{x}{F},\underset{x}{F}\right] + d\,\overset{x}{u}\otimes\underset{x}{U} = 0. \text{ x not summed,} \tag*{(1.8)}$$

Then V_n is called Sasakian 3 – structure metric manifold.

A 3 – structure metric manifold V_n is called a 3- structure co-symplectic manifold if the following relations hold:

$$\left(D_X\,\underset{x}{F}\right)Y = \underset{x}{\overset{y}{A}}\left(X\right)\underset{y}{F}\,Y, \tag*{(1.9)a}$$

$$\underset{x}{\overset{y}{A}} + \underset{y}{\overset{x}{A}} = 0. \tag*{(1.9)b}$$

In consequence of equations (1.9), we have

$$\left(D_X\,\overset{x}{u}\right)\left(Y\right) = -\underset{y}{\overset{x}{A}}\left(X\right)\overset{y}{u}\left(Y\right), \tag*{(1.10)a}$$

The equation (1.10)a may be replaced by

$$\left(D_X\,\underset{x}{U}\right) = \underset{x}{\overset{y}{A}}\left(X\right)\underset{y}{U}. \tag*{(1.10)b}$$

A 3 – structure metric manifold V_n is called a 3 – structure nearly co-symplectic manifold, if

$$\left(D_X \underset{x}{F}\right) X = \overset{y}{\underset{x}{A}}(X) \underset{y}{F} X, \tag{1.11)a}$$

$$\overset{y}{\underset{x}{A}} + \overset{x}{\underset{y}{A}} = 0. \tag{1.11)b}$$

The equation (1.11)a implies

$$\left(D_X \overset{x}{u}\right)(Y) - \left(D_{\underset{x}{F}Y} \overset{x}{u}\right)\left(\underset{x}{F} X\right) = -\overset{x}{\underset{l}{A}}(X)\overset{l}{u}(Y) + \varepsilon_{xqp} \overset{q}{\underset{x}{A}}\left(\underset{x}{F} Y\right)\overset{p}{u}(X), \text{ x not summed.} \tag{1.12}$$

ALGEBRAIC RELATIONS BETWEEN 3- STRUCTURE METRIC MANIFOLDS

Theorem 2.1: If we put

$$'\underset{x}{F}(X,Y) = g\left(\underset{x}{F} X, Y\right) = -'\underset{x}{F}(Y,X). \tag{2.1}$$

Then

$$'\underset{x}{F}\left(\underset{x}{F} X, \underset{x}{F} Y\right) = '\underset{x}{F}(X,Y), \tag{2.2)a}$$

i.e. $'\underset{x}{F}$ is hybrid in X and Y.

$$'\underset{x}{F}\left(\underset{x}{F} X, Y\right) = -'\underset{x}{F}\left(X, \underset{x}{F} Y\right). \tag{2.2)b}$$

Proof: Applying $\underset{x}{F}$ on X and Y in equation (2.1) and using equations (1.1) a, (1.1) c and (1.2) a, we get the equation (2.2)a. Applying $\underset{x}{F}$ on X in equation (2.1) and using equation (1.1) a then comparing the resulting equation with the equation obtained by applying $\underset{x}{F}$ on Y in equation (2.1) with the use of equation (1.2)a, we get the equation (2.2)b.

Theorem 2.2: For a 3 – structure almost Sasakian manifold, we have

$$g\left(X, D_Y \underset{x}{U}\right) = g\left(Y, D_X \underset{x}{U} - 2\underset{x}{F} X\right), \tag{2.3)a}$$

$$g\left(\underset{y}{F} Y, D_X \underset{x}{U}\right) - 2\varepsilon_{xyz} g\left(\underset{z}{F} X, Y\right) = g\left(X, D_{\underset{y}{F}Y} \underset{x}{U} - 2\overset{y}{u}(Y)\underset{x}{U} + 2\underset{xy}{\delta} Y\right), \tag{2.3)b}$$

$$\left(D_X \overset{x}{u}\right)\underset{x}{U} = \left(D_{\underset{x}{U}} \overset{x}{u}\right) X, \tag{2.3)c}$$

$$\left(D_{\underset{x}{F}X} \overset{x}{u}\right) Y + \left(D_X \overset{x}{u}\right)\underset{x}{F} Y = \left(D_Y \overset{x}{u}\right)\underset{x}{F} X + \left(D_{\underset{x}{F}Y} \overset{x}{u}\right) X, \tag{2.3)d}$$

$$\left(D_X \overset{x}{u}\right) Y + \left(D_{\underset{x}{F}Y} \overset{x}{u}\right)\underset{x}{F} X = \left(D_{\underset{x}{F}X} \overset{x}{u}\right)\underset{x}{F} Y + \left(D_Y \overset{x}{u}\right) X, \tag{2.3)e}$$

$$g\left(\underset{y}{U}, D_Y \underset{x}{U}\right) = g\left(Y, D_{\underset{y}{U}} \underset{x}{U} - 2\varepsilon_{xyz} \underset{z}{U}\right). \tag{2.3)f}$$

Proof: From equations (1.2)b and (1.3), we have

$$2\, g\left(\underset{x}{F} X, Y\right) = g\left(Y, D_X \underset{x}{U}\right) - g\left(X, D_Y \underset{x}{U}\right). \tag{2.4}$$

Equation (2.4) gives the equation (2.3)a. Applying $\underset{y}{F}$ on Y in equation (2.4) and using equation (1.2) in the resulting equation, we get the equation (2.3)b. Replacing Y by $\underset{x}{U}$ in equation (2.1), we get

$$'\underset{x}{F}\left(X, \underset{x}{U}\right) = 0. \tag{2.5}$$

Putting Y= $\underset{x}{U}$ in equation (1.3) and using equation (2.5), we get the equation (2.3)c. Applying $\underset{x}{F}$ on X and Y alternatively in equation (1.3) and using equation (2.2)b, we get the equation (2.3)d. Applying $\underset{x}{F}$ on X and Y in equation (1.3) and using equations (2.1) and (2.2)a, we get the equation (2.3)e. Putting X= $\underset{y}{U}$ in equation (2.3)a and using equation (1.1)b, we get the equation (2.3)f.

Theorem 2.3: For a K- contact 3- structure manifold, we have

$$2\underset{x}{F}\underset{y}{U}=\left[\underset{x}{U},\underset{y}{U}\right], \tag{2.6)a}$$

$$g\left(\underset{x}{U},\underset{y}{U}\right)=\varepsilon_{xyz}\underset{z}{F}-\underset{x}{F}\underset{y}{F}+\overset{y}{u}\otimes\underset{x}{U}, \tag{2.6)b}$$

$$\left(D_X\overset{x}{u}\right)\underset{x}{U}=0, \tag{2.6)c}$$

$$\left(D_{\underset{x}{U}}\overset{x}{u}\right)X=0, \tag{2.6)d}$$

$$g\left(\underset{y}{U},D_Y\underset{x}{U}\right)=g\left(Y,D_{\underset{y}{U}}\underset{x}{U}-\left[\underset{x}{U},\underset{y}{U}\right]\right), \tag{2.6)e}$$

$$\left(D_Y\overset{x}{u}\right)\left(\left[\underset{x}{U},\underset{y}{U}\right]\right)=2\varepsilon_{xyz}\left(D_{\underset{z}{U}}\overset{x}{u}\right)Y+2\left(D_{\underset{y}{U}}\overset{x}{u}\right)\underset{x}{F}Y-2\left(D_{\underset{x}{F}Y}\overset{x}{u}\right)\underset{y}{U}, \tag{2.6)f}$$

Proof: From equations (1.6)a and (1.1)b, we get the equation (2.6)a. Using (1.5)a in equation (1.1)a, we get the equation (2.6)b. Using equation (2.3)c in equation (1.5)b, we get the equations (2.6)c and (2.6)d. Using equation (1.6)a in equation(2.3)f, we get the equation (2.6)e. Putting X= $\underset{y}{U}$ in equation (2.3)d and using equations (2.6)a and (1.1)c, we get the equation (2.6)f.

Theorem 2.4: For a Sasakian 3 – structure metric manifold, we have

$$\left[\underset{x}{F},\underset{x}{F}\right]+2\left(D_X\overset{x}{u}\right)(Y)\underset{x}{U}=0, \tag{2.7)a}$$

$$\left[\underset{x}{F},\underset{x}{F}\right]+2\,{}'\underset{x}{F}(X,Y)\underset{x}{U}=0, \tag{2.7)b}$$

$$2\varepsilon_{xyz}\left(\overset{x}{\underset{z}{\delta}}\underset{x}{U}-3\underset{z}{U}\right)+\left(\left(D_{\underset{x}{U}}\overset{x}{u}\right)\underset{y}{U}-\left(D_{\underset{y}{U}}\overset{x}{u}\right)\underset{x}{U}\right)\underset{x}{U}=0, \tag{2.7)c}$$

$$\left[D_X\underset{x}{U},\underset{x}{F}Y\right]-[X,Y]+\overset{x}{u}([X,Y])\underset{x}{U}-\underset{x}{F}\left[D_X\underset{x}{U},Y\right]-\underset{x}{F}\left[X,\underset{x}{F}Y\right]+2\,{}'\underset{x}{F}(X,Y)\underset{x}{U}=0, \tag{2.7)d}$$

$$\underset{x}{F}\left[D_X\underset{x}{U},\underset{x}{U}\right]=\overset{x}{u}\left(\left[X,\underset{x}{U}\right]\right)\underset{x}{U}-\left[X,\underset{x}{U}\right], \tag{2.7)e}$$

$$\overset{y}{u}\left(\left[\underset{x}{F}X,\underset{x}{F}Y\right]\right)-\overset{y}{u}([X,Y])=\varepsilon_{xyz}\left\{\overset{z}{u}\left(\left[\underset{x}{F}X,Y\right]\right)+\overset{z}{u}\left(\left[X,\underset{x}{F}Y\right]\right)\right\}-\overset{y}{\underset{x}{\delta}}\left\{\overset{x}{u}([X,Y])+2\left(D_X\overset{x}{u}\right)Y\right\}, \tag{2.7)f}$$

$$\varepsilon_{xyz}\overset{z}{u}\left(\left[D_X\underset{x}{U},\underset{x}{U}\right]\right)=\overset{y}{\underset{x}{\delta}}\overset{x}{u}\left(\left[X,\underset{x}{U}\right]\right)-\overset{y}{u}\left(\left[X,\underset{x}{U}\right]\right). \tag{2.7)g}$$

Proof: From equations (1.3) and (1.5)b, we have

$${}'\underset{x}{F}(X,Y)=\left(D_X\overset{x}{u}\right)Y=-\left(D_Y\overset{x}{u}\right)X, \tag{2.8}$$

Using equation (2.8) in equation (1.8), we get the equations (2.7)a and (2.7)b .

From equation (1.8), we have

$$\left[\underset{x}{F}X,\underset{x}{F}Y\right]-[X,Y]+\overset{x}{u}([X,Y])\underset{x}{U}-\underset{x}{F}\left[\underset{x}{F}X,Y\right]-\underset{x}{F}\left[X,\underset{x}{F}Y\right]+\left(\left(D_X\overset{x}{u}\right)Y-\left(D_Y\overset{x}{u}\right)X\right)\underset{x}{U}=0. \tag{2.9}$$

Putting X = $\underset{x}{U}$ and Y = $\underset{y}{U}$ in equation (2.9) and then using equations (1.1)b, (1.1)d and (1.6), we get the equation (2.7)c. using equations (1.7)a and (1.3) in equation (2.9) , we get the equation (2.7)d. Putting Y =$\underset{x}{U}$ in equation (2.7)d and using equations (1.1)b and (2.5), we get the equation (2.7)e. Applying $\overset{y}{u}$ on equation (2.9) and using equations (2.8), (1.1)c and (1.1)d, we get the equation (2.7)f. Applying $\overset{y}{u}$ on equation (2.7)e and and using equations (1.1)c and (1.1)d, we get the equation (2.7)g.

Theorem 2.5: A 3- structure co-symplectic manifold is 3- structure almost Sasakian manifold, if

$$\overset{y}{\underset{x}{A}}(X)\,'\underset{y}{F}(Y,Z)+\overset{y}{\underset{x}{A}}(Y)\,'\underset{y}{F}(Z,X)+\overset{y}{\underset{x}{A}}(Z)\,'\underset{y}{F}(X,Y)=0. \tag{2.10}$$

Proof: From equations (1.9)a, we have

$$\left(D_X\,'\underset{x}{F}\right)(Y,Z)=\overset{y}{\underset{x}{A}}(X)\,'\underset{y}{F}(Y,Z). \tag{2.11}$$

Writing similar equations by cyclic permutations of X, Y and Z in the equation (2.11), adding the resulting equations, we get

$$\left(D_X\,'\underset{x}{F}\right)(Y,Z)+\left(D_Y\,'\underset{x}{F}\right)(Z,X)+\left(D_Z\,'\underset{x}{F}\right)(X,Y)=\overset{y}{\underset{x}{A}}(X)\,'\underset{y}{F}(Y,Z)+\overset{y}{\underset{x}{A}}(Y)\,'\underset{y}{F}(Z,X)+\overset{y}{\underset{x}{A}}(Z)\,'\underset{y}{F}(X,Y). \tag{2.12}$$

Using equation (2.10) in equation (2.12), we get

$$\left(D_X\,'\underset{x}{F}\right)(Y,Z)+\left(D_Y\,'\underset{x}{F}\right)(Z,X)+\left(D_Z\,'\underset{x}{F}\right)(X,Y)=0. \tag{2.13}$$

Differentiating equation (1.3), we get equation (2.13). Hence the statement.

Theorem 2.6: A 3- structure co-symplectic manifold is 3- structure almost Sasakian manifold, if

$$\overset{x}{\underset{y}{A}}(Y)\overset{y}{u}(X)-\overset{x}{\underset{y}{A}}(X)\overset{y}{u}(Y)=2\,'\underset{x}{F}(X,Y). \tag{2.14}$$

Proof: Interchanging X and Y in equation (1.10)a and then subtracting the resulting equation from equation (1.10)a, we get

$$\left(D_X\overset{x}{u}\right)Y-\left(D_Y\overset{x}{u}\right)X=\overset{x}{\underset{y}{A}}(Y)\overset{y}{u}(X)-\overset{x}{\underset{y}{A}}(X)\overset{y}{u}(Y). \tag{2.15}$$

Using equation (2.14) in equation (2.15), we get

$$\left(D_X\overset{x}{u}\right)Y-\left(D_Y\overset{x}{u}\right)X=2\,'\underset{x}{F}(X,Y).$$

Which shows that the manifold is 3- structure almost Sasakian manifold.

Theorem 2.7: A 3- structure co-symplectic manifold is K- contact 3- structure metric manifold, if

$$\overset{x}{\underset{y}{A}}(Y)\overset{y}{u}(X)+\overset{x}{\underset{y}{A}}(X)\overset{y}{u}(Y)=0. \tag{2.16}$$

Proof: From equation (1.9)a, we have

$$g\left(D_X\underset{x}{F}Y-\underset{x}{F}D_XY,Z\right)=\overset{y}{\underset{x}{A}}(X)\,g\left(\underset{y}{F}Y,Z\right). \tag{2.17}$$

Putting $Y=\underset{x}{U}$ and $Z=\underset{y}{U}$ in equation (2.17) and using equations (1.1)b and (1.10)b, we get

$$2\,\overset{y}{\underset{x}{A}}(X)\,\varepsilon_{xyz}\,g\left(\underset{z}{U},\underset{y}{U}\right)=0. \tag{2.18}$$

From equation (1.10)a, we get

$$\left(D_X \overset{x}{u}\right)Y+\left(D_Y \overset{x}{u}\right)X=-\overset{x}{\underset{y}{A}}(X)\overset{y}{u}(Y)-\overset{x}{\underset{y}{A}}(Y)\overset{y}{u}(X). \tag{2.19}$$

Using equation (2.16) in equation (2.19), we get

$$\left(D_X \overset{x}{u}\right)Y+\left(D_Y \overset{x}{u}\right)X=0. \tag{2.20}$$

Equations (2.18) and (2.20) give the statement.

Theorem 2.8: A 3- structure co-symplectic manifold is Sasakian 3 – structure metric manifold, if

$$\overset{x}{\underset{y}{A}}(X)\overset{y}{u}(Y)\underset{x}{U}-\overset{x}{\underset{y}{A}}(Y)\overset{y}{u}(X)\underset{x}{U}=\left[\underset{x}{F},\underset{x}{F}\right]. \tag{2.21}$$

Proof: Multiplying equation (2.15) by $\underset{x}{U}$ and then using equation (2.21) in the resulting equation, we get

$$\left\{\left(D_X \overset{x}{u}\right)Y-\left(D_Y \overset{x}{u}\right)X\right\}\underset{x}{U}+\left[\underset{x}{F},\underset{x}{F}\right]=0. \tag{2.22}$$

Which shows that the manifold is Sasakian 3 – structure metric manifold.

Theorem 2.9: A 3- structure co-symplectic manifold is Sasakian 3 – structure metric manifold, if

$$\overset{y}{\underset{x}{A}}\left(\underset{x}{F}X\right)\underset{y}{F}Y-\overset{y}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\underset{y}{F}X-\overset{y}{\underset{x}{A}}(X)\underset{x}{F}\underset{y}{F}Y+\overset{y}{\underset{x}{A}}(Y)\underset{x}{F}\underset{y}{F}X+\left\{\overset{x}{\underset{y}{A}}(Y)\overset{y}{u}(X)-\overset{x}{\underset{y}{A}}(X)\overset{y}{u}(Y)\right\}\underset{x}{U}=0. \tag{2.23}$$

Proof: From equation (1.9)a, we have

$$D_X\underset{x}{F}Y-\underset{x}{F}D_XY=\overset{y}{\underset{x}{A}}(X)\underset{y}{F}Y. \tag{2.24}$$

Applying $\underset{x}{F}$ on equation (2.24), we get

$$\underset{x}{F}D_X\underset{x}{F}Y-\underset{x}{F}^2D_XY=\overset{y}{\underset{x}{A}}(X)\underset{x}{F}\underset{y}{F}Y, \tag{2.25)a}$$

Similarly, we have

$$D_{\underset{x}{F}X}\underset{x}{F}Y-\underset{x}{F}D_{\underset{x}{F}X}Y=\overset{y}{\underset{x}{A}}\left(\underset{x}{F}X\right)\underset{y}{F}Y, \tag{2.25)b}$$

$$D_{\underset{x}{F}Y}\underset{x}{F}X-\underset{x}{F}D_{\underset{x}{F}Y}X=\overset{y}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\underset{y}{F}X, \tag{2.25)c}$$

$$\underset{x}{F}D_Y\underset{x}{F}X-\underset{x}{F}^2D_YX=\overset{y}{\underset{x}{A}}(Y)\underset{x}{F}\underset{y}{F}X. \tag{2.25)d}$$

From equations (2.25) and (1.10)a, we get

$$\begin{aligned}\left[\underset{x}{F},\underset{x}{F}\right]+\left\{\left(D_X \overset{x}{u}\right)Y-\left(D_Y \overset{x}{u}\right)X\right\}\underset{x}{U}&=\overset{y}{\underset{x}{A}}\left(\underset{x}{F}X\right)\underset{y}{F}Y-\overset{y}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\underset{y}{F}X-\overset{y}{\underset{x}{A}}(X)\underset{x}{F}\underset{y}{F}Y\\&+\overset{y}{\underset{x}{A}}(Y)\underset{x}{F}\underset{y}{F}X+\left\{\overset{x}{\underset{y}{A}}(Y)\overset{y}{u}(X)-\overset{x}{\underset{y}{A}}(X)\overset{y}{u}(Y)\right\}\underset{x}{U}.\end{aligned} \tag{2.26}$$

Using equation (2.23) in equation (2.26), we get

$$\left[\underset{x}{F},\underset{x}{F}\right]+\left\{\left(D_X \overset{x}{u}\right)Y-\left(D_Y \overset{x}{u}\right)X\right\}\underset{x}{U}=0.$$

Hence the statement.

Theorem 2.10: If a 3- structure co-symplectic manifold is 3- structure almost Sasakian manifold, then we have

$$2\left(D_X \,{}'\underset{x}{F}\right)(Y,Z)=\overset{y}{\underset{x}{A}}(Y)\left\{\overset{y}{\underset{z}{A}}(X)\overset{z}{u}(Z)-\overset{y}{\underset{x}{A}}(Z)\overset{x}{u}(X)\right\}+\overset{y}{\underset{x}{A}}(Z)\left\{\overset{y}{\underset{x}{A}}(Y)\overset{x}{u}(X)-\overset{y}{\underset{x}{A}}(X)\overset{x}{u}(Y)\right\}. \tag{2.27}$$

Proof: From equation (1.9)a, we have

$$\left(D_X \,{}'\underset{x}{F}\right)(Y,Z)=\overset{y}{\underset{x}{A}}(X)\,{}'\underset{y}{F}(Y,Z). \tag{2.28}$$

Using equation (2.10) in equation (2.28), we get

$$\left(D_X \,{}'\underset{x}{F}\right)(Y,Z)=-\overset{y}{\underset{x}{A}}(Y)\,{}'\underset{y}{F}(Z,X)-\overset{y}{\underset{x}{A}}(Z)\,{}'\underset{y}{F}(X,Y). \tag{2.29}$$

Now, using equations (1.3) and (1.10)a in equation(2.29), we get the equation (2.27).

Theorem 2.11: A nearly co-symplectic manifold is 3 – structure contact Riemannian manifold, if

$$\begin{aligned}2\,{}'\underset{x}{F}(X,Y)=&\left(D_{\underset{x}{F}Y}\overset{x}{u}\right)\left(\underset{x}{F}X\right)-\left(D_{\underset{x}{F}X}\overset{x}{u}\right)\left(\underset{x}{F}Y\right)-\overset{x}{\underset{l}{A}}(X)\overset{l}{u}(Y)+\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\overset{p}{u}(X)\\&+\overset{x}{\underset{l}{A}}(Y)\overset{l}{u}(X)-\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}X\right)\overset{p}{u}(Y).\end{aligned} \tag{2.30}$$

Proof: Interchanging X and Y in equation (1.12) and then subtracting the resulting equation from equation (1.12), We get

$$\begin{aligned}\left(D_X\overset{x}{u}\right)Y-\left(D_Y\overset{x}{u}\right)X=&\left(D_{\underset{x}{F}Y}\overset{x}{u}\right)\left(\underset{x}{F}X\right)-\left(D_{\underset{x}{F}X}\overset{x}{u}\right)\left(\underset{x}{F}Y\right)-\overset{x}{\underset{l}{A}}(X)\overset{l}{u}(Y)+\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\overset{p}{u}(X)\\&+\overset{x}{\underset{l}{A}}(Y)\overset{l}{u}(X)-\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}X\right)\overset{p}{u}(Y).\end{aligned} \tag{2.31}$$

Now, using equation (2.30) in equation (2.31), we get

$$\left(D_X\overset{x}{u}\right)Y-\left(D_Y\overset{x}{u}\right)X=2\,{}'\underset{x}{F}(X,Y).$$

Hence the statement.

Theorem (2.12: A nearly co-symplectic manifold is K- contact 3- structure metric manifold, if

$$\begin{aligned}\left(D_{\underset{x}{F}Y}\overset{x}{u}\right)\left(\underset{x}{F}X\right)+\left(D_{\underset{x}{F}X}\overset{x}{u}\right)\left(\underset{x}{F}Y\right)=&\overset{x}{\underset{l}{A}}(X)\overset{l}{u}(Y)-\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\overset{p}{u}(X)\\&+\overset{x}{\underset{l}{A}}(Y)\overset{l}{u}(X)-\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}X\right)\overset{p}{u}(Y).\end{aligned} \tag{2.32}$$

Proof: Interchanging X and Y in equation (1.12) and then adding the resulting equation from equation (1.12), we get

$$\begin{aligned}&\left(D_X\overset{x}{u}\right)Y+\left(D_Y\overset{x}{u}\right)X-\left(D_{\underset{x}{F}Y}\overset{x}{u}\right)\left(\underset{x}{F}X\right)-\left(D_{\underset{x}{F}X}\overset{x}{u}\right)\left(\underset{x}{F}Y\right)\\&=-\overset{x}{\underset{l}{A}}(X)\overset{l}{u}(Y)+\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\overset{p}{u}(X)-\overset{x}{\underset{l}{A}}(Y)\overset{l}{u}(X)+\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}X\right)\overset{p}{u}(Y).\end{aligned} \tag{2.33}$$

Using equation (2.32) in equation (2.33), we get

$$\left(D_X\overset{x}{u}\right)Y+\left(D_Y\overset{x}{u}\right)X=0.$$

Which shows that the manifold is K- contact 3- structure metric manifold.

Theorem 2.13: A nearly co-symplectic manifold is Sasakian 3 – structure metric manifold, if

$$\left[\underset{x}{F},\underset{x}{F}\right]=\left\{\left(D_{\underset{x}{F}X}\overset{x}{u}\right)\left(\underset{x}{F}Y\right)-\left(D_{\underset{x}{F}Y}\overset{x}{u}\right)\left(\underset{x}{F}X\right)\right\}\underset{x}{U}+\overset{x}{\underset{l}{A}}(X)\overset{l}{u}(Y)\underset{x}{U}-\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}Y\right)\overset{p}{u}(X)\underset{x}{U}$$
$$-\overset{x}{\underset{l}{A}}(Y)\overset{l}{u}(X)\underset{x}{U}+\varepsilon_{xqp}\overset{q}{\underset{x}{A}}\left(\underset{x}{F}X\right)\overset{p}{u}(Y)\underset{x}{U}. \tag{2.34}$$

Proof: Multiplying equation (2.31) by $\underset{x}{U}$ and then using equation (2.34) in the resulting equation, we get

$$\left\{\left(D_X\overset{x}{u}\right)Y-\left(D_Y\overset{x}{u}\right)X\right\}\underset{x}{U}=-\left[\underset{x}{F},\underset{x}{F}\right]. \qquad (2.35)\text{a}$$

or

$$\left[\underset{x}{F},\underset{x}{F}\right]+d\overset{x}{u}\otimes\underset{x}{U}=0. \qquad (2.35)\text{b}$$

The equation (2.35)b shows that the manifold is Sasakian 3 – structure metric manifold.

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