International Journal of Mathematical Archive-6(6), 2015, 52-62
Available online through www.ijma.info ISSN 2229 – 5046

THERMAL RADIATION EFFECT ON OSCILLATING PLATE TEMPERATURE FLOW OF MICROPOLAR VISCOUS DISSIPATIVE FLUID PAST A VERTICAL POROUS PLATE IN PRESENCE OF COUPLE STRESSES

G. SUDHA*, J. SUCHARITHA

***Department of Mathematics, Mahatma Gandhi Institute of Technology (M. G. I. T), Gandipet, Hyderabad, 500075, Telangana State, India.**

Department of Mathematics, University College of Science, Osmania University, Hyderabad, 500007, Telangana State, India.

(Received On: 28-05-15; Revised & Accepted On: 17-06-15)

ABSTRACT

The present work is devoted to investigate the effect of thermal radiation on unsteady two – dimensional oscillatory flow of a polar electrically conducting viscous incompressible Bossinesq fluid past an infinite vertical plate whose temperature varied periodically about a mean constant non – zero value with time in presence of couple stresses and viscous dissipation. The Rosseland approximation is used to describe radiative heat transfer in the limit of optically thick fluids. The governing equations of this class of polar fluids are known to exhibit a boundary layer phenomenon. The dimensionless governing equations for this investigation are reduced to a system of coupled partial differential equations using an efficient finite difference method, and equations are solved numerically. The influence of various flow parameters of the flow field has been discussed and explained graphically. Further, the results obtained under the limiting conditions were found to be in good agreement with the existing one.

Keywords: *Thermal radiation, Micropolar and Couple stress fluid, Viscous dissipation, Finite difference method.*

NOMENCLATURE:

(x', y')	Distances along and perpendicular to the plate respectively
t'	Dimensional time
g	Acceleration due to gravity
u'	Dimensional velocity
T'	Dimensional temperature
T'_w	Temperature at the wall
T'_∞	Temperature far away from the plate
B_o	Strength of a magnetic field
j'	Dimensional micro – inertia
q_r	The radiative heat flux
n'	Dimensional frequency of Oscillation
k^*	The mean absorption coefficient
(u, v)	Components of velocities along and perpendicular to the plate respectively
y	Depth of the fluid
t	Dimensionless time
U_∞	Non – dimensional free stream velocity
u'_p	Dimensional plate velocity
K'	Dimensional Permeability of the porous medium
K	Non – dimensional Permeability of the porous medium
U_p	Non – dimensional plate velocity
j	Non dimensional micro – inertia
I	Scalar constant
$\Pr$	Prandtl number
M	Hartmann number
Ec	Eckert number
C_p	Specific heat at constant pressure
Gr	Thermal Grashof number
R	Thermal radiation parameter
C_f	Skin friction coefficient at the surface of the plate
Nu	Rate of heat transfer (or) Nusselt number

Corresponding Author: **G. Sudha***

Greek Symbols:

Ω Angular velocity component
σ Electrical conductivity
ξ Thermal diffusivity
ρ Fluid Density
ν Fluid kinematic viscosity
ν_r Fluid kinematic rotational viscosity
β^* Coefficient of volumetric expansion of the working fluid
α Fluid thermal diffusivity
ω' Dimensional angular velocity vector
κ Thermal conductivity
ε Scalar constant (≤ 1)
$\bar{\sigma}$ Stefan – Boltzmann constant
ω Angular velocity vector
θ Non dimensional temperature
μ Fluid dynamic Viscosity

Superscript:

' Differentiation w.r.t. to y

Subscripts:

w Wall condition
∞ Free stream condition

1. INTRODUCTION

The problem of micropolar fluids through porous media has many applications, such as porous rocks, foams and foamed solids, aerogels, alloys, polymer blends, and micro emulsions. In recent years, many authors have studied unsteady free convection flow of a micropolar fluid with or without a magnetic field through a porous medium. For example, Agarwal and Dhanapal [1] obtained a numerical solution to study the fully developed free convective flow between two parallel walls with suction (or injection) embedded in a micropolar fluid. Chamkha *et al.* [2] analyzed the fully developed free convective flow of a micropolar fluid in vertical parallel plate channel with asymmetric heating by numerically and analytically. The closed form analytic solutions for the flow and heat transfer characteristics of micropolar fluid in a vertical channel are given by Cheng [3]. Prathap Kumar *et al.* [4] studied the problem of fully developed free convective flow in a vertical channel, partially filled with micropolar fluid. Muthuraj and Srinivas [5] investigated fully developed MHD flow of a micropolar and viscous fluid in a vertical porous space using HAM. Damesh *et al.* [6] have studied the combined effect of heat generation or absorption and first order chemical reaction to micropolar fluid flows over a uniform stretched surface. Rahman and Al – Lawatia [6] studied the effect of higher order chemical reaction on micropolar fluid flow on a power law permeable stretched sheet with variable concentration in a porous medium.

At high temperature, thermal radiation can be significantly affect the heat transfer and the temperature distribution of a micropolar fluid in a channel. Heat transfer by simultaneous free convection and thermal radiation in the case of a micropolar fluid has not gained as much attention. This is unfortunate because thermal radiation plays an important role in determining the overall surface heat transfer in situations where convective heat transfer coefficients are small, as is the case in free convection where such situations are common in space technology [8]. Dulal Pal and Babulal Talukdar [9] studied the effect of thermal radiation on an unsteady hydromagnetic convective heat and mass transfer for a viscous fluid past a semi – infinite vertical moving plate embedded in a porous media in the presence of heat absorption and first – order chemical reaction of the species by using Perturbation technique. Yanhai Lin *et al.* [10] examined radiation effects on Marangoni convection flow and heat transfer in pseudo – plastic non – Newtonian nanofluids driven by a temperature gradient. The surface tension is assumed to vary linearly with temperature and the solutions are obtained numerically by the shooting method. The steady laminar natural convection along a vertical isothermal plate with linear or non – linear Rosseland radiation is investigated by Asterios Pantokratoras [11]. Hall effects on natural convection of participating MHD with thermal radiation are investigated numerically by Jing – Kui Zhang *et al.* [12]. An external uniform magnetic field is applied on a square cavity which is filled with participating magnetic fluid. The full filled fluid has characteristics of gray, absorbing, emitting, scattering and electrically conducting. All walls of the cavity are opaque and diffusively reflection. Seth *et al.* [13] investigated the effects of Hall current and rotation on unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible and optically thick radiating fluid past an impulsively moving vertical plate embedded in a fluid saturated porous medium, when temperature of the plate has a temporarily ramped profile, is carried out. Exact solution of the governing equations is obtained in closed form by Laplace transform technique. The effect of thermal radiation on a steady two – dimensional natural convection laminar flow of viscous incompressible optically thick fluid along a vertical flat plate with streamwise sinusoidal surface temperature has been investigated by Mamun Molla *et al.* [14] using the appropriate variables, the basic governing equations are transformed to convenient form and then solved numerically employing two efficient methods, namely, the Implicit Finite Difference method (IFD) together with the Keller box scheme and Straight Forward Finite Difference (SFFD) method. Combined effects of Soret (thermal – diffusion) and Dufour (diffusion – thermo) on mixed convection over a stretching sheet embedded in a saturated porous medium in the presence of thermal radiation and first – order chemical reaction are studied by Dulal Pal and Hiranmoy Mondal [15].

In the above non – Newtonian MHD studies, couple stress fluids have not been considered. Couple stress fluid theory was introduced by Stokes [16] and is among the polar non – Newtonian fluid theory which considers couple stresses in addition to the classical Cauchy stresses in viscous fluid dynamics. It is the simplest generalization of the classical theory of fluids which allows for polar effects such as the presence of couple stresses and body couples. Recently, a number of researchers have investigated couple stress fluid flows owing to the significance of such fluids in chemical engineering applications including polymer–thickened oils, liquid crystals, polymeric suspensions[17] and physiological fluid mechanics [18]. Couple stress fluids are also important in the tribology of thrust bearings [19] and the lubrication of engine rod bearings [20]. Couple stress fluids are much simpler than micropolar fluids [21], since they possess no microstructure at the kinematic level and therefore the kinematics of such fluids is totally described using the velocity field. A transport phenomenon in couple stress fluids has received considerable attention. Umavathi *et al.* [22] studied mathematically the steady laminar fully developed flow and heat transfer in a horizontal channel consisting of a couple – stress fluid sandwiched between two clear viscous fluids, showing that the effect of the couple stress parameter is to promote the motion of the fluid. Srinivasacharyulu and Odelu [23] used the quasilinearization method to investigate numerically the incompressible laminar flow of a couple stress fluids in a porous channel with expanding or contracting walls, assuming symmetric injection or suction along the walls. Patil and Kulkarni [24] used a volume–averaging technique to examine the two–dimensional oscillatory natural convection flow of an incompressible polar fluid through a porous medium bounded by an infinite vertical porous plate with oscillating suction and temperature at the wall, identifying a multiple boundary layer structure near the wall. Zueco and Anwar Beg [25] studied the pulsatile flow of couple stress fluid and Eyring – Powell fluid in a rigid channel with wall transpiration, using the network electrical method. Hydromagnetic flows of couple stress fluids have also stimulated some interest owing to the facility of controlling such flows with transverse magnetic fields. Ramana Murthy *et al.* [26] examined the steady hydromagnetic flow of a conducting, incompressible couple stress fluid in an annular region between two concentric rotating vertical circular cylinders, with porous lining inside the outer cylinder, imposing an external radial magnetic field.

The purpose of the present paper is to report numerical results for the problem of the micropolar fluid behaviour on unsteady two – dimensional oscillatory electrically conducting viscous incompressible Bossinesq fluid flow past an infinite vertical plate whose temperature varied periodically about a mean constant non – zero value with time. Thermal radiation, couple stresses and viscous dissipation have been considered for high speed fluid. The governing equations of the problem contain the non – partial differential equations which are transformed by similarity technique into dimensionless coupled partial differential equations. The obtained dimensionless equations are solved numerically by finite difference technique. In general, this study is very complicated to solve. Therefore, it is necessary to investigate in detail the distributions of velocity, microrotation and temperature across the boundary layer in addition to the surface skin friction. Such a study has important applications in the manufacture of electro – conductive polymers and other electrically conducting non – Newtonian liquids and has thus far not appeared in the literature.

2. MATHEMATICAL FORMULATION

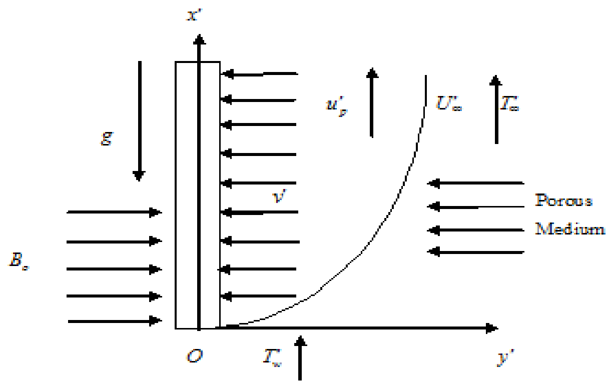

Figure-1. Physical Model and coordinate system of the problem

Consider an unsteady, incompressible, two – dimensional free convective micropolar Bossinesq thermal radiating fluid flow past an infinite vertical plate whose temperature varied periodically about a mean constant non – zero value with time in presence of couple stresses and viscous dissipation. Choose the coordinate system such that $x'-$ axis is along the vertical plate and $y'-$ axis normal to the plate. The physical model and coordinate system are shown in figure 1. The plate is maintained at temperature T'_w. It is assumed that there is no applied voltage which implies the absence of electric field. A uniform magnetic field of magnitude B_o is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. Viscous and Darcy's resistance terms are taken into account with constant permeability of the porous medium. Using the Boussinesq's and boundary layer approximations, the governing equations for the micropolar fluid are given by

Equation of Continuity:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Equation of linear momentum:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = (\nu + \nu_r)\frac{\partial^2 u'}{\partial y'^2} + g\beta^*(T' - T'_\infty) - \nu\left(\frac{u'}{K'}\right) - \left[\frac{\sigma\mu^2}{\rho}B_o^2\right]u' + 2\nu_r\left(\frac{\partial \omega'}{\partial y'}\right) \tag{2}$$

Equation of Angular momentum:

$$\frac{\partial \omega'}{\partial t'} + v'\frac{\partial \omega'}{\partial y'} = \left(\frac{\gamma^*}{I}\right)\frac{\partial^2 \omega'}{\partial y'^2} \tag{3}$$

Equation of Energy:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \zeta\frac{\partial^2 T'}{\partial y'^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \zeta\frac{\partial q_r}{\partial y'} \tag{4}$$

Under these assumptions, the appropriate boundary conditions for the velocity and temperature fields are:

$$\left.\begin{array}{l} t' \le 0: \; u' = 0,\; \omega' = 0,\; T' = T'_\infty \; for\ all\ y' \\ t' > 0: \begin{cases} u' = u'_p,\; \dfrac{\partial \omega'}{\partial y'} = -\dfrac{\partial^2 u'}{\partial y'^2},\; T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{i\Omega' t'} \; at\ y' = 0 \\ u' = U'_\infty = U_o\left(1 + \varepsilon e^{i\Omega' t'}\right),\; \omega' \to 0,\; T' \to T'_\infty \; at\ y' \to \infty \end{cases} \end{array}\right\} \tag{5}$$

We assume the velocity v' depends on time like

$$v' = -V_o\left(1 + \varepsilon A e^{i\Omega' t'}\right) \tag{6}$$

Where A is a real positive constant, ε and εA small less than unity and V_o is a scale of suction velocity which has non – zero positive constant. Further, we invoke the Rosseland approximation [27], for the radiative flux in equation (4) where

$$q_r = -\frac{4\bar{\sigma}}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{7}$$

It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T'_∞ and neglecting higher – order terms. This results in the following approximation:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{8}$$

On the strength of equations (7) and (8), equation (4) becomes

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \zeta\frac{\partial^2 T'}{\partial y'^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{16\bar{\sigma}T'^3_\infty\kappa}{3\rho C_p k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{9}$$

We now introduce the dimensionless variables, as follows:

$$\left.\begin{array}{l} u = \dfrac{u'}{V_o},\; v = \dfrac{v'}{V_o},\; y = \dfrac{V_o^2 y'}{\nu},\; t = \dfrac{t' V_o^2}{\nu},\; U_\infty = \dfrac{U'_\infty}{V_o},\; U_p = \dfrac{u'_p}{V_o},\; \omega = \dfrac{\nu}{V_o^2}\omega',\; \theta = \dfrac{T' - T'_\infty}{T'_w - T'_\infty}, \\ \Omega = \dfrac{\Omega'\nu}{V_o^2},\; \alpha = \dfrac{\nu_r}{\nu},\; \gamma = \dfrac{I\nu}{\gamma^*},\; K = \dfrac{K' V_o^2}{\nu^2},\; \Pr = \dfrac{\nu\rho C_p}{\kappa} = \dfrac{\nu}{\zeta},\; Gr = \dfrac{\nu g\beta^*(T'_w - T'_\infty)}{\rho V_o^2}, \\ M = \dfrac{\sigma B_o^2 \nu}{\rho V_o^2},\; R = \dfrac{16\bar{\sigma}T'^3_\infty}{3k^*(T'_w - T'_\infty)},\; Ec = \dfrac{V_o^2}{C_p(T'_w - T'_\infty)} \end{array}\right\} \tag{10}$$

Furthermore, the spin – gradient viscosity γ which defines the relationship between the coefficients of viscosity and micro – inertia, is given by

$$\gamma = \left(\mu + \frac{A^*}{2} \right) j' = \mu\, j' \left(1 + \frac{\beta}{2} \right) \tag{11}$$

Where β denotes the dimensionless viscosity ratio, defined as follows:

$$\beta = \frac{A^*}{\mu} \tag{12}$$

in which A^* is the coefficient of gyro – viscosity (or vortex viscosity). On account of equation (10), our governing equations (2), (3) and (9) become

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\Omega t}\right) \frac{\partial u}{\partial y} = (1+\beta)\frac{\partial^2 u}{\partial y^2} + (Gr)\theta - Nu + 2\beta\left(\frac{\partial \omega}{\partial y}\right) \tag{13}$$

$$\frac{\partial \omega}{\partial t} - \left(1 + \varepsilon A e^{i\Omega t}\right) \frac{\partial \omega}{\partial y} = \frac{1}{\gamma}\frac{\partial^2 \omega}{\partial y^2} \tag{14}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{i\Omega t}\right) \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}(1+R)\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{15}$$

Where $N = M + \dfrac{1}{K}$. The boundary conditions (5) are then given by the following dimensionless form:

$$\left.\begin{array}{l} t \le 0: \ \{u = 0,\ \omega = 0,\ \theta = 0 \quad for\ all\ y \\ t > 0: \begin{cases} u = U_p,\ \dfrac{\partial \omega}{\partial y} = -\dfrac{\partial^2 u}{\partial y^2},\ \theta = 1 + \varepsilon e^{i\Omega t} \ \ at\ \ y = 0 \\ u \to U_\infty,\ \omega \to 0,\ \theta \to 0\ as\ y \to \infty \end{cases} \end{array}\right\} \tag{16}$$

Where all the variables have been given in the nomenclature. For practical engineering applications and the design of chemical engineering systems, quantities of interest include the following Skin friction coefficient, Nusselt number and Sherwood number are useful to compute. By virtue of equations (13) – (15), we obtain the streamwise velocity, microrotation, temperature and concentration in the boundary layer. We can now calculate the skin friction coefficient at the surface of the porous plate, which is given by

$$C_f = \frac{\tau_w^*}{\rho U_o V_o} = \left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{17}$$

We can also calculate the heat transfer coefficient at the wall of the plate in terms of Nusselt number as follows:

$$Nu = \frac{x'}{\left(T'_w - T'_\infty\right)}\left[\frac{\partial T'}{\partial y'}\right]_w \Rightarrow Nu\,\mathrm{Re}_x^{-1} = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0}$$

(18)

Where $\mathrm{Re}_x^{-1} = \dfrac{V_o x'}{\nu}$ is Reynolds number.

The mathematical formulation of the problem is now completed. Equations (13)–(15) present a coupled non – linear system of partial differential equations and are to be solved by using initial and boundary conditions (16). However, exact solutions are difficult, whenever possible. Hence, these equations are solved by the Crank – Nicholson method.

3. METHOD OF SOLUTION

We shall solve the system of partial differential equations numerically using the finite difference technique and equations (13) – (15) yield.

$$\left(\frac{u_i^{j+1} - u_i^j}{\Delta t}\right) - B\left(\frac{u_{i+1}^j - u_i^j}{\Delta y}\right) = (1+\beta)\left(\frac{u_{i+1}^j - 2u_i^j + u_{i-1}^j}{(\Delta y)^2}\right) + (Gr)\theta_i^j + N\left[u_i^j\right] + 2\beta\left(\frac{\omega_{i+1}^j - \omega_i^j}{\Delta y}\right) \tag{19}$$

$$\left(\frac{\omega_i^{j+1} - \omega_i^j}{\Delta t}\right) - B\left(\frac{\omega_{i+1}^j - \omega_i^j}{\Delta y}\right) = \frac{1}{\gamma}\left(\frac{\omega_{i+1}^j - 2\omega_i^j + \omega_{i-1}^j}{(\Delta y)^2}\right) \tag{20}$$

$$\left(\frac{\theta_i^{j+1}-\theta_i^j}{\Delta t}\right)-B\left(\frac{\theta_{i+1}^j-\theta_i^j}{\Delta y}\right)=\frac{1}{\Pr}(1+R)\left(\frac{\theta_{i+1}^j-2\theta_i^j+\theta_{i-1}^j}{(\Delta y)^2}\right)+(Ec)\left(\frac{u_{i+1}^j-u_i^j}{\Delta y}\right)^2 \qquad (21)$$

Where $B=1+\varepsilon Ae^{i\Omega t}$, the indices i and j refer to y and t respectively.

The initial and boundary conditions (16) yield.

$$\left.\begin{array}{l} t\le 0:\ \{u_i^j=0,\ \omega_i^j=0,\ \theta_i^j=0 \quad for\ all\ y \\ t>0:\begin{cases} u_i^j=U_p,\ \left(\dfrac{\omega_{i+1}^j-\omega_i^j}{\Delta y}\right)=-\left(\dfrac{u_{i+1}^j-2u_i^j+u_{i-1}^j}{(\Delta y)^2}\right),\ \theta_i^j=1+\varepsilon e^{i\Omega t}\ at\ y=0 \\ u_i^j\to U_\infty,\ \omega_i^j\to 0,\ \theta_i^j\to 0\ as\ y\to\infty \end{cases}\end{array}\right\} \qquad (22)$$

The term consistency applied to a finite difference procedure means that the procedure may in fact approximate the solution of the partial differential equation under study and not the solution of any other partial differential equation. The consistency is measured in terms of the difference between a differential equation and a difference equation. Here, we can write

$$\frac{\partial u}{\partial t}=\frac{u_i^{j+1}-u_i^j}{\Delta t}+O(\Delta t),\ \frac{\partial \omega}{\partial t}=\frac{\omega_i^{j+1}-\omega_i^j}{\Delta t}+O(\Delta t),\ \frac{\partial \theta}{\partial t}=\frac{\theta_i^{j+1}-\theta_i^j}{\Delta t}+O(\Delta t)$$

$$\frac{\partial u}{\partial y}=\frac{u_{i+1}^j-u_i^j}{\Delta y}+O(\Delta y),\ \frac{\partial \omega}{\partial y}=\frac{\omega_{i+1}^j-\omega_i^j}{\Delta y}+O(\Delta y),\ \frac{\partial \theta}{\partial y}=\frac{\theta_{i+1}^j-\theta_i^j}{\Delta y}+O(\Delta y)$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{u_{i+1}^j-2u_i^j+u_{i-1}^j}{(\Delta y)^2}+O(\Delta y)^2,\ \frac{\partial^2 \omega}{\partial y^2}=\frac{\omega_{i+1}^j-2\omega_i^j+\omega_{i-1}^j}{(\Delta y)^2}+O(\Delta y)^2 \text{ and}$$

$$\frac{\partial^2 \theta}{\partial y^2}=\frac{\theta_{i+1}^j-2\theta_i^j+\theta_{i-1}^j}{(\Delta y)^2}+O(\Delta y)^2$$

For consistency of equation (19), we estimate

$$\left\{\left(\frac{u_i^{j+1}-u_i^j}{\Delta t}\right)-B\left(\frac{u_{i+1}^j-u_i^j}{\Delta y}\right)-(1+\beta)\left(\frac{u_{i+1}^j-2u_i^j+u_{i-1}^j}{(\Delta y)^2}\right)-(Gr)\theta_i^j+N\left[u_i^j\right]-2\beta\left(\frac{\omega_{i+1}^j-\omega_i^j}{\Delta y}\right)\right\}$$
$$-\left\{\frac{\partial u}{\partial t}-B\frac{\partial u}{\partial y}-(1+\beta)\frac{\partial^2 u}{\partial y^2}-(Gr)\theta+N(u)-2\beta\left(\frac{\partial \omega}{\partial y}\right)\right\}_{i,\ j}=O(\Delta t)+O(\Delta y) \qquad (23)$$

For consistency of equation (20), we estimate

$$\left\{\left(\frac{\omega_i^{j+1}-\omega_i^j}{\Delta t}\right)-B\left(\frac{\omega_{i+1}^j-\omega_i^j}{\Delta y}\right)-\frac{1}{\gamma}\left(\frac{\omega_{i+1}^j-2\omega_i^j+\omega_{i-1}^j}{(\Delta y)^2}\right)\right\}-\left\{\frac{\partial \omega}{\partial t}-B\frac{\partial \omega}{\partial y}-\frac{1}{\gamma}\frac{\partial^2 \omega}{\partial y^2}\right\}_{i,\ j}=O(\Delta t)+O(\Delta y) \qquad (24)$$

Similarly with respect to equation (21), we estimate

$$\left\{\left(\frac{\theta_i^{j+1}-\theta_i^j}{\Delta t}\right)-B\left(\frac{\theta_{i+1}^j-\theta_i^j}{\Delta y}\right)-\frac{1}{\Pr}(1+R)\left(\frac{\theta_{i+1}^j-2\theta_i^j+\theta_{i-1}^j}{(\Delta y)^2}\right)-(Ec)\left(\frac{u_{i+1}^j-u_i^j}{\Delta y}\right)^2\right\}$$
$$-\left\{\frac{\partial \theta}{\partial t}-B\frac{\partial \theta}{\partial y}-\frac{1}{\Pr}(1+R)\frac{\partial^2 \theta}{\partial y^2}-(Ec)\left(\frac{\partial u}{\partial y}\right)^2\right\}_{i,\ j}=O(\Delta t)+O(\Delta y) \qquad (25)$$

Here, the right hand side of equations (23) – (25) represents truncation error as $\Delta t\to 0$ with $\Delta y\to 0$, the truncation error tends to zero. Hence our explicit scheme is consistent. Here Δy, Δt are mesh sizes along y and time direction respectively. The infinity taken as $i=1$ to n and the equations (23), (24) and (25) are solved under the boundary conditions (22), following the tri diagonal system of equations are obtained.

$$A_i X_i = B_i\ (i=1 \text{ to } n) \qquad (26)$$

Where A_i is the tri diagonal matrix of order $n \times n$ and X_i, B_i are the column matrices having n components. The above system of equations has been solved by Thomas Algorithm (Gauss elimination method), for velocity, microrotation and temperature. In order to prove the convergence of the finite difference scheme, the computations are carried out for different values of Δt. But the Crank – Nicholson method is unconditionally stable. By changing the value of Δt there is no change in the study state condition. So, the finite difference scheme is convergent and stable.

4. RESULTS AND DISCUSSIONS

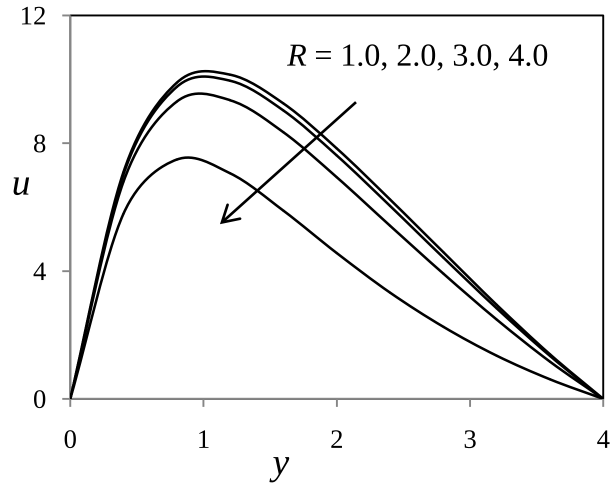

Figure-2. The effect of Thermal radiation parameter on velocity profiles

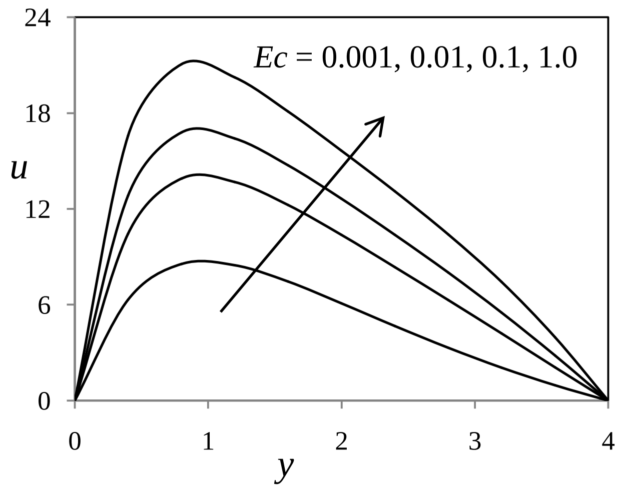

Figure-3. The effect of Viscous dissipation (Eckert number) on velocity profiles

The formulation of the problem that accounts for the effect of thermal radiation on unsteady two–dimensional oscillatory flow of a polar electrically conducting viscous incompressible Bossinesq fluid past an infinite vertical plate whose temperature varied periodically about a mean constant non – zero value with time in presence of couple stresses and viscous dissipation. The Rosseland approximation is used to describe radiative heat transfer in the limit of optically thick fluids. The dimensionless governing equations for this investigation are reduced to a system of coupled partial differential equations using an efficient finite difference method, and equations are solved numerically. In the calculations, the boundary condition for $y \to \infty$ is approximated by $y_{\max} = 4$, which is sufficiently large for the velocity to approach the relevant stream velocity. Figs. (2) – (7) show representative plots of the streamwise velocity and microrotation as well as temperature profiles and surface skin - friction for a micropolar fluid with the fixed flow conditions $\Pr = 0.71$, $K = 2.0$, $\beta = 0.2$, $\varepsilon = 0.1$, $A = 0.5$ and $t = 1.0$, while Gr, M, R and Ec are varied over a range, which are listed in the figured legend. From figure (2) we observe a decrease in the velocity with increase in the radiation parameter. Generally we observe that the velocity increases rapidly starting from the plate, attains a peak value before decreasing almost exponentially away from the plate. From figure (3), we observe the same pattern as the previous figure except here we see that increase in the viscous dissipation heating leads to an increase in the velocity. Figures (4) and (5) depict the effects of free convection and Hartmann number on the angular velocity profile from where we observe a decrease in the angular velocity when either the free convection parameter or the Hartmann number is increased.

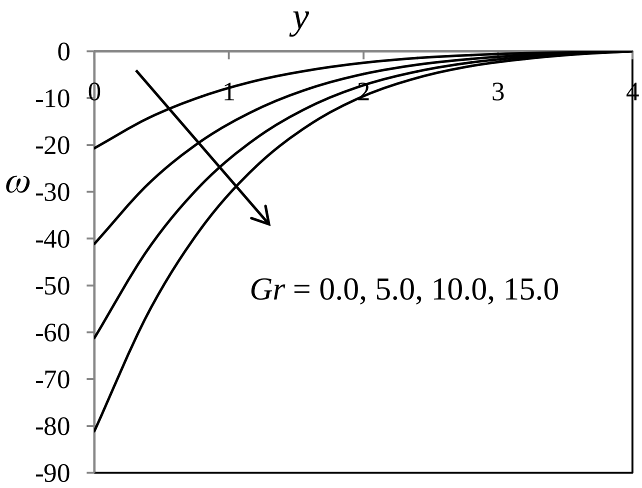

Figure-4. Effect of free convection parameter (Grashof number) on the angular velocity profiles

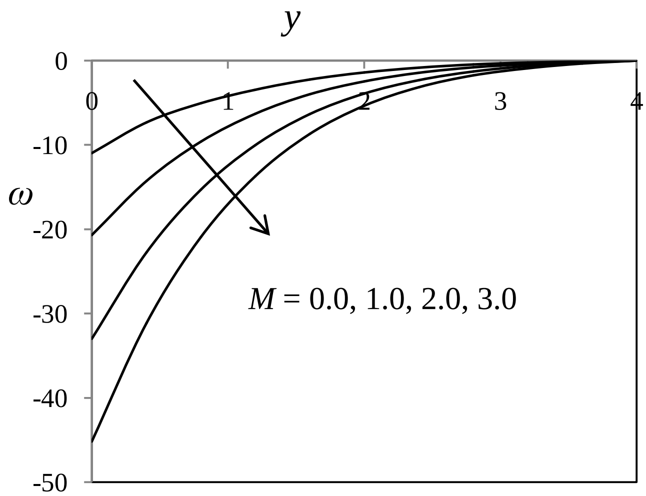

Figure-5. Effect of Hartmann number (Magnetic field) on the angular velocity profiles

Figures (6) and (7), respectively, show the temperature distribution and the skin – friction, where we observe an increase in temperature with increase in radiation and an increase in skin – friction with increase in viscous dissipation heating. These cases are for ($Gr > 0$) which will correspond to cooling of the plate by free convection currents. In the other case of heating of the plate by free convection currents ($Gr < 0$), these observed effects are reversed (graphs not shown). Our observations are in agreement with the findings of Raptis [28] and Cookey *et al.* [29], and they compliment the findings of Ogulu *et al.* [30] where the effect of rotation on the velocity is not discussed, and in Jain and Tanja [31].

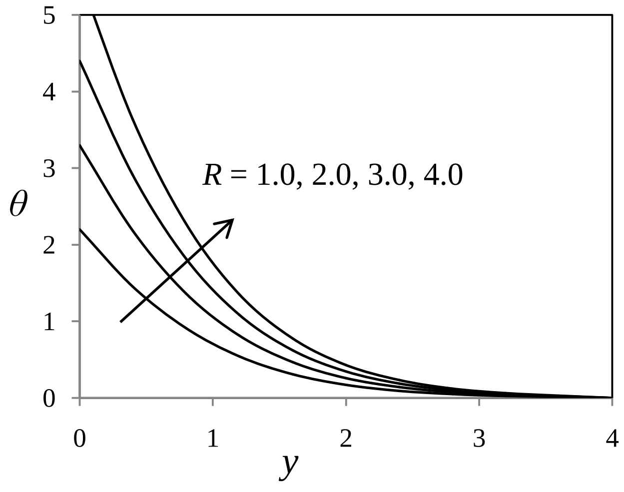

Figure-6. The effect of Thermal radiation parameter on temperature profiles

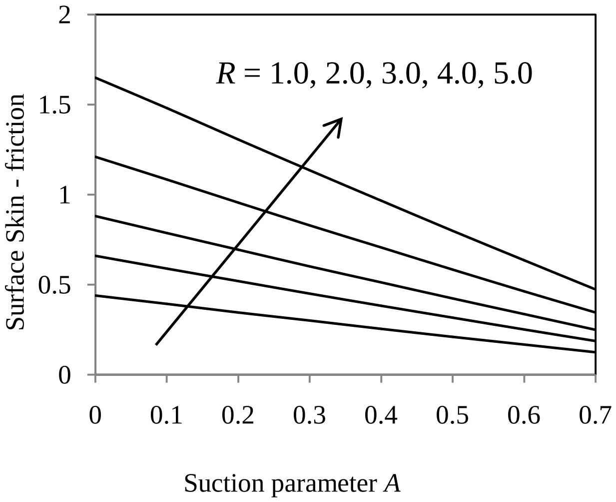

Figure-7. Surface skin friction against suction parameter A for different values of the thermal radiation parameter

Table-1: Skin – friction results (C_f) for the values of Gr, M, Pr, $R, K, Ec, \beta, \varepsilon, A$ and t

Gr	M	Pr	R	K	Ec	β	ε	A	t	C_f
2.0	2.0	0.71	2.0	2.0	0.001	0.2	0.1	0.5	1.0	2.10365987
4.0	2.0	0.71	2.0	2.0	0.001	0.2	0.1	0.5	1.0	2.22481532
2.0	4.0	0.71	2.0	2.0	0.001	0.2	0.1	0.5	1.0	1.99826501
2.0	2.0	7.0	2.0	2.0	0.001	0.2	0.1	0.5	1.0	1.98036547
2.0	2.0	0.71	4.0	2.0	0.001	0.2	0.1	0.5	1.0	1.99065841
2.0	2.0	0.71	2.0	4.0	0.001	0.2	0.1	0.5	1.0	2.19605778
2.0	2.0	0.71	2.0	2.0	0.100	0.2	0.1	0.5	1.0	2.11480695
2.0	2.0	0.71	2.0	2.0	0.001	0.4	0.1	0.5	1.0	2.09158711
2.0	2.0	0.71	2.0	2.0	0.001	0.2	0.2	0.5	1.0	1.99268122
2.0	2.0	0.71	2.0	2.0	0.001	0.2	0.1	1.0	1.0	2.15611478
2.0	2.0	0.71	2.0	2.0	0.001	0.2	0.1	0.5	2.0	2.14302871

Table-2: Rate of heat transfer $\left(Nu\left(\mathrm{Re}_x^{-1}\right)\right)$ values for different values of Pr, R, Ec, ε, A and t

Pr	R	Ec	ε	A	t	$Nu\left(\mathrm{Re}_x^{-1}\right)$
0.71	2.0	0.001	0.1	0.5	1.0	1.55497842
7.00	2.0	0.001	0.1	0.5	1.0	1.45136921
0.71	4.0	0.001	0.1	0.5	1.0	1.60493215
0.71	2.0	0.100	0.1	0.5	1.0	1.57220492
0.71	2.0	0.001	0.2	0.5	1.0	1.47016524
0.71	2.0	0.001	0.1	1.0	1.0	1.61305478
0.71	2.0	0.001	0.1	0.5	2.0	1.59302641

The profiles for skin – friction (C_f) due to velocity under the effects of Gr, M, Pr, $R, K, Ec, \beta, \varepsilon, A$ and t are presented in table – 1. From this table, it is to note that an increase in Gr, K, R, Ec, A and t leads to exert greater skin – friction on the boundary whereas M, Pr, β and ε reduce it. The profiles for Nusselt number $\left(Nu\left(\mathrm{Re}_x^{-1}\right)\right)$ due to temperature under the effects of Pr, R, Ec, ε, A and t are presented in the table – 2. From this table – 2, it is seen that an increase in Pr and ε to decrease in $Nu\left(\mathrm{Re}_x^{-1}\right)$ and an increase in R, Ec, A and t lead to increase in $Nu\left(\mathrm{Re}_x^{-1}\right)$.

5. CONCLUSIONS

In this study numerical solutions are obtained for the problem of the flow of a micropolar fluid past a vertical porous plate in the presence of couple stresses and radiation, where the temperature of the plate is assumed to oscillate about a mean value. We have done the analysis and discussion of the effect of material parameters on the temperature, velocity distributions, the skin – friction at the plate and the angular velocity. It is seen that the skin friction is greatly affected by viscous dissipation heat as well as the radiative heat transfer. Increase in the radiation parameter results in a decrease in the velocity while an increase in the viscous dissipation heat leads to an increase in the velocity. We also conclude that increases in free convection and the magnetic parameters both lead to a decrease in the angular velocity.

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Source of support: Nil, Conflict of interest: None Declared