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ON $I_{s\hat{g}}$ -CONTINUITY IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce and study the notions of $I_{s\hat{g}}$-closed sets, $I_{s\hat{g}}$-continuity, $I_{s\hat{g}}$-irresolute, $I_{s\hat{g}}$-connected, $I_{s\hat{g}}$-normal in ideal topological spaces.

Keywords: *$I_{s\hat{g}}$-closed, $I_{s\hat{g}}$-continuity, $I_{s\hat{g}}$-irresolute, $I_{s\hat{g}}$-connected and $I_{s\hat{g}}$-normal.*

1. INTRODUCTION AND PRELIMINARIES

An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) A ∈I and B ⊆ A implies B ∈ I, (2) A ∈ I and B ∈ I implies A ∪ B ∈ I. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A ⊆ X, A*(I,τ) = {x ∈ X: A ∩ U ∉I for every U∈τ (X, x)} is called the local function of A with respect to I and τ [8]. We simply write A* in case there is no chance for confusion. A kuratowski closure operator cl*(.) for a topology τ*(I, τ) called the *- topology, finer than τ is defined by cl*(A) = A ∪ A* [13]. If A ⊆ X, cl(A) and int(A) will respectively, denote the closure and interior of A in (X, τ).

Definition 1.1: A subset A of a topological space (X, τ) is called

1. g-closed [9], if cl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2. g*-closed[14], if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
3. $\hat{g}$ -closed [15], if cl(A) ⊆ U whenever A ⊆ U and U is semi open in (X, τ).
4. gs-closed [2], if scl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
5. sg-closed [5], if scl (A) ⊆ U whenever A ⊆ U and U is semi open in (X, τ).
6. s$\hat{g}$-closed [11], if scl (A) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open in (X, τ).
   Complements of the above mentioned closed sets are called their respective open sets.

Definition 1.2: A subset A of an ideal topological spaces (X, τ, I) is said to be

1. semi-I-closed [7], if int(cl*(A)) ⊆ A
2. I_{gs}-closed [10], if sIcl(A) ⊆ U whenever A ⊆ U and U is open in X.
3. I_{sg}-closed [10], if sIcl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
   The complements of the above mentioned closed sets are called their respective open sets.

Definition 1.3: A function f: (X, τ, I) → (Y, σ) is said to be

1. g-continuous [3], if for every open set v ∈ σ, f^{-1}(v) is g-open in (X, τ).
2. gs-continuous [5], if for every open set v ∈ σ, f^{-1}(v) is gs-open in (X, τ).
3. sg-continuous [12], if for every open set v ∈ σ, f^{-1}(v) is sg-open in (X, τ).
4. gp-continuous [1], if for every open set v ∈ σ, f^{-1}(v) is gp-open in (X, τ).
5. gsp-continuous [6], if for every open set v ∈ σ, f^{-1}(v) is gsp-open in (X, τ).

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2. $I_{s\hat{g}}$ -CLOSED SETS

Definition 2.1: A subset A of a space (X, τ, I) is called $I_{s\hat{g}}$-closed, if sIcl(A) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open.

Theorem 2.2: Every closed set is $I_{s\hat{g}}$-closed but not conversely.

Proof: Let A be a closed set. Let U be $\hat{g}$-open such that A ⊆ U. Since A is closed, sIcl(A) ⊆ cl(A) = A ⊆ U. Hence A is $I_{s\hat{g}}$-closed.

Example 2.3: Let X = {a, b, c},τ = { Φ ,{a},{a,b},X} and I = {Φ,{a}}. Here {a, c} is $I_{s\hat{g}}$-closed but not closed.

Theorem 2.4: Every $I_{s\hat{g}}$-closed is s$\hat{g}$-closed but not conversely.

Proof: Let A be $I_{s\hat{g}}$-closed set of (X, τ, I). Let U be $\hat{g}$-open such that
U ⊇ sIcl(A) = A ∪ int*(cl(A)) ⊆ A ∪ int(cl(A)) = scl(A). This shows that A is s$\hat{g}$-closed.

Example 2.5: Let X = {a, b, c},τ = {Φ,{a},{b,c},X} and I = {Φ,{a},{b},{a,b}}. Here {b} and {c} are s$\hat{g}$-closed but not $I_{s\hat{g}}$-closed.

Theorem 2.6: Every $I_{s\hat{g}}$-closed is gs-closed but not conversely.

Proof: Let A be $I_{s\hat{g}}$-closed set of (X, τ, I). Let U be any open set such that A ⊆ U. Since every open set is $\hat{g}$-open. scl(A) ⊆ U. Hence A is gs-closed set.

Example 2.7: Let X = {a, b, c}, τ = {Φ,{a},{b,c}, X}} and I = { Φ,{a},{b},{a,b}}. Here {a, b} is gs-closed but not $I_{s\hat{g}}$-closed.

Theorem 2.8: The union of two $I_{s\hat{g}}$-closed set is $I_{s\hat{g}}$-closed set.

Proof: Assume that A and B are $I_{s\hat{g}}$-closed in (X, τ, I). Let U be $\hat{g}$-open such that A ∪ B⊆ U. Then A ⊆ U and B ⊆ U. Since A and B are $I_{s\hat{g}}$-closed, sIcl(A) ⊆ U, sIcl(B) ⊆ U. sIcl(A ∪ B) = sIcl(A) ∪ sIcl(B) ⊆ U. That is sIcl(A ∪ B) ⊆ U. Hence A ∪ B is $I_{s\hat{g}}$-closed in (X, τ, I).

Theorem 2.9: Let A be $I_{s\hat{g}}$-closed set of (X, τ, I). Then sIcl(A) − A does not contain a nonempty set.

Proof: Let A be $I_{s\hat{g}}$-closed set and F be a $\hat{g}$ -closed set contained in sIcl(A). Then F^c is $\hat{g}$-open set, such that A ⊆ F^c. Since A is $I_{s\hat{g}}$-closed set. sIcl(A) ⊆ F^c. Thus F ⊆ $(sIcl(A))^c$. Also F ⊆ sIcl(A) − A.

Therefore F ⊆ $(scl(A))^c$ ∩ sIcl(A) = Φ. Hence F = Φ.

Remark 2.10: Suppose I = {Φ}, then $I_{s\hat{g}}$-closed sets coincides with s$\hat{g}$-closed set.

Theorem 2.11: Let (X, τ, I) be an ideal space. Then either {x} is $\hat{g}$-closed or $\{x\}^c$ is $I_{s\hat{g}}$-closed for every x ∈ X.

Proof: Suppose that {x} is not $\hat{g}$-closed in X, then $\{x\}^c$ is not $\hat{g}$-open and that only $\hat{g}$-open set containing $\{x\}^c$ is the space X itself. That is $\{x\}^c$ ⊆ X. Therefore sIcl(A) ⊆ X and so $\{x\}^c$ is a $I_{s\hat{g}}$-closed.

Theorem 2.12: Let A be a $I_{s\hat{g}}$-closed in (X, τ, I). Then A is semi-I-closed iff sIcl(A) − A is closed.

Proof:
Necessity: Let A be an $I_{s\hat{g}}$-closed and semi-I-closed. Then sIcl(A)=A and so sIcl(A) − A = Φ which is closed.

Sufficiency: Since A is $I_{s\hat{g}}$-closed set by Theorem 2.9, sIcl(A) − A contains no nonempty closed set. But sIcl(A) − A is closed. This implies that sIcl(A) − A = Φ. That is sIcl(A) = A. Hence A is semi-I-closed.

Theorem 2.13: Every $I_{s\hat{g}}$-closed is g-closed, g*-closed, sg-closed, gp-closed and gsp-closed but not conversely.

Example 2.14: Let X = {a, b, c},τ = {Φ,{a},{b,c}, X} and I = {Φ,{a},{b},{a,b}}. Here {a,b} is g-closed, g*-closed, sg-closed, gp-closed and gsp-closed but not $I_{s\hat{g}}$-closed.

3. $I_{s\hat{g}}$ − Continuity

Definition 3.1: A function f: (X, τ, I) → (Y, σ) is said to be $I_{s\hat{g}}$ continuous, if $f^{-1}(v)$ is $I_{s\hat{g}}$-closed in (X, τ, I) for every closed set v in (Y, σ).

Theorem 3.2: For a function f: (X, τ, I) → (Y, σ), the following hold.

1. Every continuous function is $I_{s\hat{g}}$- continuous.
2. Every $I_{s\hat{g}}$- continuous function is s$\hat{g}$- continuous.
3. Every $I_{s\hat{g}}$ - continuous function is gs - continuous.

Proof

(i) Let f be a continuous function and v be a closed set in (Y, σ). Then $f^{-1}(v)$ is closed in (X, τ, I). Since every closed set is $I_{s\hat{g}}$- closed, $f^{-1}(v)$ is $I_{s\hat{g}}$- closed in (X, τ, I). Hence f is $I_{s\hat{g}}$- continuous.

(ii) Let f be a $I_{s\hat{g}}$- continuous function and v be a closed set in (Y, σ). Then $f^{-1}(v)$ is $I_{s\hat{g}}$- closed in (X, τ, I). Since every $I_{s\hat{g}}$- closed set is s$\hat{g}$- closed set, $f^{-1}(v)$ is s$\hat{g}$- closed in (X, τ, I). Hence f is s$\hat{g}$- continuous.

(iii) Let f be a $I_{s\hat{g}}$- continuous function and v be a closed set in (Y, σ). Then $f^{-1}(v)$ is $I_{s\hat{g}}$- closed in (X, τ, I). Since every $I_{s\hat{g}}$- closed set is gs- closed set, $f^{-1}(v)$ is gs- closed in (X, τ, I). Hence f is gs- continuous.

The above theorem need not be true as seen from the following examples.

Examples 3.3:

(i) Let X = Y = {a, b, c}, τ = {Φ, {a}, X}, σ = {Φ, {b}, {a, b}, Y} and I = {Φ, {a}, {c}, {a, c}}. Let the function f: (X, τ, I) → (Y, σ) is defined by f(a) = a, f(b) = b, f(c) = c. Then the function f is $I_{s\hat{g}}$- continuous but not continuous.

(ii) Let X = Y = {a, b, c}, τ = {Φ, {a}, {a, b}, X}, σ = {Φ, {b}, Y} and I = {Φ, {a}}. Let the function f: (X, τ, I) → (Y, σ) is defined by f(a) = b, f(b) = c, f(c) = a. Then the function f is s$\hat{g}$- continuous but not $I_{s\hat{g}}$- continuous.

(iii) Let X = Y = {a, b, c}, τ = {Φ, {a}, {b, c}, X}, σ = {Φ, {c}, {a, c}, Y} and I = {Φ, {a}, {b}, {a,b}}. Let the function f: (X, τ, I) → (Y, σ) be the identity function. Then the function f is sg-continuous but not $I_{s\hat{g}}$- continuous.

Theorem 3.4: For a function f: (X, τ, I) → (Y, σ), the following hold.

(i) Every $I_{s\hat{g}}$- continuous function is g-continuous.
(ii) Every $I_{s\hat{g}}$- continuous function is g*-continuous.
(iii) Every $I_{s\hat{g}}$- continuous function is sg-continuous.
(iv) Every $I_{s\hat{g}}$- continuous function is gp-continuous.
(v) Every $I_{s\hat{g}}$- continuous function is gsp-continuous.

Proof: It is obvious.

The above theorem need not be true as seen from the following examples.

Examples 3.5:

(i) Let X = Y = {a, b, c}, τ = {Φ, {a}, {b,c}, X}, σ = {Φ, {a}, {a,b}, Y} and I = {Φ, {a}, {c}, {a,c}}. Let the function f:(X, τ, I) → (Y, σ) is defined by f(a) = b, f(b) = c, f(c) = a. Then the function f is g-continuous but not $I_{s\hat{g}}$-continuous.

(ii) Let X = Y = {a, b, c}, τ = { Φ, {a}, {a,b},{a,c}, X}, σ = { Φ, {a}, Y} and I = {Φ, {a}, {b}, {a,b}}. Let the function f: (X, τ, I) → (Y, σ) be the identity function. Then f is g*- continuous but not $I_{s\hat{g}}$- continuous.

(iii) Let X = Y = {a, b, c}, τ = {Φ, {b}, {a,b}, X}, σ = {Φ, {c}, Y} and I = {Φ, {b}}. Let the function f: (X, τ, I) → (Y, σ) be the identity function. Then f is sg-continuous but not $I_{s\hat{g}}$- continuous.

(iv) Let X = Y = {a, b, c}, τ = { Φ, {a}, {a,c}, X}, σ = {Φ, {c}, Y} and I = {Φ, {a}}. Let the function f: (X, τ, I) → (Y, σ) be the identity function. Then the function f is gp and gsp- continuous but not $I_{s\hat{g}}$-continuous.

Theorem 3.6: A map f: (X, τ, I) → (Y, σ) is $I_{s\hat{g}}$-continuous iff the inverse image of every closed set in (Y, σ) is $I_{s\hat{g}}$- closed in (X, τ, I).

Proof:

Necessary: Let v be an open set in (Y, σ). Since f is $I_{s\hat{g}}$- continuous, $f^{-1}(v^C)$ is $I_{s\hat{g}}$- closed in (X, τ, I). But $f^{-1}(v^C) = X - f^{-1}(v)$. Hence $f^{-1}(v)$ is $I_{s\hat{g}}$- closed in (X, τ, I).

Sufficiency: Assume that the inverse image of every closed set in (Y,) is $I_{s\hat{g}}$- closed in (X, τ, I). Let v be a closed set in (Y, σ). By our assumption $f^{-1}(v^C) = X - f^{-1}(v)$ is $I_{s\hat{g}}$- closed in (X, τ, I), which implies that $f^{-1}(v)$ is $I_{s\hat{g}}$- closed in (X, τ, I). Hence f is $I_{s\hat{g}}$- continuous.

Remark 3.7:

(i) The union of any two $I_{s\hat{g}}$- continuous function is $I_{s\hat{g}}$continuous.
(ii) The intersection of any two $I_{s\hat{g}}$- continuous function is need not be $I_{s\hat{g}}$- continuous.
(iii) Suppose I = {Φ}, then the notion of $I_{s\hat{g}}$- continuous consides with s$\hat{g}$- continuous.

Definition 3.8: A function f:(X, τ, I_1) → (Y, σ, I_2) is said to be $I_{s\hat{g}}$-irresolute, if $f^{-1}(v)$ is $I_{s\hat{g}}$-closed in (X, τ, I_1) for every $I_{s\hat{g}}$-closed set v in (Y, σ, I_2).

Example 3.9: Let X = Y = {a, b, c}, τ = {Φ, X, {a}, {a, b}}, I_1={ Φ,{a}} and σ = {Φ, Y, {b}} I_2 = {Φ, {a}, {b}, {a, b}}. Then the function f: (X, τ, I_1) → (Y, σ, I_2) defined by f(a) = c, f(b) = a and f(c) = b is $I_{s\hat{g}}$-irresolute.

Theorem 3.14: Let f: (X, τ, I_1) → (Y, σ, I_2) and g:(Y, σ, I_2) →(Z, η, I_3) be any two functions. Then the following hold.

(i) g ∘ f is $I_{s\hat{g}}$ - continuous if f is $I_{s\hat{g}}$ continuous and g is continuous.
(ii) g ∘ f is $I_{s\hat{g}}$ - continuous if f is $I_{s\hat{g}}$ irresolute and g is $I_{s\hat{g}}$ continuous.
(iii) g ∘ f is $I_{s\hat{g}}$ - irresolute if f is $I_{s\hat{g}}$ irresolute and g is irresolute.

Proof:

(i) Let v be a closed set in Z. Since g is continuous, $g^{-1}(v)$ is closed in Y. $I_{s\hat{g}}$-continuous of f implies, $f^{-1}(g^{-1}(v))$ is $I_{s\hat{g}}$ -closed in X and hence g ∘ f is $I_{s\hat{g}}$ -continuous.
(ii) Let v be a closed set in Z. Since g is $I_{s\hat{g}}$ -continuous, $g^{-1}(v)$ is $I_{s\hat{g}}$ -closed in Y. Since f is $I_{s\hat{g}}$ -irresolute, $f^{-1}(g^{-1}(V))$ is $I_{s\hat{g}}$ -closed in X. Hence g ∘ f is $I_{s\hat{g}}$ -continuous.
(iii) Let v be a $I_{s\hat{g}}$ -closed in Z. Since g is $I_{s\hat{g}}$ - irresolute, $g^{-1}(v)$ is $I_{s\hat{g}}$ -closed in Y. Since f is $I_{s\hat{g}}$-irresolute, $f^{-1}(g^{-1}(v))$ is $I_{s\hat{g}}$ -closed in X. Hence g ∘ f is $I_{s\hat{g}}$ -irresolute.

Theorem 3.15: Let X = A ∪ B be a topological space with topology τ and Y be a topological space with topology σ. Let f: (A, τ/A) → (Y, σ) and g:(B, τ/B) → (Y, σ) be $I_{s\hat{g}}$-continuous maps such that f(x) = g(x) for every x ∈ A ∩ B. Suppose that A and B are $I_{s\hat{g}}$-closed sets in X. Then the combination α: (X, τ, I) → (Y, σ) is $I_{s\hat{g}}$ - continuous.

Proof: Let F be any closed set in Y. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ = C ∪ D where C = $f^{-1}(F)$ and D = $g^{-1}(F)$. But C is $I_{s\hat{g}}$ -closed in A and A is be $I_{s\hat{g}}$ -closed in X and so C is $I_{s\hat{g}}$ -closed in X. Since we have proved that if B ⊆ A ⊆ X, B is $I_{s\hat{g}}$-closed in A and A is $I_{s\hat{g}}$ -closed in X, then B is $I_{s\hat{g}}$ -closed in X. Also C ∪ D is $I_{s\hat{g}}$ -closed in X. Therefore $\alpha^{-1}(F)$ is $I_{s\hat{g}}$-closed in X. Hence α is $I_{s\hat{g}}$ -continuous.

Definition 3.16: A topological space (X, τ, I) is said to be $I_{s\hat{g}}$-connected if X cannot be written as a disjoint union of two non-empty $I_{s\hat{g}}$ - open subsets. A subset A of X is $I_{s\hat{g}}$-connected if it is $I_{s\hat{g}}$-connected as a subspace.

Theorem 3.17: If f:(X, τ, I) → (Y, σ) is $I_{s\hat{g}}$ -continuous surjection and X is $I_{s\hat{g}}$ -connected, then Y is connected.

Proof: Suppose Y = A ∪ B where A and B are disjoint open sets is Y. Since f is $I_{s\hat{g}}$-continuous and onto, X= $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty $I_{s\hat{g}}$-open sets in X, a contradiction since X is $I_{s\hat{g}}$-connected. Hence Y is connected.

Definition 3.18: An ideal space (X, τ, I) is said to be $I_{s\hat{g}}$-normal if for each pair of non-empty disjoint closed sets A and B of X, there exists disjoint $I_{s\hat{g}}$-open subsets U and V of X such that A ⊆ U and B ⊆ V.

Theorem 3.19: If f:(X, τ, I) → (Y, σ) is $I_{s\hat{g}}$ -continuous, closed injection and Y is normal, then X is $I_{s\hat{g}}$ -normal.

Proof: Let A and B be a disjoint closed subsets of X. Since f is closed and injective, f(A) and f(B) are disjoint, closed subsets of Y. Since Y is normal, there exists a disjoint open subsets U and V of X such that f(A) ⊆ U and f(B) ⊆ V.

Hence A ⊆ $f^{-1}(U)$ and B ⊆ $f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V)$ = Φ. Since f is $I_{s\hat{g}}$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $I_{s\hat{g}}$-open in X which implies X is $I_{s\hat{g}}$-normal.

REFERENCES

1. I.Arokiarani, K.Balachandran and J. Dontchev, Some characterizations of gp-irresolute and gp-continuous maps between topological spaces, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 20(1999), 93-104.
2. Arya.S.P and Nour.T.M., Characterizations of S-Normal Spaces, Indian.J.Pure,Appl.Math.,21(1990), 717-719.
3. Balachandran.K, Sundaram.P and Maki.H, On Generalized continuous maps in topological spaces, Mem. Fac. Sci.Kochi Univ. Ser. A. Math., 12(1991), 5-13.
4. Bhattacharya.K and Lahiri.B.K, Semi-generalized closed sets in topology, Indian. J. Math (3). 29(1987), 375.
5. Devi.R., Maki.H and Balachandran.K., Semi-Generalized homomorphisms and generalized semi-Homeomorphisms, Indian J., Pure. Apl. Math., 26(3) (1995), 271-284.
6. Dontchev.J., On generalizing semi-pre open sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 16(1995), 35-48.
7. E.Hatir and T.Noiri, On Decomposition of continuity via idealization, Acta Math.Hunger., 96(2002), 341-349.
8. Kuratowski, Topology, Vol.I, Academic press, Newyork(1966).
9. Levine.N., Generalized Closedsets in topology.Rend.Circ.Mat.Palermo. 19(1970), 89-96.
10. M.Rajamani and M.Anand A Study on sg-closed sets in ideal topological spaces, Mphil., Thesis(2009).
11. N. Ramya and A. Parvathi, s $\hat{g}$ -closed sets in topological spaces, Mathematics and its applications- A new wave, 2011.
12. Sundaram.P., Maki.K and Balachandran.K., Semi-Generalized continuous maps and semi $T_{\frac{1}{2}}$ spaces, Bulletine of Fukuoka university of Education, Vol.40, no.3 (1991), 33-40.
13. R.Vaidynathaswamy, Set topology, Chelsea, Publishing company, Newyork (1960).
14. Veerakumar.M.K.R.S, Between closed sets and g-closed sets, Mem. Fac.Sci.Kochio Univ.Math., 21(2000), 1-19.
15. Veerakumar.M.K.R.S, $\hat{g}$ -closed sets in topological spaces, Math, Soc, 18(2003), 99-112.

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