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A COMPARATIVE STUDY ON ZERO-TRUNCATED POISSON-LINDLEY AND QUASI POISSON-LINDLEY DISTRIBUTIONS

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ABSTRACT

In this paper, zero-truncated quasi Poisson-Lindley distribution has been introduced and investigated its statistical properties including general expression of probabilities and moments and any other related measure and compared this distribution with zero-truncated Poisson-Lindley distribution. The problem of estimation for estimating the parameters of zero-truncated quasi Poisson-Lindley distribution has been also discussed. Application of this distribution to real data is also given and fitting have been compared with zero-truncated Poisson-Lindley distribution to see its goodness of fit.

Keywords: *Poisson-Lindley distribution, quasi Poisson-Lindley distribution, zero-truncated distribution, moments, method of moments, goodness of fit.*

1. INTRODUCTION

The Poisson–Lindley distribution with the probability mass function (p.m.f.):

$$f_0(x;\theta) = \frac{\theta^2(x+\theta+2)}{(1+\theta)^{x+3}}, \quad x = 0, 1, \dots, \quad \theta > 0, \tag{1.1}$$

was introduced by Sankaran (1970) to model count data. The distribution arises from the Poisson distribution when its parameter λ follows a Lindley (1958) distribution with the probability density function (p.d.f.):

$$g(\lambda;\theta) = \frac{\theta^2}{1+\theta}(1+\lambda)e^{-\theta\lambda}, \quad \lambda > 0, \ \theta > 0 \tag{1.2}$$

Ghitany *et al.* (2007) showed that in many ways (1.2) is a better model than one based on the exponential distribution. Ghitany and Al-Mutairi (2008a) obtained size-biased Poisson-Lindley distribution with applications to some real data sets. Ghitany *et al.* (2008b) introduced zero-truncated Poisson-Lindley distribution to model count data with applications. Ghitany and Al-Mutairi (2009) discussed estimation methods for the discrete Poisson-Lindley distribution. For increasing the flexibility, Mahmoudi and Zakerzadeh (2010) obtained an extended version of Poisson-Lindley distribution known as generalized Poisson-Lindley distribution and also discussed the estimation of the parameters of the distribution by using method of moment and maximum likelihood method. A discrete two-parameter Poisson-Lindley distribution is studied by Shanker *et.al* (2013). Dutta and Borah (2014) reviewed some properties and applications of size-biased Poisson-Lindley distribution. The zero-modified form of Poisson-Lindley distribution is investigated by Dutta and Borah (2014). Recently, Dutta and Borah (2015) studied on certain recurrence relations arising in different forms of size-biased Poisson-Lindley distributions.

Truncation of a distribution occurs when a range of possible variate values either is ignored or is impossible to observe. In most common form of truncation, the zeroes are not recorded. In this case, the zero-truncated distributions can be used as a distribution for the sizes of groups. This situation occurs in applications such as the number of claims per claimant, the number of occupants per car etc.

When the data to be modeled originate from a generating mechanism that structurally excludes zero counts, the Poisson-Lindley distribution must be adjusted to count for the missing zeros. Ghitany, Al-Mutairi and Nadarajah (2008b) obtained zero-truncated Poisson-Lindley (ZTPL) distribution to model count data by considering the zero-truncated form of Poisson-Lindley distribution as

$$f(x;\ \theta) = \frac{f_0(x;\theta)}{1-f_0(0;\theta)} = \frac{\theta^2}{\theta^2+3\theta+1}\frac{(x+\theta+2)}{(1+\theta)^x}; \quad x = 1, 2, 3, \dots; \ \theta > 0. \tag{1.3}$$

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They also showed that, the ZTPL distribution is unimodal and has an increasing failure rate, i.e. since the ratio
$\frac{f(x+1;\theta)}{f(x;\theta)} = \frac{1}{(1+\theta)}\left(1+\frac{1}{x+\theta+2}\right)$

is a decreasing function in x, the ZTPL distribution is unimodal. Also, since
$\frac{f(x+2;\theta)f(x;\theta)}{f^2(x+1;\theta)} = \frac{1+(1/x+\theta+3)}{1+(1/x+\theta+2)} < 1$

$f(x;\ \theta)$ is log-concave and hence the ZTPL distribution has an increasing failure rate. [See Ghitany, Al-Mutairi and Nadarajah (2008b)]

The mean and variance of ZTPL distribution obtained by Ghitany et. al. (2008b) are given by

$$\mu = \frac{(1+\theta)^2(\theta+2)}{\theta(\theta^2+3\theta+1)}$$

and $\sigma^2 = \frac{(1+\theta)^2(\theta^3+6\theta^2+10\theta+2)}{\theta^2(\theta^2+3\theta+1)^2}$ respectively.

In this paper, we proposed the zero-truncated quasi Poisson–Lindley (ZTQPL) distribution and find its distributional properties and then compared them with zero-truncated Poisson-Lindley distribution investigated by Ghitany *et al* (2008b).

2. ZERO-TRUNCATED QUASI POISSON-LINDLEY DISTRIBUTION

The quasi Poisson-Lindley (QPL) distribution is a two parameter discrete mixture distribution with p.m.f

$$P_1(x;\alpha,\theta) = \frac{\theta}{\alpha+1}\frac{[\alpha+\theta(1+\alpha)+\theta x]}{(1+\theta)^{x+2}};\ x = 0,1,2,\dots,\theta > 0,\alpha > -1 \qquad (2.1)$$

arises from the Poisson distribution when its parameter follows a Shanker and Mishra (2013a) quasi Lindley distribution with probability density function (p.d.f)

$$f(x;\alpha,\theta) = \frac{\theta}{\alpha+1}(\alpha+\theta x)e^{-\theta x},\ \ x > 0,\ \theta > 0,\alpha > \ -1 \qquad (2.2)$$

It can be seen that, the Poisson-Lindley distribution (1.1) of Sankaran (1970) is a particular case of QPL distribution (2.1) at $\alpha = \theta$. The model (2.1) is a more generalized and more flexible than the Poisson-Lindley distribution of Sankaran (1970).

Shanker and Mishra (2013b) obtained quasi Poisson-Lindley distribution by compounding Poisson distribution with quasi Lindley distribution and showed that, quasi Poisson-Lindley (QPL) distribution is a better model than the Poisson-Lindley distribution of Sankaran (1970) for analyzing different types of count data. A Size-biased form of QPL distribution is investigated by Shanker and Mishra (2013c) by size biasing the quasi Poisson-Lindley distribution. They have been also discussed methods of parameter estimation for estimating the parameters of the size-biased QPL distribution. Recently, Borah and Dutta (2015) investigated certain properties of two-parameter quasi Poisson-Lindley distribution along with its applications to some real data set.

The probability mass function (p.m.f) of zero-truncated quasi Poisson-Lindley (ZTQPL) distribution with parameters α and θ obtained as

$$P_T(x;\ \alpha,\theta) = \frac{p_1(x;\alpha,\theta)}{1-p_1(0;\alpha,\theta)} = \frac{\theta}{[\alpha+\theta(\alpha+2)+1]}\frac{[\alpha+\theta(1+\alpha)+\theta x]}{(1+\theta)^x};\ \ x = 1,2,3,\dots;\ \theta > 0;\ \alpha > -1 \qquad (2.3)$$

while θ is the scale and α the shape parameter. Simply denote it by ZTQPL (α,θ). The ZTQPL distribution can be also derived as a mixture of zero-truncated Poisson distribution when its parameter λ follows quasi Lindley distribution with probability density function (2.2). It is also seen that, zero-truncated Poisson-Lindley (ZTPL) distribution (1.3) is a particular case of ZTQPL distribution at $\alpha = \theta$.

Since,

$$\frac{P_T(x+1;\ \alpha,\theta)}{P_T(x;\ \alpha,\ \theta)} = \frac{1}{1+\theta}\left[1+\frac{\theta}{\{\alpha+\theta(1+\alpha)+\theta x\}}\right]$$

is a decreasing function in x, the ZTQPL distribution is unimodal. [see Johnson, Kotz and Kemp (2005)]

3. STATISTICAL PROPERTIES AND RELATED MEASURES

In this Section, some important statistical Properties and some related measures for ZTQPL distribution will be derived.

3.1 Probability Generating Function

If x follows ZTQPL (α,θ), then the probability generating function (p.g.f) of x is obtained as

$$g(t)=\sum_{x=1}^{\infty}t^x P_T(x;\ \alpha,\theta)=\frac{\theta}{\alpha+\theta(\alpha+2)+1}\sum_{x=1}^{\infty}\left(\frac{t}{1+\theta}\right)^x[\alpha+\theta(1+\alpha)+\theta x]$$

$$=\frac{\theta}{\alpha+\theta(\alpha+2)+1}\left[\{\alpha+\theta(1+\alpha)\}\sum_{x=1}^{\infty}\left(\frac{t}{1+\theta}\right)^x+\theta\sum_{x=1}^{\infty}x\left(\frac{t}{1+\theta}\right)^x\right]$$

$$=\frac{\theta}{\alpha+\theta(\alpha+2)+1}\left[\{\alpha+\theta(1+\alpha)\}\left(\frac{t}{1+\theta-t}\right)+\frac{\theta(1+\theta)t}{(1+\theta-t)^2}\right]$$

$$=\frac{\theta t\,[(1+\theta)(\theta+2\theta+\theta\alpha)-t(\alpha+\theta+\theta\alpha)]}{[\alpha+\theta(\alpha+2)++1](1+\theta-t)^2}\ ;t>0 \tag{3.1}$$

Note that, for $\alpha\rightarrow\theta$ this $g(t)$ reduces to the pgf of ZTPL distribution of Ghitany *et al.* (2008b) which is given as

$$g(t)=\frac{\theta^2 t[(1+\theta)(\theta+3)-(\theta+2)t]}{(\theta^2+3\theta+1)(1+\theta-t)^2};\ \theta>0,t>0.$$

The recursive relation for probabilities of ZTQPL distribution with pgf (3.1) is given as

$$p_r=\frac{1}{(1+\theta)^2}[2(1+\theta)p_{r-1}-p_{r-2}];\ \ r>2 \tag{3.2}$$

where, $p_1=\frac{\theta(\alpha+2\theta+\theta\alpha)}{(1+\theta)[\alpha+\theta(\alpha+2)+1]}$ and $p_2=\frac{\theta(\alpha+3\theta+\theta\alpha)}{(1+\theta)^2[\alpha+\theta(\alpha+2)+1]}$ so on.

If $\alpha\rightarrow\theta$, in the probabilities of ZTQPL distribution then these probabilities are same as that of ZTPL distribution of Ghitany *et al.* (2008b) which are given below as

$p_1=\frac{\theta^2(\theta+3)}{(1+\theta)(\theta^2+3\theta+1)}$ and $p_2=\frac{\theta^2(\theta+4)}{(1+\theta)^2(\theta^2+3\theta+1)}$ etc.

The general form of r^{th} order probability is given by

$$p_r=\frac{\theta[\alpha+(r+1)\theta+\theta\alpha]}{(1+\theta)^r[\alpha+\theta(\alpha+2)+1]};r=1,2,3,... \tag{3.3}$$

The higher order probabilities can be obtained very easily by using either relation (3.2) or (3.3).

3.2 Moments and related measures

In this section, r^{th} order factorial moment and moments about origin and central moments of ZTQPL distribution can be obtained as follows:

The factorial moment generating function of ZTQPL distribution is

$$m(t)=\frac{\theta(1+\theta)(1+t)(\alpha+2\theta+\theta\alpha)-\theta(1+t)^2(\alpha+\theta+\alpha\theta)}{(\theta-t)^2[\alpha+\theta(\alpha+2)+1]};\ t>0,\theta>0,\alpha>-1 \tag{3.4}$$

Note that, the factorial moment generating function of ZTPL distribution is a particular form of factorial moment generating function of ZTQPL distribution, i.e. if $\alpha\rightarrow\theta$ in (3.4) then it reduces to the factorial moment generating function of ZTPL distribution, which is given by

$$m(t)=\frac{\theta^2(1+t)[(1+\theta)(\theta+3)-(\theta+2)(1+t)]}{(\theta-t)^2(\theta^2+3\theta+1)};\ t>0,\theta>0 \tag{3.5}$$

The factorial moment recursive relation of ZTQPL distribution is given as

$$\mu'_{(r)}=\frac{1}{\theta^2}\left[2\theta r\mu'_{(r-1)}-r(r-1)\mu'_{(r-2)}\right];\ r>2 \tag{3.6}$$

where, $\mu'_{(1)}=\frac{(1+\theta)^2(\alpha+2)}{\theta[\alpha+\theta(\alpha+2)+1]}$ and $\mu'_{(2)}=\frac{(1+\theta)^2 2(\alpha+3)}{\theta^2[\alpha+\theta(\alpha+2)+1]}$

The more general expression of r^{th} order factorial moment is given by

$$\mu'_{(r)}=\frac{(1+\theta)^2 r!(\alpha+r+1)}{\theta^r[\alpha+\theta(\alpha+2)+1]};r=1,2,3,... \tag{3.7}$$

After obtaining the first four factorial moments and then using the relationship between factorial moments and moments about origin, the first four moments about origin of ZTQPL distribution are given as

$$\mu'_1=\frac{(1+\theta)^2(\alpha+2)}{\theta[\alpha+\theta(\alpha+2)+1]}$$

$$\mu'_2=\frac{(1+\theta)^2[(2+\alpha)(\theta+2)+2]}{\theta^2[\alpha+\theta(\alpha+2)+1]}$$

$\mu'_3=\frac{(1+\theta)^2[(2+\alpha)\theta^2+(6\alpha+18)\theta+(6\alpha+24)]}{\theta^3[\alpha+\theta(\alpha+2)+1]}$ and

$$\mu_4^{'} = \frac{(1+\theta)^2[(2+\alpha)\theta^3 + 14(\alpha+3)\theta^2 + 36(\alpha+4)\theta + 24(\alpha+5)]}{\theta^4[\alpha+\theta(\alpha+2)+1]}$$

Note that, if $\alpha \to \theta$ these raw moments are same as that of ZTPL distribution (1.3) of Ghitany *et al.* (2008b).

Hence, the mean and variance of ZTQPL distribution is given as

$$\mu = \frac{(1+\theta)^2(\alpha+2)}{\theta[\alpha+\theta(\alpha+2)+1]} \tag{3.8}$$

and $$\sigma^2 = \frac{(1+\theta)^2[(1+\theta)(\alpha^2+4\alpha)+\theta\alpha+6\theta+2]}{\theta^2[\alpha+\theta(\alpha+2)+1]^2} \tag{3.9}$$

If $\alpha \to \theta$ in (3.8) and (3.9), then the mean and variance of ZTQPL distribution is same as that of ZTPL distribution obtained by Ghitany *et al.* (2008b) which is mentioned in above introduction section.

The index of dispersion for ZTQPL distribution is given by

$$\gamma = \frac{\sigma^2}{\mu} = \frac{\alpha(1+\theta)(\alpha+4)+\theta\alpha+6\theta+2}{\theta(\alpha+2)[\alpha+\theta(\alpha+2)+1]} \tag{3.10}$$

it follows that the ZTQPL distribution is over-dispersed ($\sigma^2 > \mu$) for all values of (α, θ), and equi-dispersed ($\sigma^2 = \mu$) for large amount of θ. For$\alpha = \theta$, the index of dispersion of ZTQPL distribution reduces to index of dispersion of ZTPL distribution [see Ghitany *et al.* (2008b)], which is given by

$$\gamma = \frac{\theta^3+6\theta^2+10\theta+2}{\theta(\theta+2)(\theta^2+3\theta+1)} \tag{3.11}$$

The coefficient of variation (CV) is the ratio of the standard deviation to the mean. The higher the coefficient of variation, the greater the level of dispersion around the mean. It is generally expressed as a percentage. Without units, it allows for comparison between distributions of values whose scales of measurement are not comparable. The CV of ZTQPL distribution is:

$$CV = \frac{(1+\theta)[\alpha(\alpha+4)+2]+\theta(\alpha+4)}{\theta(\alpha+2)[\alpha+\theta(\alpha+2)+1]} \tag{3.12}$$

If $\alpha \to \theta$, then it is same as that of ZTPL distribution .

The distribution of a random variable is often characterized in terms of its moment generating function (mgf), a real function whose derivatives at zero are equal to the moments of the random variable. Moment generating functions have great practical relevance not only because they can be used to easily derive moments, but also because a probability distribution is uniquely determined by its mgf. The mgf of ZTQPL distribution can be derived as:

$$M_x(t) = \sum_{x=1}^{\infty} e^{tx}\, P_T(x;\, \alpha, \theta) = \frac{\theta e^t[(1+\theta)(\theta+2\theta+\theta\alpha)-e^t(\alpha+\theta+\theta\alpha)]}{[\alpha+\theta(\alpha+2)++1](1+\theta-e^t)^2} \tag{3.13}$$

The ZTQPL distribution is more generalized and more flexible than the one parameter ZTPL distribution of Ghitany *et. al* (2008b) for analyzing different types of count data.

4. ESTIMATION OF PARAMETER

One of the most important property of a distribution is the problem of estimation of the parameter. In case of ZTPL distribution, the single parameter θ was estimated by using method of moment estimation procedure [see Ghitany et al. (2008b). To estimate the parameters of ZTQPL distribution we have been used an adhoc method, i.e; estimate the two parameters in terms of ratio of the first two probabilities and mean of the distribution.

In case of ZTQPL distribution,

$$p_1 = \frac{\theta(\alpha+2\theta+\theta\alpha)}{(1+\theta)[\alpha+\theta(\alpha+2)+1]}$$

$$p_2 = \frac{\theta(\alpha+3\theta+\theta\alpha)}{(1+\theta)^2[\alpha+\theta(\alpha+2)+1]}$$

The ratio of these two probabilities gives

$$\frac{p_2}{p_1} = \frac{1}{(1+\theta)}\frac{(\alpha+3\theta+\theta\alpha)}{(\alpha+2\theta+\theta\alpha)}$$

$$\Rightarrow (1+\theta)\alpha = \left[\frac{\theta p_1}{(1+\theta)p_2 - p_1} - 2\theta\right] \text{ [after calculation]} \tag{4.1}$$

Again, the mean of ZTQPL distribution gives

$$\mu = \frac{(1+\theta)^2(\alpha+2)}{\theta[\alpha+\theta(\alpha+2)+1]}$$

$$\Rightarrow (1+\theta)\alpha = \frac{2(1+\theta)^2-\theta(1+2\theta)\mu}{[\theta\mu-(1+\theta)]} \text{ [after calculation]} \quad (4.2)$$

Now equating the both sides of (4.1) and (4.2), we have a quadratic equation in terms of θ as

$$A\theta^2 + B\theta + C = 0$$

where,

$A = p_1(\mu-1) + p_2(\mu-2)$
$B = p_2(\mu-4) + p_1(1-\mu)$
$C = 2(p_1 - p_2)$

which gives the estimator of the parameter θ as

$$\hat{\theta} = \frac{-B+\sqrt{B^2-4AC}}{2A} \quad (4.3)$$

After obtaining the estimator $\hat{\theta}$ of θ, the other parameter α can be estimated either from

$$\hat{\alpha} = \frac{1}{(1+\theta)}\left[\frac{\theta p_1}{(1+\theta)p_2-p_1} - 2\theta\right] \quad (4.4)$$

or,

$$\hat{\alpha} = \frac{2(1+\theta)^2-\theta(1+2\theta)\mu}{(1+\theta)[\theta\mu-(1+\theta)]} \quad (4.5)$$

5. APPLICATION AND GOODNESS OF FIT

In order to examine the flexibility and to see the application of ZTQPL distribution, two sets of reported data taken from Ghitany *et.al* (2008b) are considered. The first data set represents the immunogold assay data of Cullen *et al.* (1990) for which ZTPL distribution was fitted by Ghitany *et al.* (2008b). Cullen *et al.* (1990) gave counts of sites with 1, 2, 3, 4 and 5 particles from immunogold assay data. The counts were 122, 50, 18, 4, 4. The second data set represents animal abundance data of Keith and Meslow (1968). In a study carried out by Keith and Meslow (1968), snowshoe hares were captured over 7 days. There were 261 hares caught over 7 days. Of these, 188 were caught once, 55 were caught twice, 14 were caught three times, 4 were caught four times, and 4 were five times.

We are interested in testing the null hypothesis H_0: "Number of attached particles and number of snowshoe hares is a ZTQPL random variable" verses the alternative hypothesisH_1: "Number of attached particle and number of snowshoe hares is not a ZTQPL random variable".

The expected frequencies of ZTQPL distribution along with the fitted ZTPL distribution estimated parameters and computed χ^2 and p −value are shown in Table 1 and Table 2. It is clear from the tables that, the null hypothesis H_0 cannot be rejected; indeed, the close agreement between the observed and expected frequencies suggests that the ZTQPL distribution provides a "good fit" to the two data sets.

Table-1: Observed vs. expected frequencies of immunogold assay data [Cullen *et al.* (1990)]

Number of attached particles	Observed Frequency $\bar{x} = 1.576$	ZTPL Ghitany et al. (2008b) $\hat{\theta} = 2.185$	ZTQPL $\hat{\theta} = 2.890$ $\hat{\alpha} = -0.242$
1	122	124.8	121.9
2	50	46.8	50.2
3	18	17.1	17.9
4	4	6.1	5.9
5	4	3.2	2.1
Total	198	198.0	198.0
χ^2		0.511	0.017
$d.f$		2	2
$p-value$		0.76	0.99

Table-2: Observed vs. expected frequencies of animal abundance data [Keith and Meslow (1968)]

Number of snowshoe hare	Observed frequency	ZTPL $\hat{\theta} = 3.101$	ZTQPL $\hat{\theta} = 2.606$ $\hat{\alpha} = 7.647$
1	184	187.4	183.1
2	55	53.5	54.8
3	14	14.8	16.4
4	4	4.2	4.9
5	4	1.1	1.8
Total	261	261.0	261.0
χ^2		1.522	0.608
$d.f$		2	2
$p-value$		0.47	0.74

6. CONCLUSION

The paper has introduced a zero-truncated quasi Poisson-Lindley distribution for analyzing different types of count data to which one-parameter zero-truncated Poisson-Lindley distribution is a particular case. The paper has derived some statistical properties of the distribution including problem of parameter estimation and compared them with the properties of particular form. An application of ZTQPL distribution is illustrated to immunogold assay data and animal abundance data.

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