International Journal of Mathematical Archive-6(6), 2015, 11-16
Available online through www.ijma.info ISSN 2229 – 5046

TWO GENERALIZED (θ, θ)-DERIVATIONS ON PRIME RINGS

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(Received On: 13-05-15; Revised & Accepted On: 15-06-15)

ABSTRACT

Let R be an associative ring with center $Z(R)$. In this paper, we will extend the notation of a generalized (θ,θ)-derivation F associated with (θ,θ)-derivation d, to, two generalized (θ,θ)-derivations F and G associated with the same(θ,θ)-derivation d, to obtain either $d = 0$ or $\theta\big(F(R)\big) \subseteq Z(R)$ of prime rings under certain conditions. Throughout this paper, F will always denote onto map.

Key words: *prime ring, derivation, generalized derivation, (θ,θ)-derivation, generalized (θ,θ)-derivation.*

INTRODUCTION

This paper consists of two sections. In section one, we recall some basic definitions and other concepts, which will be used in our paper, we explain these concepts by examples, remarks. In section two Yass in [1] extend the notation of a generalized derivation F associated with derivation d in [2], to, two generalized derivations F and G associated with the same derivation d. In this paper we extend this, to, two generalized (θ,θ)-derivation F and G associated with the same (θ, θ)-derivation d, to obtain either $d = 0$ or $\theta\big(F(R)\big) \subseteq Z(R)$ of prime rings under certain conditions.

1. BASIC CONCEPTS

Definition 1.1: [2] A ring R is called a prime ring if for any $a, b \in R$, $aRb = \{0\}$, implies that either $a = 0$ or $b = 0$.

Examples 1.2: [2]
1. Any domain is a prime ring.
2. Any matrix ring over an integral domain is a prime ring.

Definition 1.3: [3] Let R be a ring. Define a lie product [.,.] on as follows $[x, y] = xy - yx$, for all $x, y \in$ R.

Properties 1.4: [3] Let R be a ring. Then for all $x, y, z \in R$, we have
1. $[x\,,yz] = y\,[x\,,z] + [x\,,y]z$
2. $[xy\,,z] = x\,[y\,,z] + [x\,,z]y$
3. $[x + y, z] = [x\,,z] + [y\,,z]$
4. $[x, y + z] = [x\,,y] + [x\,,z]$

Definition 1.5: [4] Let R be a ring. Define a Jordan product on R as follows $aob = ab + ba$, for all $a, b \in R$.

Properties 1.6: [4] Let R be a ring. Then for all $x, y, z \in R$, we have
1. $xo(yz) = (xoy)z - y[x, z] = y(xoz) + [x, y]z$
2. $(xy)oz = x(yoz) - [x, z]y = (xoz)y + x[y, z]$

Definition 1.7: [4] Let R be a ring, the center of R denoted by $Z(R)$ and is defined by: $Z(R) = \{x \in R : xr = rx,$ for all $r \in R\}$

Definition 1.8: [1] Let R be a ring, then an additive map $d: R \to R$ is called derivation, if:
$d(xy) = d(x)y + x\,d(y)$, for all $x, y \in R$.

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Example 1.9: [1] Let $R = \{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}, a, b \in N$, where N is the ring of integers} be a ring of 2×2 matrices with respect to usual addition and multiplication.

Let $d : R \to R$, defined by $d \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$, for all $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \in R$.

Then d is a derivation of R.

Definition 1.10: [2] Let R be a ring. An additive mapping $F : R \to R$ is called a generalized derivation associated with d if there exists a derivation $d : R \to R$, such that

$$F(xy) = F(x)\, y + x\, d(y), \text{ for all } x, y \in R.$$

Example 1.11: [2] Let $R = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}, a, b, c \in Z,$ the set of integers}. The additive maps $F, d : R \to R$ define the following:

$F \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & a+c \\ 0 & 0 \end{pmatrix}$, for all $a, b, c \in Z$.

And $d \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & a-c \\ 0 & 0 \end{pmatrix}$, for all $a, b, c \in Z$.

Then F is a generalized derivation of R associated with d.

Definition 1.12: [3] Let R be a ring. An additive mapping $d : R \to R$ is called a (θ,θ)-derivation, where $\theta : R \to R$ is a mapping of R, if

$$d(xy) = d(x)\theta(y) + \theta(x)\, d(y), \text{ for all } x, y \in R.$$

2. TWO GENERALlZED (θ,θ)-DERIVATIONS

Definition 2.1: Let R be a ring. An additive mapping $F : R \to R$ is called a generalized (θ,θ)-derivation associated with d, where $\theta : R \to R$ is a mapping of R, if there exists a (θ,θ)-derivation $d : R \to R$, such that

$$F(xy) = F(x)\theta(y) + \theta(x)\, d(y), \text{ for all } x, y \in R.$$

Example 2.2 : Let $R = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}, a, b, c \in Z,$ the set of integers. The additive maps $F, d : R \to R$ define the following:

$F \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & a+c \\ 0 & 0 \end{pmatrix}$, for all $a, b, c \in Z$, and

$d \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & a-c \\ 0 & 0 \end{pmatrix}$, for all $a, b, c \in Z$, and

Suppose that $\theta : R \to R$ is a mapping such that

$\theta \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$, for all $a, b, c \in Z$.

It is clear that d is a (θ,θ)-derivation of R.

Then F is a generalized (θ,θ)-derivation of R associated with d.

Theorem 2.3: Let R be a prime ring. If R admits a nonzero generalized (θ,θ)-derivations F and G associated with the same (θ,θ)-derivation d, where θ is an automorphism of R, such that $[\theta(F(x)), G(y)] = 0$, for all $x, y \in R$, then either

$$d = 0 \text{ or } \theta(F(R)) \subseteq Z(R).$$

Proof: Replace y by yz in our hypotheses holds

$$G(y)\,[\theta(F(x)), \theta(z)] + \theta(y)[\theta(F(x)), d(z)] + [\theta(F(x)), \theta(y)]\, d(z) = 0, \text{ for all } x, y, z \in R \quad (1)$$

Replace z by $z\,F(x)$ in (1), we get:

$$G(y)[\theta(F(x)), \theta(z)\,\theta(F(x))] + \theta(y)[\theta(F(x)), d(z\,F(x))] + [\theta(F(x), \theta(y)]\, d(z\,F(x)) = 0, \text{ for all } x, y, z \in R \quad (2)$$

This can be rewritten as:

$$G(y)\,[\theta\,(F(x)), \theta(z)]\,\theta(F(x)) + \theta(y)[\theta(F(x)), d(z)]\,\theta(F(x)) + \theta(y)\,\theta(z)[\theta(F(x)), d(F(x))]$$
$$+\theta(y)[\theta(F(x)), \theta(z)]d(F(x)) + [\theta(F(x)), \theta(y)]\, d(z)\,\theta(F(x)) + [\theta(F(x)), \theta(y)]\theta(z)\, d(F(x)) = 0,$$
$$\text{for all } x, y, z \in R \quad (3)$$

Right multiplication of (1) by $\theta(F(x))$, to get:
$G(y)[\theta(F(x)),\theta(z)]\,\theta(F(x))+\theta(y)[\theta(F(x)),d(z)]\,\theta(F(x))+[\theta(F(x)),\theta(y)]\,d(z)\,\theta(F(x))=0$,
for all $x,y,z\in R$ (4)

From (3) and (4) one obtains:
$\theta(y)\,\theta(z)[\theta(F(x)),d(F(x))]+\theta(y)[\theta(F(x)),\theta(z)]d(F(x))+[\theta(F(x)),\theta(y)]\theta(z)\,d(F(x))=0$,
for all $x,y,z\in R$ (5)

Now, replace y by ty in (5), to get:
$\theta(t)\,\theta(y)\,\theta(z)[\theta(F(x)),d(F(x))]+\theta(t)\,\theta(y)[\theta(F(x)),\theta(z)]d(F(x))+\theta(t)[\theta(F(x)),\theta(y)]\theta(z)\,d(F(x))$
$+[\theta(F(x)),\theta(t)]\,\theta(y)\,\theta(z)\;d(F(x))=0$, for all $x,y,t,z\in R$ (6)

Left multiplication of (5) by $\theta(t)$, gives:
$\theta(t)\,\theta(y)\,\theta(z)[\theta(F(x)),d(F(x))]+\theta(t)\,\theta(y)[\theta(F(x)),\theta(z)]d(F(x))+\theta(t)[\theta(F(x)),\theta(y)]\theta(z)\,d(F(x))=0$,
for all $x,y,t,z\in R$ (7)

From (6) and (7), we obtain:
$[\theta(F(x)),\theta(t)]\,\theta(y)\,\theta(z)\,d(F(x))=0$ for all $x,y,t,z\in R$ (8)

Since R is prime, then we get:
either $[\theta(F(x)),\theta(t)]=0$, for all $x,t\in R$
or $\theta(z)d(F(x))=0$, for all $x,z\in R$

In the first case, we get $\theta(F(R))\subseteq Z(R)$.

And in the second case, we get R $d(F(x))=0$, for all $x\in R$, since R is prime and F is onto, we get $d=0$.

Theorem 2.4: Let R be a prime ring. If R admits a nonzero generalized (θ,θ)-derivations F and G associated with the same (θ,θ)-derivation d, where θ is an automorphism of R, such that $\theta(F(x))\;o\;G(y)=0$, for all $x,y\in R$, then either $d=0$ or $\theta(F(R))\subseteq Z(R)$.

Proof: By hypotheses, we have :
$\theta(F(x))o\,G(y)=0$, for all $x,y\in R$ (1)

Replacing y by yz in (1), we get:
$(\theta(F(x))o\,G(y))\,\theta(z)-G(y)[\theta(F(x)),\theta(z)]+(\theta(F(x))o\,\theta(y))\,d(z)-\theta(y)\,[\theta(F(x)),d(z)]=0$,
for all $x,y,z\in R$ (2)

From (1) and (2), we obtain:
$(\theta(F(x))o\,\theta(y))\;d(z)-\theta(y)[\theta(F(x)),d(z)]-G(y)\,[\theta(F(x)),\theta(z)]=0$, for all $x,y,z\in R$ (3)

Replacing z by F(x) in (3), to get
$(\theta(F(x))o\,\theta(y))\;d(F(x))-\theta(y)[\theta(F(x)),d(F(x))]=0$, for all $x,y\in R$ (4)

Again, replacing y by zy in (4), gives:
$\theta(z)\,(\theta(F(x))o\,\theta(y))\,d(F(x))+[\theta(F(x)),\theta(z)]\,\theta(y)d(F(x))-\theta(z)\,\theta(y)[\theta(F(x)),d(F(x))]=0$,
for all $x,y,z\in R$ (5)

Left multiplication of (4) by $\theta(z)$, to get:
$\theta(z)\,(\theta(F(x))o\,\theta(y))\,d(F(x))-\theta(z)\theta(y)\,[\theta(F(x)),d(F(x))]=0$, for all $x,y,z\in R$ (6)

From (5) and (6), one obtains:
$[\theta(F(x)),\theta(z)]\;\theta(y)d\,(F(x))=0$, for all $x,y,z\in R$ (7)

By primeness of, we obtain:
either, $[\theta(F(x)),\theta(z)]=0$, for all $x,z\in R$ and hence $\theta(F(R))\subseteq Z(R)$.
or, $d\,(F(x))=0$, for all $x\in R$, and since F is onto, we get $d=0$.

Theorem 2.5: Let R be a prime ring. If R admits a nonzero generalized (θ,θ)-derivations F and G associated with the same (θ,θ)-derivation d, where θ is an automorphism of R, such that
$\theta\big(F(x)\big)\ o\ G(y)=\theta(x)\ o\ \theta(y)$, for all $x,y\in R$, then either $d=0$ or $\theta(F(R))\subseteq Z(R)$.

Proof: we have

$$\theta\left(F(x)\right) o\ G(y)=\theta(x)\ o\ \theta(y),\ \text{for all } x,y\in R \quad (1)$$

Replacing y by yz in (1), we get:

$$(\theta(\mathrm{F(x)})\ o\ \mathrm{G(y)})\ \theta(\mathrm{z}) - \mathrm{G(y)}[\theta(F(x)),\theta(z)] + \big(\theta\big(F(x)\big)\ o\ \theta(y)\big)d(z)-\theta(y)\left[\theta\big(F(x)\big),d(z)\right]$$
$$=\big(\theta(x)\ o\ \theta(y)\big)\theta(z)-\theta(y)[\theta(x),\theta(z)],\ \text{for all } \mathrm{x,y,z}\in\mathrm{R} \quad (2)$$

Combining (1) and (2), we get:

$$-G(y)\left[\theta\big(F(x)\big),\theta(z)\right]+\Big(\theta\big(F(x)\big)o\ \theta(y)\Big)\ d(z)-\theta(y)\left[\theta\big(F(x)\big),d(z)\right]\theta(y)[\theta(x),\theta(z)]=0,\ \text{for all } x,y,z\in R \quad (3)$$

Replacing z by $F(x)$ in (3), we get:

$$\Big(\theta\left(F(x)\right)o\ \theta(y)\Big)\ d\left(F(x)\right)-\theta(y)\left[\theta\left(F(x)\right),d(F(x))\right]+\theta(y)\ [\theta(x),\ \theta\left(F(x)\right)]=0,\ \text{for all } x,y,z\in R \quad (4)$$

Again, replacing y by ry in (4), we obtain:

$$\Big(\theta(r)\ (\theta(F(x))o\ \theta(y))+[\theta\ (F(x)),\theta(r)]\theta(y)\Big)d\left(F(x)\right)-\theta(r)\ \theta(y)$$
$$[\theta\big(F(x)\big),\mathrm{d(F(x)}]+\theta(r)\theta(y)\left[\theta(x),\theta\big(F(x)\big)\right]=0,\ \textit{for all}\ x,\mathrm{y},\mathrm{r}\in\mathrm{R} \quad (5)$$

Left multiplication of (4) by $\theta(r)$, to get:

$$\theta(r)\Big(\theta\big(F(x)\big)o\ \theta(y)\Big)d\left(F(x)\right)-\theta(r)\ \theta(y)\left[\theta\big(F(x)\big),d\left(F(x)\right)\right]+\theta(r)\ \theta(y)\left[\theta(x),\theta\big(F(x)\big)\right]=0,$$
for all $x,y,r\in R$ (6)

From (5) and (6), we obtain:

$$\left[\theta\big(F(x)\big),\theta(r)\right]\theta(y)d\left(F(x)\right)=0,\ \text{for all } x,y,r\in R \quad (7)$$

Since R is prime, we get:
either, $\left[\theta\big(F(x)\big),\theta(r)\ \right]=0$, for all $x,r\in R$ and

Hence $\theta\big(F(R)\big)\subseteq Z(R)$.
Or, $d\left(F(x)\right)=0$, for all $x\in R$, and since F is onto, we get $d=0$,

Theorem 2.6: Let R be a prime ring. If R admits a nonzero generalized (θ,θ)-derivations F and G associated with the same (θ,θ)-derivation d, where θ is an automorphism of R, such that $\left[\theta\big(F(x)\big),G(y)\right]$=$[\theta(x),\theta(y)]$, for all $x,y\in R$, then either $d=0$ or $\theta(F(R))\subseteq Z(R)$.

Proof: we have

$$\left[\theta\big(F(x)\big),G(y)\right]=[\theta(x),\theta(y)],\ \text{for all } x,y\in R \quad (1)$$

Replacing y by yz in (1), we get:

$$G(y)\left[\theta\left(F(x)\right),\theta(z)\ \right]+\left[\theta\left(F(x)\right),G(y)\right]\theta(z)+\theta(y)\left[\theta\big(F(x)\big),d(z)\right]+\left[\theta\big(F(x)\big),\theta(y)\right]d(z)$$
$$=\theta(y)\ [\theta(x),\theta(z)]+[\theta(x),\theta(y)]\theta(z),\ \text{for all } x,y,z\in R \quad (2)$$

Combining (1) with (2), we get:

$$G(y)\left[\theta\left(F(x)\right),\theta(z)\right]+\theta(y)\left[\theta\left(F(x)\right),d(z)\right]+\left[\theta\left(F(x)\right),\theta(y)\right]d(z)-\theta(y)[\theta(x),\theta(z)\ ]=0,$$
for all $x,y,z\in R$ (3)

Replacing z by $F(x)$ in (3), we get:

$$\theta(y)\left[\theta\left(F(x)\right),d\big(F(x)\big)\right]+\left[\theta\left(F(x)\right),\theta(y)\right]d\big(F(x)\big)-\theta(y)[\theta(x),\theta\left(F(x)\right)]=0,\ \text{for all } x,y\in R \quad (4)$$

Again, replace y by ty in (4), we obtain:

$$\theta(t)\theta(y)\left[\theta\left(F(x)\right),d\big(F(x)\big)\right]+\theta(t)\left[\theta\left(F(x)\right),\theta(y)\right]d\big(F(x)\big)+\left[\theta\left(F(x)\right),\theta(t)\right]\theta(y)\ d\big(F(x)\big)$$
$$-\theta(t)\theta(y)\left[\theta\ (x),\theta\big(F(x)\big)\right]=0,\ \text{for all } x,y,t\in R \quad (5)$$

Left multiplication of (4) by $\theta(t)$, to get:
$$\theta(t)\theta(y)\big[\theta\left(F(x)\right), d\big(F(x)\big)\big] + \theta(t)\big[\theta\left(F(x)\right), \theta(y)\big]d\big(F(x)\big) - \theta(t)\theta(y)\big[\theta(x), \theta\left(F(x)\right)\big] = 0, \text{ for all } x, y, t \in R \quad (6)$$

From (5) and (6), one obtains:
$$\big[\theta\left(F(x)\right), \theta(t)\big]\, \theta(y) d\big(F(x)\big) = 0, \text{ for all } x, y, t \in R \quad (7)$$

By primeness of R, we obtain:
either, $\big[\theta\left(F(x)\right), \theta(t)\big] = 0$, for all $x, t \in R$ and

Hence $\theta\left(F(R)\right) \subseteq Z(R)$
or, $d\big(F(x)\big) = 0$, and since F is onto, we get $d = 0$.

Theorem 2.7: Let R be a prime ring. If R admits a nonzero generalized (θ,θ)-derivations F and G associated with the same (θ,θ)-derivation d, where θ is an automorphism of R, such that $\big[\theta\big(F(x)\big), G(y)\big] = \theta(x)\, o\, \theta(y)$, for all $x, y \in R$, then either $d = 0$ or $\theta(F(R)) \subseteq Z(R)$.

Proof: we have
$$\big[\theta\big(F(x)\big), G(y)\big] = \theta(x) o\, \theta(y), \text{ for all } x, y \in R \quad (1)$$

Replacing y by yz in (1), we get:
$$\big[\theta\left(F(x)\right), G(y)\big]\theta(z) + G(y)\big[\theta\big(F(x)\big), \theta(z)\big] + \big[\theta\big(F(x)\big), \theta(y)\big]d(z) + \theta(y)\big[\theta\big(F(x)\big), d(z)\big]$$
$$= \big(\theta(x) o\, \theta(y)\big)\theta(z) - \theta(y)[\theta(x), \theta(z)], \text{ for all } x, y, z \in R \quad (2)$$

Combining (1) with (2), we get:
$$G(y)\big[\theta\left(F(x)\right), \theta(z)\big] + \big[\theta\big(F(x)\big), \theta(y)\big]\, d(z) + \theta(y)\big[\theta\big(F(x)\big), d(z)\big] + \theta(y)[\theta(x), \theta(z)] = 0, \text{ for all } x, y, z \in R \quad (3)$$

Replace z by $z\, F(x)$ in (3), we get:
$$G(y)\big[\theta\big(F(x)\big), \theta(z)\big]\, \theta\big(F(x)\big) + \big[\theta\big(F(x)\big), \theta(y)\big]\, d(z)\, \theta\big(F(x)\big)$$
$$+\big[\theta\big(F(x)\big), \theta(y)\big]\theta(z)\, d\big(F(x)\big) + \theta(y)\big[\theta\big(F(x)\big), d(z)\big]\, \theta\big(F(x)\big)$$
$$+ \theta(y)\theta(z)\big[\theta\big(F(x)\big), d\big(F(x)\big)\big] + \theta(y)\big[\theta\big(F(x)\big), \theta(z)\big]d\big(F(x)\big)$$
$$+ \theta(y)\theta(z)\big[\theta(x), \theta\big(F(x)\big)\big] + \theta(y)[\theta(x), \theta(z)]\, \theta\big(F(x)\big) = 0, \text{ for all } x, y, z \in R \quad (4)$$

Right multiplication of (3) by $\theta\big(F(x)\big)$, to get:
$$G(y)\big[\theta\big(F(x)\big), \theta(z)\big]\, \theta\big(F(x)\big) + \big[\theta\big(F(x)\big), \theta(y)\big]\, d(z)\, \theta\big(F(x)\big)$$
$$+ \theta(y)\big[\theta\big(F(x)\big), d(z)\big]\, \theta\big(F(x)\big) + \theta(y)[\theta(x), \theta(z)]\, \theta\big(F(x)\big) = 0, \text{ for all } x, y, z \in R \quad (5)$$

From (4) and (5), one obtains:
$$\big[\theta\big(F(x)\big), \theta(y)\big]\, \theta(z)\, d\big(F(x)\big) + \theta(y)\theta(z)\, \big[\theta\big(F(x)\big), d\big(F(x)\big)\big]$$
$$+\theta(y)\big[\theta\big(F(x)\big), \theta(z)\big]\, d\big(F(x)\big) + \theta(y)\theta(z)\big[\theta(x), \theta\big(F(x)\big)\big] = 0, \text{ for all } x, y, z \in R \quad (6)$$

Now, replace y by ry in (6), we get: $\theta(r)\big[\theta\big(F(x)\big), \theta(y)\big]\, \theta(z)\, d\big(F(x)\big) + \big[\theta\big(F(x)\big), \theta(r)\big]\, \theta(y)\theta(z)$
$d\big(F(x)\big) + \theta(r)\theta(y)\theta(z)\big[\theta\big(F(x)\big), d\big(F(x)\big)\big] + \theta(r)\, \theta(y)\big[\theta\big(F(x)\big), \theta(z)\big]$
$d\big(F(x)\big) + \theta(r)\theta(y)\theta(z)\big[\theta(x), \theta\big(F(x)\big)\big] = 0$, for all $x, y, r, z \in R$ (7)

Left multiplication of (6) by $\theta(r)$, to get:
$$\theta(r)\big[\theta\big(F(x)\big), \theta(y)\big]\, \theta(z)\, d\big(F(x)\big) + \theta(r)\theta(y)\theta(z)\big[\theta\big(F(x)\big), d\big(F(x)\big)\big]$$
$$+\theta(r)\theta(y)\big[\theta\big(F(x)\big), \theta(z)\big]\, d\big(F(x)\big) + \theta(r)\theta(y)\theta(z)\big[\theta(x), \theta\big(F(x)\big)\big] = 0 \text{ for all } x, y, r, z \in R \quad (8)$$

From (7) and (8), we get:
$$\big[\theta\big(F(x)\big), \theta(r)\big]\theta(y)\theta(z)d\big(F(x)\big) = 0, \text{ for all } x, y, r, z \in R \quad (9)$$

By primeness of R , (9) gives
either, $\big[\theta\big(F(x)\big), \theta(r)\big] = 0$, for all $x, r \in R$ and thus $\theta\big(F(R)\big) \subseteq Z(R)$
or $d\big(F(x)\big) = 0$, for all $x \in R$ and since F is onto, we get $d = 0$.

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Source of support: Nil, Conflict of interest: None Declared