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BALANCED DOMINATION NUMBER OF SOME GRAPHS

[1]S. CHRISTILDA*, [2]P. NAMASIVAYAM
[1]Department of Mathematics,
Sadakathullah Appa college, Tirunelveli – 627011, Tamil Nadu, India.

[2]PG and Research Department of Mathematics,
The MDT Hindu College, Tirunelveli – 627010, Tamil Nadu, India.

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ABSTRACT

Let G=(V,E) be a graph. A Subset D of V is called a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. The Domination number γ(G) of G is the cardinality of the minimum dominating set of G. Let G = (V, E) be a graph and let f be a function that assigns to each vertex of V to a set of values from the set {1,2,.......k} that is, f: V(G) → {1,2,.....k} such that for each u, v ∈ V(G), f(u) ≠ f(v), if u is adjacent to v in G. Then the dominating set D ⊆ V(G) is called a balanced dominating set if $\sum_{u\epsilon D} f(u) = \sum_{v\epsilon V-D} f(v)$. *In this paper, we determine the balanced domination number for complete graph, complete bipartite graph and wheels.*

***Keywords**: Balanced domination, Bipartite, Complete, Independent.*

***Mathematics Subject Classification**: 05C69.*

1. BALANCED DOMINATION

Let G = (V, E) be a graph and let f be a function that assigns to each vertex of V to a set of values from the set {1,2,.......k} that is, f:V(G) → {1,2,.....k} such that for each u, v ϵ V(G), f(u)≠f(v), if u is adjacent to v in G. Then the set D ⊆ V (G) is called a balanced dominating set if $\sum_{u\epsilon D} f(u) = \sum_{v\epsilon V-D} f(v)$

The balanced domination number $\gamma_{bd}(G)$ is the minimum cardinality of the balanced dominating set.

The set D ⊆V (G) is called strong balanced dominating set if $\sum_{u\epsilon D} f(u) \geq \sum_{v\epsilon V-D} f(v)$. Also the set D ⊆ V (G) is called weak balanced dominating set if $\sum_{u\epsilon D} f(u) \leq \sum_{v\epsilon V-D} f(v)$.

The sum of the values assigned to each vertex of G is called the total value of G.

Hence Total value = f (V) =$\sum_{v\epsilon V(G)} f(v)$.

Theorem 1.1: Let G be a graph with n vertices. Then G has a balanced dominating set iff f (V) = $\sum_{v\epsilon V(G)} f(v)$ is even.

Proved in [6].

Theorem 1.2: Let G be a graph with n vertices. Then G has no balanced dominating set iff f (V) = $\sum_{v\epsilon V(G)} f(v)$ is odd.

Proved in [6].

Note: Since we divide the graph G into 2 sets of vertices having equal values, we get two balanced dominating set for every graph G.

Theorem 1.3: For a graph G, 0≤ $\gamma_{bd}(G) \leq \frac{n}{2}$.

Proof:
Case (i): if f (V) is odd, then G has no balanced dominating set.

Corresponding Author: [1]S. Christilda, [1]Department of Mathematics,
Sadakathullah Appa college, Tirunelveli – 627011, Tamil Nadu, India.*

Therefore, $\gamma_{bd}(G) = 0$.

Case (ii): if f(V) is even, then G has balanced dominating set.

Also every graph G has 2 balanced dominating sets say D_1 and D_2.
$|D_1| + |D_2| = n$

If $|D_1| = |D_2|$, we get $2|D_1| = n$, $|D_1| = \frac{n}{2}$

Therefore, $\gamma_{bd}(G) = \frac{n}{2}$.

If $|D_1| > |D_2|$, then D_2 is the minimal balanced dominating set.

If $|D_1| = |D_2|$, we get $|D_1| = \frac{n}{2}$

Since D_2 is minimal, $|D_2| < \frac{n}{2}$, therefore $\gamma_{bd}(G) = |D_2| < \frac{n}{2}$.

If $|D_1| < |D_2|$, then D_1 is the minimal balanced dominating set.

Since D_1 is minimal, $|D_1| < \frac{n}{2}$, therefore $\gamma_{bd}(G) = |D_1| < \frac{n}{2}$.

In three cases, we get $\gamma_{bd}(G) \leq \frac{n}{2}$. Hence the theorem.

2. BALANCED DOMINATION NUMBER OF COMPLETE GRAPH

Complete graph	Labeling of vertices	γ_{bd}
K_2	**{1,2}**	1
K_3	**{1,2,3}**	1
K_4	**{1,2,3,4}**	2
K_5	{1,2,3,4,5}	0
K_6	{1,2,3,4,5,6}	0
K_7	**{1,2,3,4,5,6,7}**	3
K_8	**{1,2,3,4,5,6,7,8 }**	3
K_9	{1,2, 3,4,5,6,7,8 ,9}	0
K_{10}	{1,2, 3,4,5,6,7,8 ,9,10}	0
K_{11}	**{1,2, 3,4,5,6,7,8 ,9,10,11}**	4
K_{12}	**{1,2, 3,4,5,6,7,8 ,9,10,11,12}**	4
K_{13}	{1,2, 3,4,5,6,7,8 ,9,10,11,12,13}	0
K_{14}	{1,2, 3,4,5,6,7,8 ,9,10,11,12,13,14}	0
K_{15}	**{1,2,3,4,5,6,7,8 ,9,10,11,12,13,14,15}**	5
K_{16}	**{1,2,3,4,5,6,7,8 ,9,10,11,12,13,14,15,16}**	5
K_{17}	{1,2, 3,4,5,6,7,8 ,9,10,11,12,13,14,15,16,17}	0
K_{18}	{1,2, 3,4,5,6,7,8 ,9,10,11,12,13,14,15,16,17,18}	0
K_{19}	**{1,2,3,4,5,6,7,8 ,9,10,11,12,13,14,15,16,17,18,19}**	6
K_{20}	**{1,2,3,4,5,6,7,8 ,9,10,11,12,13,14,15,16,17,18,19,20}**	6

Table-2.1.1

Theorem 2.1: For a complete graph G with n vertices, if $\sum_{v \epsilon V(G)} f(v)$ is even then $\sum_{u \epsilon D} f(u) = \frac{n(n+1)}{4}$.

Proved in [6].

Result 2.2: For complete graphs K_{2n+1} and K_{2n+2} (n= 2,4,6,8,……..) , $\gamma_{bd} = 0$.

3. BALANCED DOMINATION NUMBER OF COMPLETE BIPARTITE GRAPH

The complete bipartite graphs can be partitioned into 2 sets of non-adjacent vertices, so we can assign values to vertices of each partition by one value. That is, we have the values {1, 2} and there are exactly 2 possible labeling of vertices.

But we get a balanced dominating set for complete bipartite graph only if $f(V) = \sum_{v \epsilon V(G)} f(v)$ is even.

Complete bipartite graph	Labeling of vertices	γ_{bd}
$K_{1,1}$	{1,2}	0
$K_{1,2}$	L_1: {1,2,2} L_2: **{1,1,2}**	1
$K_{1,3}$	L_1: {1,2,2,2} L_2: {1,1,1,2}	0
:$K_{1,4}$	L_1: {1,2,2,2,2} L_2: **{1,1,1,1,2}**	2
$K_{2,1}$	L_1: **{1,1,2}** L_2: {1,2,2}	1
$K_{2,2}$	L:**{1,1,2,2}**	2
$K_{2,3}$	L_1: **{1,1,2,2,2}** L_2: {1,1,1,2,2}	2
$K_{3,3}$	L:{1,1,1,2,2,2}	0
$K_{3,4}$	L_1: {1,1,1,2,2,2,2} L_2: **{1,1,1,1,2,2,2}**	3
$K_{4,2}$	L_1: {1,1,1,1,2,2} L_2: **{1,1,2,2,2,2}**	2
$K_{4,4}$	L:**{1,1,1,1,2,2,2,2}**	3
$K_{5,1}$	L_1: {1,1,1,1,1,2} L_2: {1,2,2,2,2,2}	0
$K_{5,2}$	L_1: **{1,1,2,2,2,2,2}** L_2: {1,1,1,1,1,2,2}	3
$K_{5,3}$	L_1: {1,1,1,1,1,2,2,2} L_2: {1,1,1,2,2,2,2,2}	0

Table-3.1.1

Theorem 3.2: Let G be a complete bipartite graph $K_{m,n}$ (m, n≥1), Then G has balanced dominating set if

i) m is odd & n is even

ii) m is even & n is odd

iii) both m and n are even.

Proof: Let G be a complete bipartite graph $K_{m,n}$.

i) m is odd & n is even

For a complete bipartite graph, f(u),(u∈ V(G)) must be 1 or 2.

Therefore, there must be m 1's and n 2's (or) n 1's and m 2's.

Therefore, f(V) = $\sum_{v\epsilon V(G)} f(v)$ = n 1's + m 2's
= even + even =even

Therefore, f(V) = $\sum_{v\epsilon V(G)} f(v)$ is even.

By theorem 1.1, G has balanced dominating set.

ii) m is even & n is odd

For a complete bipartite graph, f (u), (u∈ V (G)) must be 1 or 2.

Therefore, there must be m 1's and n 2's (or) n 1's and m 2's.

Therefore, f (V) = $\sum_{v\epsilon V(G)} f(v)$ = m 1's + n 2's
= even + even =even

Therefore, f (V) = $\sum_{v\epsilon V(G)} f(v)$ is even.

By theorem 1.1, G has balanced dominating set.

iii) both m and n are even

For a complete bipartite graph, f (u), (u∈ V (G)) must be 1 or 2.

Therefore, there must be m 1's and n 2's (or) n 1's and m 2's.

Therefore, f (V) = $\sum_{v\epsilon V(G)} f(v)$ = m 1's + n 2's (or) n 1's + m 2's
= even + even =even

Therefore, f (V) = $\sum_{v\epsilon V(G)} f(v)$ is even.

By theorem 1.1, G has balanced dominating set.

Theorem 3.3: Let G be a complete bipartite graph $K_{m,\, n}$ (m, n≥1), Then G has no balanced dominating set if both m and n are odd.

Proof: Let G be a complete bipartite graph $K_{m,n}$.

Let both m and n be odd.

For a complete bipartite graph, f(u),(u∈ V(G)) must be 1 or 2.Therefore,there must be m 1's and n 2's (or) n 1's and m 2's.

We know that odd number of 1's gives odd number and any number of 2's must be even.

Therefore, f(V) = $\sum_{v\epsilon V(G)} f(v)$ = m 1's + n 2's (or) n 1's + m 2's
= odd + even =odd
Therefore, f(V) = $\sum_{v\epsilon V(G)} f(v)$ is odd.

By theorem 1.2, G has no balanced dominating set.

4. WHEELS

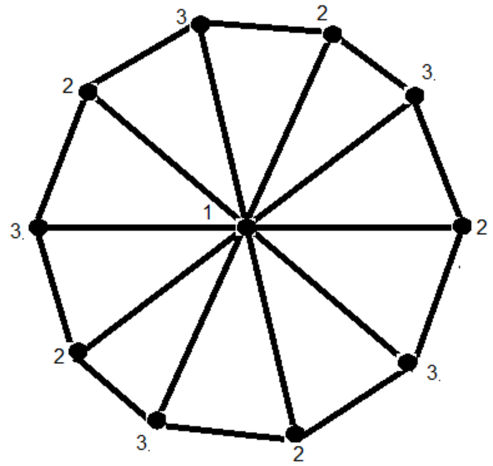
A Wheel on n vertices W_n is a graph with n vertices x_1, x_2,…….x_n with x_1 having degree n-1 and all the other vertices having degree 3.

Wheel graph	Labeling of vertices	γ_{bd}
W_3	**{1,2,3}**	1
W_4	**{1,2,3,4}**	2
W_5	L_1:{1,2,2,3,3}	2
	L_2:**{1,1,2,3,3}**	
	L_3:{1,1,2,2,3}	
W_6	L_1:{1,2,2,3,3,4}	2
	L_2:{1,2,3,3,4,4}	
	L_3:{1,1,2,3,4,4}	
	L_4:**{1,2,2,3,4,4}**	
W_7	L_1:**{1,2,2,2,3,3,3}**	3
	L_2:**{1,1,1,2,3,3,3}**	
	L_3:**{1,1,1,2,2,2,3}**	
W_8	L_1:**{1,2,2,2,3,3,3,4}**	3
	L_2:**{1,2,3,3,3,4,4,4}**	
	L_3:**{1,2,2,2,3,4,4,4}**	
	L_4:**{1,1,1,2,3,4,4,4}**	
	L_5:**{1,1,1,2,3,3,3,4}**	
	L_6:**{1,1,1,2,2,2,3,4}**	
W_9	L_1:{1,2,2,2,2,3,3,3,3}	4
	L_2: **{1,1,1,1,2,3,3,3,3}**	
	L_3:{1,1,1,1,2,2,2,2,3}	
W_{10}	L_1:{1,2,2,2,2,3,3,3,3,4}	4
	L_2: {1,1,1,1,2,2,2,2,3,4}	
	L_3:**{1,1,1,1,2,3,3,3,3,4}**	
	L_4:{1,2,3,3,3,3,4,4,4,4}	
	L_5: **{1,2,2,2,2,3,4,4,4,4}**	
	L_6:{1,1,1,1,2,3,4,4,4,4}	
W_{11}	L_1:**{1,2,2,2,2,2,3,3,3,3,3}**	5

Table-4.1.1

Result 4.2: For Wheel graph W_n, $\gamma_{bd}(G) = \frac{\Delta}{2}$ if n is odd.

Example 4.3: Consider the wheel graph W_{11}(n is odd)



$$D = \{3, 3, 3, 3, 1\}$$
$$\sum_{u \epsilon D} f(u) = 13 = \sum_{v \epsilon V-D} f(v)$$
$$\gamma_{bd}(W_{11}) = 5 = \frac{\Delta}{2}.$$

5. INDEPENDENT BALANCED DOMINATION

A set S of vertices in a graph G is a independent balanced dominating set if S is a balanced dominating set and the set of vertices S is independent.

The independent balanced domination number $\gamma_{ibd}(G)$ is the minimum cardinality of the independent balanced dominating set.

Theorem 5.1: Let G be a complete bipartite graph $K_{m,\,n}$ (m>n), then G has two independent balanced dominating sets if m =2n.

Proof: Let G be a complete bipartite graph. G can be partitioned into 2 sets S_1 and S_2 with $|S_1|$=m, $|S_2|$=n & each set of vertices have labeling 1 and 2.

Also S_1 and S_2 are independent.

If m=2n, give the labeling 1 to each of vertices of S_2 and 2 to each of vertices of S_1.

Therefore, we get $\sum_{u\epsilon S1} f(u) = \sum_{v\epsilon S2} f(v)$ and both the set S_1 and S_2 are independent.

Therefore G has two independent balanced dominating sets.

Theorem 5.2: Let G be a complete bipartite graph $K_{m,n}$ and if m =2n then $\gamma_{ibd}(G)$ = n.

Proof: Let m =2n.

By theorem 5.1, G has 2 independent balanced dominating set S_1 and S_2. And $|S_1|$=m, $|S_2|$= n .

Since $\gamma_{ibd}(G)$ is the minimum cardinality of the independent balanced dominating set,
$\gamma_{ibd}(G)$ = min{m ,n}.

Since m>n, $\gamma_{ibd}(G)$ = n.

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