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WEAK AND SEMI COMPATIBLE MAPS IN FUZZY PROBABILISTIC METRIC SPACE USING IMPLICIT RELATION

Rajesh Shrivastav, Satyendra Nath , Vivek Patel and vanita Ben Dhagat*

Benajeer Govt. College, Bhopal (M.P.)

E-mail: vanita1_dhagat@yahoo.co.in

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ABSTRACT

The concept fuzzy probabilistic metric space is given and proves the existence of unique common fixed point of four self-maps with weak compatibility semi compatibility satisfying an implicit relation. At the end we provide examples in support of the results.

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Key words and Phrases: *Fuzzy probabilistic space, Weak compatible mapping, Semi-compatible mapping, Implicit function, common fixed point.*

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1. INTRODUCTION:

Menger [2] in 1942 introduced the notation of the probabilistic metric space. The probabilistic generalization of metric space appears to be well adopted for the investigation of physical quantities and physiological thresholds.

Cho et al. [1] introduced the notation of semi compatible maps in a- topological space. According to them a pair of self-maps (S, T) to be semi compatible if condition (i) $Sy = Ty \Rightarrow STy = TSy$; (ii) the sequence $\{x_n\}$ in X and $x \in X$, $\{Sx_n\} \to x$, $\{Tx_n\} \to x$ then $STx_n = Tx$ as $n \to \infty$, hold. We define semi compatible self-maps in probabilistic metric space by (ii) only. Popa in [3] used the family Φ of implicit function to find the fixed points of two pairs of semi compatible maps in a d complete topological space, where Φ be the family of real continuous function $\phi : (R^+)^4 \to R$ satisfying the properties

(G_h) for every $u \geq 0$, $v \geq 0$ with $\phi(u,v,u,v) \geq 0$ or $\phi(u,v,v,u) \geq 0$ we have $u \geq v$.

(G_u) $\phi(u,u,1,1) \geq 0$ implies that $u \geq 1$

The main object of this paper is to define Fuzzy probabilistic metric space and prove fixed point theorem in the setting of fuzzy probabilistic metric space using weak compatibility, semi compatibility and an implicit relation. At the end examples in support of the results.

2. PRELIMINARIES:

Let us define and recall some definitions:

Definition: 2.1 A fuzzy probabilistic metric space (FPM space) is an ordered pair (X,F_α) consisting of a nonempty set X and a mapping F_α from XxX into the collections of all distribution functions $F_\alpha \in R$ for all $\alpha \in [0,1]$. For $x, y \in X$ we denote the distribution function $F_\alpha(x,y)$ by $F_{\alpha(x,y)}$ and $F_{\alpha(x,y)}(u)$ is the value of $F_{\alpha(x,y)}$ at u in R.
The functions $F_{\alpha(x,y)}$ for all $\alpha \in [0,1]$ assumed to satisfy the following conditions:

(a) $F_{\alpha(x,y)}(u) = 1\ \forall\ u > 0$ iff $x = y$,
(b) $F_{\alpha(x,y)}(0) = 0\ \forall\ x, y$ in X,
(c) $F_{\alpha(x,y)} = F_{\alpha(y,x)}\ \forall\ x, y$ in X,
(d) If $F_{\alpha(x,y)}(u) = 1$ and $F_{\alpha(y,z)}(v) = 1$ then $F_{\alpha(x,z)}(u+v) = 1\ \forall\ x, y, z$ in X and $u, v > 0$

***Corresponding author: vanita Ben Dhagat*, *E-mail: vanita1_dhagat@yahoo.co.in**
Benajeer Govt. College, Bhopal (M.P.)

Definition: 2.2 A commutative, associative and non-decreasing mapping t: $[0,1] \times [0,1] \to [0,1]$ is a t-norm if and only if $t(a,1)=a$ for all $a \in [0,1]$, $t(0,0)=0$ and $t(c,d) \geq t(a,b)$ for $c \geq a$, $d \geq b$]

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Definition: 2.3 A Fuzzy Menger space is a triplet (X,F_α,t), where (X,F_α) is a FPM-space, t is a t-norm and the generalized triangle inequality

$$F_{\alpha(x,z)}(u+v) \geq t\,(F_{\alpha(x,z)}(u), F_{\alpha(y,z)}(v)) \quad \text{holds for all x, y, z in X u, v > 0 and } \alpha. \in [0,1]$$

The concept of neighborhoods in Fuzzy Menger space is introduced as

Definition: 2.4 Let (X,F_α,t) be a Fuzzy Menger space. If $x \in X$, $\varepsilon > 0$ and $\lambda \in (0,1)$, then (ε,λ) - neighborhood of x, called $U_x(\varepsilon,\lambda)$, is defined by

$$U_x(\varepsilon,\lambda) = \{y \in X: F_{\alpha(x,y)}(\varepsilon) > (1-\lambda)\}$$

An (ε,λ)-topology in X is the topology induced by the family $\{U_x(\varepsilon,\lambda): x \in X, \varepsilon > 0, \alpha. \in [0,1] \text{and } \lambda \in (0,1)\}$ of neighborhood.

Remark: If t is continuous, then Fuzzy Menger space (X,F_α,t) is a Housdroff space in (ε,λ)-topology.

Let (X,F_α,t) be a complete Fuzzy Menger space and $A \subset X$. Then A is called a bounded set if

$$\lim_{u \to \infty} \inf_{x,y \in A} F_{\alpha(x,y)}(u) = 1$$

Definition: 2.5 A sequence $\{x_n\}$ in (X,F_α,t) is said to be convergent to a point x in X if for every $\varepsilon>0$and $\lambda>0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $x_n \in U_x(\varepsilon,\lambda)$ for all $n \geq N$ or equivalently $F_\alpha(x_n, x; \varepsilon) > 1-\lambda$ for all $n \geq N$ and $\alpha \in [0,1]$.

Definition: 2.6 A sequence $\{x_n\}$ in (X,F_α, t) is said to be cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $F_\alpha(x_n,x_m; \varepsilon) > 1-\lambda \ \forall\, n, m \geq N$ for all $\alpha \in [0,1]$.

Definition: 2.7 A Fuzzy Menger space (X,F_α,t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all $\alpha \in [0,1]$.

Definition: 2.8 Let (X,F_α,t) be a Fuzzy Menger space. Two mappings f, g :$X \to X$ are said to be weakly comptable if they commute at coincidence point for all $\alpha \in [0,1]$.

Lemma: 1 Let $\{x_n\}$ be a sequence in a Fuzzy Menger space (X,F_α,t), where t is continuous and $t(p,p) \geq p$ for all $p \in [0,1]$, if there exists a constant k(0,1) such that for all $p > 0$ and $n \in N$

$$F_\alpha(x_n, x_{n+1}; kp) \geq F_\alpha(x_{n-1},x_n; p),$$

for all $\alpha \in [0,1]$ then $\{x_n\}$ is cauchy sequence.

Lemma: 2 If (X,d) is a metric space, then the metric d induces, a mapping F_α: $XxX \to L$ defined by $F_\alpha(p, q) = H_\alpha(x-d(p, q))$, $p, q \in R$ for all $\alpha \in [0,1]$. Further if t: $[0,1] \times [0,1] \to [0,1]$ is defined by $t(a,b) = \min\{a,b\}$, then (X,F_α,t) is a Fuzzy Menger space. It is complete if (X,d) is complete.

3. MAIN RESULTS:

Theorem 3.1 Let (X,F_α,t) be a complete Fuzzy Menger space, where t is continuous and $t(p,p) \geq p$ for all p and α in [0,1]. Let A, B, S and T be self mappings from X into itself such that

(i) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$;
(ii) the pair (A,S) is semi compatible and (B,T) is weak compatible;
(iii) one of A or S is continuous; for some $\phi \in \Phi$, there exist $k \in (0,1)$ such that for all $x, y \in X$ and $p>0$
(iv) $\phi(t(F_\alpha(Ax,By,kp)), t(F_\alpha(Sx,Ty,p)), t(F_\alpha(Ax,Sx,p)), t(F_\alpha(By,Ty,kp))) \geq 0$;
(v) $\phi(t(F_\alpha(Ax,By,kp)), t(F_\alpha(Sx,Ty,p)), t(F_\alpha(Ax,Sx,kp)), t(F_\alpha(By,Ty,p))) \geq 0$

then A, B, S and T have unique common fixed point in X.

Proof: Let x_0 be any arbitrary point of X, as $A(X)\subseteq T(X)$ and $B(X)\subseteq S(X)$ there exists x_1,x_2 in X such that $Ax_0=Tx_1$, $Bx_1=Sx_2$. Inductively, construct sequences $\{y_n\}$, and $\{x_n\}$ in X such that $y_{2n+1} = Ax_{2n} = Tx_{2n+1}$, $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$, for n = 0,1,2,….. .

Now by (iv)

$\phi(t(F_\alpha(Ax_{2n},Bx_{2n+1},kp)), t(F_\alpha(Sx_{2n},Tx_{2n+1},p)), t(F_\alpha(Ax_{2n},Sx_{2n},p)), t(F_\alpha(Bx_{2n+1},Tx_{2n+1},kp))) \geq 0$

$\Rightarrow \phi(t(F_\alpha(y_{2n+1}, y_{2n+2},kp)), t(F_\alpha(y_{2n}, y_{2n+1},p)), t(F_\alpha(y_{2n+1}, y_{2n},p)), t(F_\alpha(y_{2n+2}, y_{2n+1},kp))) \geq 0$

By (G_h)

$t(F_\alpha(y_{2n+2}, y_{2n+1},kp)) \geq t(F_\alpha(y_{2n+1}, y_{2n},p))$

$\Rightarrow F_\alpha(y_{2n+2}, y_{2n+1},kp) \geq F_\alpha(y_{2n+1}, y_{2n},p)$

Again putting $x = x_{2n+2}$ and $y = x_{2n+1}$ in (V), we have

$\phi(t(F_\alpha(y_{2n+3}, y_{2n+2}, kp)), t(F_\alpha(y_{2n+1}, y_{2n+2},p)), t(F_\alpha(y_{2n+3}, y_{2n+2}, kp)), t(F_\alpha (y_{2n+1}, y_{2n+2}, p))) \geq 0$

By (G_h)

$F_\alpha (y_{2n+3}, y_{2n+2}, kp) \geq F_\alpha (y_{2n+2}, y_{2n+1}, p)$

Hence by Lemma 1, $\{y_n\}$ is cauchy sequence in X. Therefore $\{y_n\}$ converge to u in X. Therefore its subsequences $\{Ax_{2n}\}$, $\{Tx_{2n+1}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n+2}\}$ also converge to u.

Case: 1 If S is continuous, we have

$SAx_{2n} \to Su$, $SSx_{2n} \to Su$

So, weak compatibility of the pair (A,S) gives $ASx_{2n} \to Su$ as $n \to \infty$

Step :(i) By putting $x = Sx_{2n}$, $y = x_{2n+1}$ in (IV) , we obtain that

$\phi(t(F_\alpha (ASx_{2n}, B x_{2n+1}, kp)), t(F_\alpha(SSx_{2n}, T x_{2n+1}, p)), t(F_\alpha(ASx_{2n}, SSx_{2n}, p)), t(F_\alpha(B x_{2n+1}, T x_{2n+1}, kp))) \geq 0$

Now letting $n \to \infty$ and by the continuity of the t- norm, we have

$\phi(t(F_\alpha (Su, u, kp)), t(F_\alpha (Su, u, p)), t(F_\alpha (Su, Su, p)), t(F_\alpha (u, u, kp))) \geq 0$

$\Rightarrow \quad \phi(t(F_\alpha (Su, u, kp)), t(F_\alpha (Su, u, p)), 1,1) \geq 0$

Now as ϕ is non-decreasing in the first argument, we have

$\Rightarrow \quad \phi(t(F_\alpha (Su, u, p)), t(F_\alpha (Su, u, p)), 1,1) \geq 0$

Using (Gu), we get $F_\alpha (Su, u, p) \geq 1$, for all $p>0$, which gives $F_\alpha(Su, u, p) =1$

$\Rightarrow Su = u$

Step: (ii) By putting x = u and $y = x_{2n+1}$ in (IV) , we obtain that

$\phi(t(F_\alpha (Au, Bx_{2n+1}, kp)), t(F_\alpha (Su, Tx_{2n+1},p)), t(F_\alpha (Au,Su,p)), t(F_\alpha (B x_{2n+1},T x_{2n+1},kp))) \geq 0$

On taking limit $n \to \infty$ and as Su = u & Bx_{2n+1},$T x_{2n+1} \to u$, we get

$$\phi(t(F_\alpha (Au, u, kp)), 1, t(F_\alpha (Au, u, p)), 1) \geq 0$$

Now as ϕ is non-decreasing in the first argument, we have

$$\phi(t(F_\alpha (Au, u, p)), 1, t(F_\alpha (Au, u, p)), 1) \geq 0$$

Using (G_h), we get F_α (Au, u, p) ≥ 1, for all p>0, which gives F_α (Au, u, p) =1,

⇒ Au = u = Su.

Step: (iii) By (I) A(X) ⊆ T(X), there exists w in X such that Au= u=Su =Tw.

By putting x = x_{2n} and y = w in (IV), we obtain that

$\phi(t(F_\alpha(Ax_{2n},Bw,kp)), t(F_\alpha(Sx_{2n},Tw,p)), t(F_\alpha(Ax_{2n},Sx_{2n},p)), t(F_\alpha(Bw,Tw,kp)))\geq 0$

On taking limit n → ∞ and as Ax_{2n}, Sx_{2n} → u, we get

$$\phi(t(F_\alpha(u, Bw, kp)), 1, 1, t(F_\alpha(Bw, u, kp))) \geq 0$$

By using (G_h), we get F_α (u,Bw,kp) ≥1, for all p > 0, which gives F_α (u,Bw,p) =1, that is, Bw = u.

Therefore Bw = Tw = u. Since (B,T) is weak compatible, we get TBw = BTw, it implies Bu = Tu.

Step: (iv) Now putting x = u and y = u in (IV) and as Au= u = Su & Bu = Tu,

We get that

$\phi(t(F_\alpha(Au, Bu, kp)), t(F_\alpha(Su, Tu, p)), t(F_\alpha(Au, Su, p)), t(F_\alpha(Bu, Tu, kp))) \geq 0$

$\phi(t(F_\alpha(Au, Bu, kp)), t(F_\alpha(Su, Tu, p)), 1, 1)) \geq 0$

Now as ϕ is non-decreasing in the first argument, we have

⇒ $\phi(t(F_\alpha(Au, Bu, p)), t(F_\alpha(Au, Bu, p)), 1,1) \geq 0$

Using (Gu), we get F_α (Au,Bu,p)≥ 1, for all p>0, which gives F_α (Au, Bu, p) =1, that is , Au = Bu. Thus u = Au = Su = Bu = Tu.

Case: 2 If A is continuous i.e. ASx_{2n} → Au. Also the pair (A,S) is semi-compatible, therefore ASx_{2n} → Su . By the uniqueness of the limit Au = Su.

Step (v) By putting x = u and y = x_{2n+1} in (IV), we get

$\phi(t(F_\alpha(Au, Bx_{2n+1}, kp)), t(F_\alpha(Su, Tx_{2n+1}, p)), t(F_\alpha(Au, Su, p)), t(F_\alpha(Bx_{2n+1},Tx_{2n+1}, kp))) \geq 0$

On taking limit n → ∞ and as Bx_{2n+1}, Tx_{2n+1} → u, we get

$$\phi(t(F_\alpha(Au, u, kp)), 1, t(F_\alpha(Au, u, p)), 1) \geq 0.$$

Now as ϕ is non-decreasing in the first argument, we have

$\phi(t(F_\alpha(Au, u, p)), 1, t(F_\alpha(Au, u, p)), 1) \geq 0.$

Using (G_h), we have F_α (Au, u, p) ≥ 1 for all p>0, which gives u = Au.

The rest of the proof follows from step (iii) onwards of the **case 1**.

UNIQUENESS OF COMMON FIXED POINT:

Let v be another common fixed point of A, S, B and T, then

v = Av = Sv = Bv = Tv. Now putting x = u and y = v in (IV), we get

$\phi(t(F_\alpha(Au, Bv, kp)), t(F_\alpha(Su, Tv, p)), t(F_\alpha(Au, Su, p)), t(F_\alpha(Bv, Tv, kp))) \geq 0$

⇒ $\phi(t(F_\alpha(u, v, kp)), t(F_\alpha(u, v, p)), t(F_\alpha(u, u, p)), t(F_\alpha(v, v, kp))) \geq 0$
⇒ $\phi(t(F_\alpha(u, v, kp)), t(F_\alpha(u, v, p)), 1, 1)) \geq 0$

Now as ϕ is non-decreasing in the first argument, we have

$\phi(t(F_\alpha(u, v, p)), t(F_\alpha(u, v, p)), 1, 1) \geq 0$

By Using (G_h), we have $F_\alpha(u, v, p) \geq 1$ for all $p>0$, which gives $u = v$.

Corollary: 3.2 Let $(X, F\alpha, t)$ be a complete Fuzzy Menger space, where t is continuous and $t(p,p) \geq p$ for all p and α in [0,1]. Let A, B, S and T be self mappings from X into itself such that

(I) $A(X) \subseteq T(X) \cap S(X)$;
(II) the pair (A,S) is semi compatible and (A,T) is weak compatible;
(III) one of A or S is continuous;
for some $\phi \in \Phi$, there exist $k \in (0,1)$ such that for all $x, y \in X$ and $p > 0$

(IV) $\phi(t(F_\alpha(Ax,Ay,kp)), t(F_\alpha(Sx,Ty,p)), t(F_\alpha(Ax,Sx,p)), t(F_\alpha(Ay,Ty,kp))) \geq 0$;
(V) $\phi(t(F_\alpha(Ax,Ay,kp)), t(F_\alpha(Sx,Ty,p)), t(F_\alpha(Ax,Sx,kp)), t(F_\alpha(Ay,Ty,p))) \geq 0$

then A, S and T have unique common fixed point in X.

Proof: Put B = A in Theorem 3.1

Corollary: 3.3 Let (X,F_α,t) be a complete Fuzzy Menger space, where t is continuous and $t(p,p) \geq p$ for all p and α in [0,1]. Let A, B, S and T be self mappings from X into itself such that

(I) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$;
(II) the pairs (A,S) and (A,T) are semi-compatible;
(III) One of A, B, T or S is continuous;

for some $\phi \in \Phi$, there exist $k \in (0,1)$ such that for all $x, y \in X$ and $p > 0$

(IV) $\phi(t(F_\alpha(Ax,By,kp)), t(F_\alpha(Sx,Ty,p)), t(F_\alpha(Ax,Sx,p)), t(F_\alpha(By,Ty,kp))) \geq 0$;
(V) $\phi(t(F_\alpha(Ax,By,kp)), t(F_\alpha(Sx,Ty,p)), t(F_\alpha(Ax,Sx,kp)), t(F_\alpha(By,Ty,p))) \geq 0$,

then A,B, S and T have unique common fixed point in X.

Proof: As semi-compatible mappings are weak compatible, the proof follows from Theorem 3.1.

4. Examples

4.1 Let X = [0,1] and metric d is defined by $d(x,y) = |x-y|$. For each p define

$$F_\alpha(x, y, p) = \begin{cases} 1. for\ x = y \\ H_\alpha(p). for\ x \neq y \end{cases}, \text{ where } H_\alpha(p) = \begin{cases} 0\ if\ \ p \leq 0 \\ p\alpha\ if\ \ 0 < p < 1 \\ 1\ if\ \ p \geq 1 \end{cases}.$$

Clearly, (X,F_α,t) is a complete fuzzy probabilistic space where t is defined by $t(p,p) \geq p$ for all p and $\alpha \in [0,1]$. The sequence $x_n = 1/n$. Let A,B, S and T are defined as $Ax = x/6$, $Tx = x$, $Bx = x/5$ and $Sx = x/2$. If k = 1 and p = 1and α =1 . So, we see the all conditions of **theorem 3.1** are satisfied and hence 0 is the common fixed point in X.

Example 4.2 :- Let X = [0,2] and metric d is defined by $d(x,y) = |x-y|$. For $\alpha \in [0,1]$. We define

$$F_\alpha(x,y,p) = \begin{cases} \dfrac{p\alpha}{p\alpha + d(x, y)} ....if..p > 0 \\ 0.....................if..p = 0 \end{cases}$$

Also define self maps A, S, B and T as follows

Sx = $\begin{cases} \frac{1}{2} \ldots\ldots 0 \le x < 1 \\ x \ldots\ldots\ldots 1 \le x \le 2 \end{cases}$, Ax = $\begin{cases} \frac{x+4}{5} \end{cases}$, Bx = $\frac{1+x}{2}$ and

Tx = $\begin{cases} 1 \ldots\ldots 0 \le x < 1 \\ \frac{3-x}{2} \ldots\ldots\ldots 1 \le x \le 2 \end{cases}$. The sequence $\{x_n\}$ is defined as $x_n = 1 - \frac{1}{2n}$.

B1 = 1 and T1 = 1 $\Rightarrow$ TB1= BT1, clearly {B,T} is weak compatible.

$Sx_n = 1 - \frac{1}{2n}$ and $Ax_n = 1 - \frac{1}{10n}$, clearly $Ax_n \to 1$ and $Sx_n \to 1$ i.e. u=1.

$ASx_n = 1 - \frac{1}{20n}$, $SAx_n = \frac{1}{2}$. Now lim $F_\alpha(ASx_n, Su, p) = F(1,1,p) = 1$. Hence {A,S} is semi compatible but not compatible as

$$\lim F_\alpha(ASx_n, SAx_n, p) = \lim F_\alpha\left(1 - \frac{1}{20n}, \frac{1}{2}, p\right) = \frac{p\alpha}{p\alpha + \frac{1}{2}} < 1, \ \forall p, \alpha \in [0,1]$$

So, for all k∈ (0,1) and for all α∈ [0,1] we see the all conditions of **theorem 3.1** are satisfied and hence 1 is the common fixed point in X.

Example: 4.3 Let X = [0,2] and metric d is defined by $d(x,y) = \frac{|x-y|}{1+|x-y|}$. For each p define

$$F_\alpha(x, y, p) = \begin{cases} 1 \ for \ x = y \\ H_\alpha(p) \ for \ x \ne y \end{cases}, \text{ where } H_\alpha(p) = \begin{cases} 0 \ if \ p \le 0 \\ p\alpha \, d(x,y) \ldots if \ 0 < p < 1 \ \& \ \alpha \in [0,1] \\ 1 \quad if \ p \ge 1 \end{cases}.$$

Clearly, (X, F_α, t) is a complete probabilistic space where t is defined by t(p,p) ≥ p.

Ax = $\begin{cases} 1 \ldots\ldots\ldots 0 \le x \le 1 \\ \frac{4-x}{2} \ldots\ldots\ldots 1 < x \le 2 \end{cases}$, Sx = $\begin{cases} 1 \ldots\ldots x = 1 \\ \frac{x+3}{5} \ldots otherwise \end{cases}$, Bx = $\begin{cases} \frac{x}{2} \ldots\ldots 0 \le x < 1/2 \\ 1 \ldots\ldots\ldots x \ge 1/2 \end{cases}$ and

Tx = $\begin{cases} 1 \ldots\ldots\ldots 0 \le x \le 1 \\ \frac{x}{2} \ldots\ldots\ldots 1 < x \le 2 \end{cases}$. The sequence $\{x_n\}$ is defined as $x_n = 2 - \frac{1}{2n}$.

B1 = 1 and T1 = 1 $\Rightarrow$ TB1= BT1 and B2 = T2 = 1 $\Rightarrow$ TB2 = BT2. Clearly {B,T} is weak compatible. $Sx_n = 1 - \frac{1}{10n}$ and $Ax_n = 1 + \frac{1}{4n}$, clearly $Ax_n \to 1$ and $Sx_n \to 1$ .That is u=1. $ASx_n = 1$, $SAx_n = \frac{4}{5} + \frac{1}{20n}$. Now lim $F_\alpha(ASx_n, Su, p) = F_\alpha(1,1,p) = 1$. . Hence {A,S} is semi compatible but not compatible as

$$\lim F_\alpha(ASx_n, SAx_n, p) = \lim F_\alpha\left(1, \frac{4}{5} + \frac{1}{20n}, p\right) = p.\alpha.1/6 < 1.$$

So, for all k∈ (0,1) and for all α∈ [0,1] we see the all conditions of **theorem 3.1** are satisfied and hence 1 is the common fixed point in X.

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