

**ENTROPY GENERATION IN COUETTE FLOW  
THROUGH CHANNEL PLATE WITH SLIP CONDITION**

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**ABSTRACT**

*In the present paper, a viscous incompressible generalized Couette flow is considered between parallel-plate. Both channel plates are at constant temperature. We have obtained analytical solutions for entropy generation number due to heat generation and fluid friction. Effects of various pertinent parameters, such as, pressure gradient, temperature asymmetry and entropy generation number are examined and discussed graphically.*

**Key - Words:** Slip Condition, Couette flow, entropy generation.

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**INTRODUCTION**

One of the important objectives of thermal systems engineering is to analyze the utilization of thermal energy in an efficient manner. Due to entropy production most of the industrial and engineering flow processes and thermal systems are unable to work at optimal level. Therefore, it is vital to determine the factors that contributed to entropy generation in order to minimize their effects and maximize the flow system efficiency. Entropy analysis is a technique to quantify the thermodynamics irreversibility in any fluid flow process. The second law of thermodynamics states that all real processes are irreversible. Entropy generation is a measure of the account of irreversibility associated to the real processes. As entropy generation takes place, the quantity of energy decreases. From the viewpoint of thermodynamics, the decrease of entropy generation means the decrease of irreversibility and less loss of energy. Therefore, in the energy optimization problems and design of many traditional heat removal engineering devices, it is necessary to evaluate the entropy generation or energy destruction due to heat transfer and viscous friction as a function of the physical and geometrical parameters selected for the optimization analysis. This procedure, known as the Entropy Generation Minimization (EGM) method [1-2] is a thermodynamic approach of optimization of engineering systems for higher energy efficiency.

Channel flows are important and one of the fundamental flow situation is Couette flow which is useful in many engineering and industrial applications. Investigation of heat transfer in such flow has many important industrial applications, such as in heat exchangers, chemical reactors, cooling and ceramic processing. However, studies of entropy generation in such flows have been limited. Mahmud and Fraser [3], Ebray *et al.* [4], Damesh *et al.* [5] have studied the local entropy generation due to steady fully developed laminar forced convection channel flow in the presence of a transverse magnetic field. Since the pioneering work of Bejan [6] on entropy generation in convective heat transfer, it is now widely recognized that convective heat transfer problems that were previously studied using the first law of thermodynamics be re-examined in the light of the second law of thermodynamics so that thermal systems can be designed with the objective of minimizing thermodynamic irreversibility. Generally it is assumed that always there is no slip condition at the impermeable boundary wall. But in certain cases this condition replaced with partial slip boundary condition. It is seen that fluid slips at the wall when it is a rarefied gas, suspension or emulsion. It is interesting to investigate Couette flow in a parallel plate channel under fluid slippage at the wall, this is examined by Ibanez [7], Aziz [8], Mahmud [9], Berh M. [10], and Khaled and Vafai [11].

The objective of this study is to obtain the entropy generation as a function of the pertinent parameters. Analytical results for the velocity and temperature profiles, and the entropy generation are presented and discussed graphically.

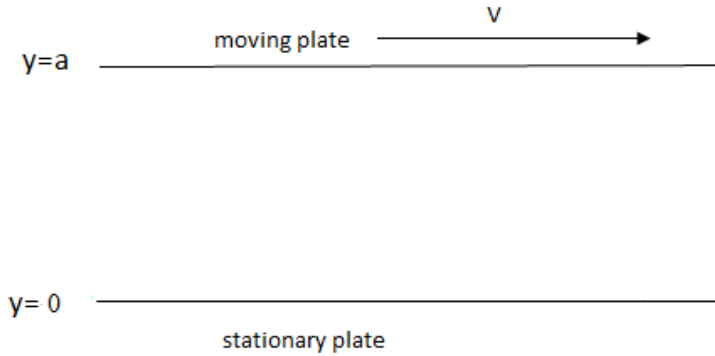
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**FORMULATION OF THE PROBLEM AND SOLUTIONS**

In the present paper we have obtained the velocity and temperature distributions in the generalized Couette flow under fluid slippage at channel plates and discussed entropy generation in the channel. The plates are separated with distance a.

The bottom plate is taken stationary while the top plate is moving with a uniform V. The fluid viscosity, μ, is assumed to be constant.



**Figure-1 Schematic Diagram**

The momentum and energy equation for the velocity and temperature field are written in dimensionless form as follows

$$\frac{d^2U}{dY^2} + P = 0 \tag{1}$$

$$\frac{d^2\theta}{dY^2} + \left(\frac{dU}{dY}\right)^2 = 0 \tag{2}$$

Where  $U = u/V$ ,  $Y = y/a$ , and  $P = -\frac{a^2}{\mu V} \frac{dp}{dx}$  is the dimensionless pressure gradient and  $\theta = \frac{kT}{\mu V^2}$  is the dimensionless temperature and k is the thermal conductivity of the fluid.

The boundary conditions in the dimensionless form are given by

$$\begin{aligned} \text{at } Y = 0, U &= \alpha \frac{\partial U}{\partial Y}, \theta = \beta \frac{\partial \theta}{\partial Y} \\ \text{at } Y = 1, U - 1 &= -\alpha \frac{\partial U}{\partial Y}, \theta = -\beta \frac{\partial \theta}{\partial Y} \end{aligned} \tag{3}$$

The solution of equation (1) and (2) subject to the boundary condition (3) are given by

$$U = -\frac{Py^2}{2} + C_1(y + \alpha) \tag{4}$$

$$\theta = -\left[ \frac{P^2 y^4}{12} + \frac{C_1 P y^3}{6} + \frac{C_1^2 y^2}{2} + C_3 y + C_4 \right] \tag{5}$$

Where,

$$\begin{aligned} C_1 &= -\frac{(P+2)}{2(2\alpha+1)} \\ C_2 &= \alpha C_1 \\ C_3 &= \frac{1}{12(2\beta+1)} \left[ -P^2(4\beta+1) + 2PC_1(3\beta+1) + 6C_1^2(2\beta+1) \right] \\ C_4 &= \beta C_3 \end{aligned}$$

**Entropy generation**

Following Bejan (1995) the dimensionless entropy generation number is

$$S_{gen}^m = \frac{k}{T^2} \left( \frac{dT}{dy} \right)^2 + \frac{\mu}{T} \left( \frac{dU}{dy} \right)^2 \tag{8}$$

Which may be expressed in dimensionless form as

$$N_s = \frac{S_{gen}''' a^2}{k} = \frac{1}{\theta^2} \left( \frac{d\theta}{dY} \right)^2 + \frac{1}{\theta} \left( \frac{dU}{dY} \right)^2 = N_{s1} + N_{s2} \quad (9)$$

Where,  $N_{s1}$  is entropy generation due to heat generations; and  $N_{s2}$  is entropy generation due to fluid friction.

### DISCUSSION

Figure 2-3 shows variation of entropy generation number due to heat generation and due to fluid friction. It is seen that as pressure increases, entropy due heat generation increases in lower half of the channel and shows reverse variation in upper half, whereas exactly reverse effect has been observed in entropy generation due to fluid friction. Figure 4 shows variation of total entropy generation for various values of velocity and thermal slip parameters.

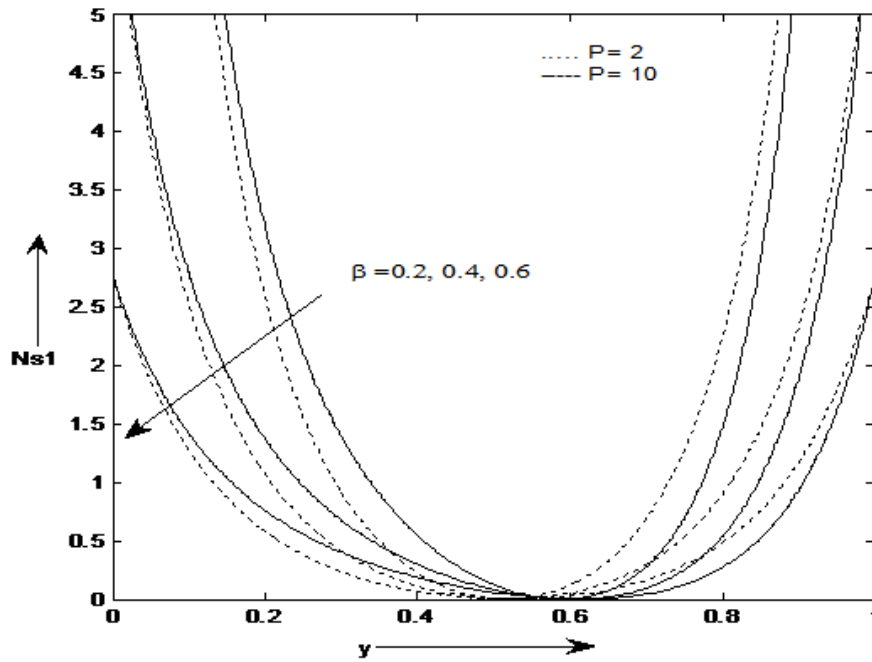


Figure -2: Entropy generation number  $N_{s1}$  vs  $y$  for  $\alpha = .2$

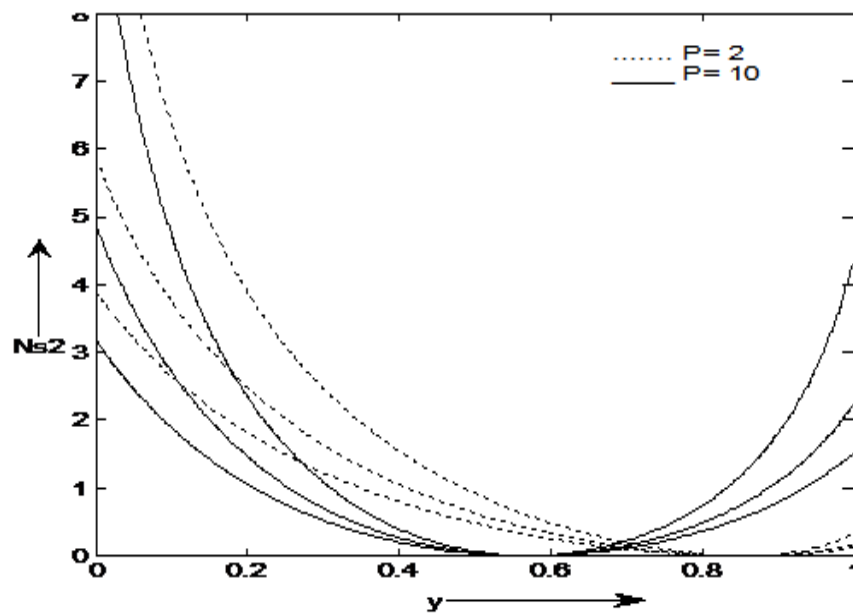


Figure-3: Entropy generation number  $N_{s1}$  vs  $y$  for  $\alpha = .2$

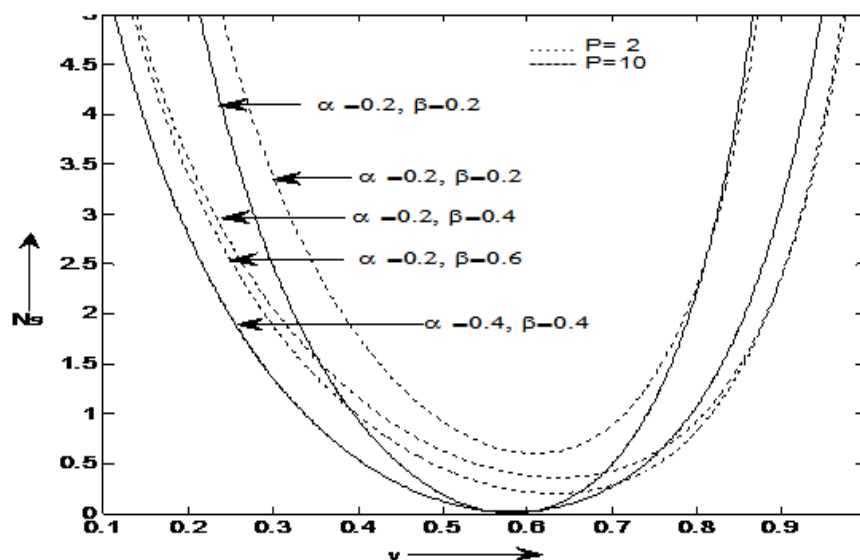


Figure -4: Total Entropy generation number  $N_s$  vs  $y$

## CONCLUSION

In the present paper we have obtained the velocity and temperature distributions in the generalized Couette flow under fluid slippage at channel plates and discussed entropy generation in the channel. We obtained that:

- as pressure increases entropy due heat generation increases in lower half of the channel and shows reverse variation in upper half.
- as pressure increases entropy due heat generation decreases in lower half of the channel and shows reverse variation in upper half.
- The effects of various values of velocity and thermal slip parameters on total entropy generation is shown graphically.

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