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# OPTIMIZING THE UNDERGROUND WATER CONFINED STEADY FLOW USING A FUZZY APPROACH

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*ABSTRACT*

*In this paper we discuss the underground water aquifers, the confined steady flow in homogeneous system where the transmissivities at all nodes inside the zone are represented by the same fuzzy number. A solution algorithm to solve the underground water confined steady flow in homogeneous system (in three dimensions) under fuzziness is considered. The purpose behind this study is optimize the sum of the water heads at each well, the total water flow, the capital and installation cost. The suggested method can be applied for planning any new aquifer development, managing the existing network of wells and planning the drilling compensation wells in partially depleted area. Numerical examples are provided to illustrate the proposed technique.*

***Keywords**: Confined steady groundwater flow, Homogeneous systems, Fuzzy multi-objective optimization.*

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## 1.1 INTRODUCTION:

More general applications utilized the model of optimizing the underground water steady flow via numerical methods such as in Woldt, Wayne et al [18]. Also, using fuzzy set method gives the parameter an imprecision in the groundwater flow models. Uncertainty modeling in health risk assessment and ground water resources management was studied by Kentel, Elcin [6].

This work considers the uncertainty which comes from the parameter imprecision. It is based on the finite difference approximation to the system as which treated via fuzziness environment. The imprecise data may be come from indirect measurements, expert judgment, or subjective to the interpretation of available information. Also, the finite difference method is used to approximate the governing equation of groundwater flow, in which aquifer parameters such as transmissivity are to be considered as a fuzzy numbers. So, the variables in the system are fuzzy instead of its crisp values, then the dependent variable (e.g. hydraulic head) is also fuzzy.

When the transmissivity is represented as a fuzzy number, the membership function of the hydraulic head outputs can be easily determined based on the analytical solution. At each level, both the transmissivity and hydraulic heads are transformed into intervals.

In this paper, we try to investigate the underground water aquifer using a multi-objective approach in homogeneous in three dimensions via the fuzzy optimization. Section 1.2 introduces the multi-objective optimization fuzzy model for the aquifer management in homogeneous system. In Section 1.3 the solution algorithm is suggested. In Section 1.4, applications with results are presented. Finally the conclusions and points for future work in this area are provided in Section 1.5.

## 1.2 MULTI-OBJECTIVE FUZZY OPTIMIZATION MODEL FOR AQUIFER MANAGEMENT IN HOMOGENEOUS SYSTEM:

The multi-objective fuzzy optimization model for the aquifer management in a homogeneous system, in three-dimensions, can be formulated as follows:

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(FMOM): $$\max \left\{ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\tilde{h}_{ijk},\ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\tilde{q}_{ijk} \right\} \tag{1.1.a}$$

$$\min \left\{ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\left[\theta\left(\tilde{h}_{ijk}\right)^{\delta}+\gamma+\beta\right] \right\} \tag{1.1.b}$$

Subject to

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\tilde{q}_{ijk} \geq Demand \tag{1.2.a}$$

$$A_{n\times n}\left(\tilde{T}\right)\tilde{h}_{n\times 1} \leq \tilde{b}_{n\times 1}+\tilde{q}_{n\times 1} \tag{1.2.b}$$

$$T,\ h,\ b,\ q \geq 0 \tag{1.2.c}$$

where $A\left(\tilde{T}\right)$ is the matrix of fuzzy head coefficients which is a function of the transmissivity,

$\tilde{h}_{n\times 1}$ is a fuzzy vector of unknown head values at each node,

$\tilde{b}_{n\times 1}$ is a fuzzy vector containing the boundary head conditions,

$\tilde{q}_{n\times 1}$ is a fuzzy vector which associated with the pumping rate,

$\theta = 5543$, $\delta = 0.299$ , $\gamma$ is the per-well drilling cost (\$/well), and $\beta$ is the pump cost (\$/pump),

~ represents the presence of fuzzy numbers within the matrices or vectors. Thus, model output will be expressed by membership functions that describe the head values as fuzzy variables.

**Definition: 1.1**

A real fuzzy number $\tilde{J}$ is a continuous fuzzy subset from the real line *R* whose triangular membership function $\mu_{\tilde{J}}(J)$ is defined by a continuous mapping from *R* to the closed interval [0,1], where

(1) $\mu_{\tilde{J}}(J) = 0$ for all $J \in (-\infty, a_1]$,
(2) $\mu_{\tilde{J}}(J)$ is strictly increasing on $J \in [a_1, m]$,
(3) $\mu_{\tilde{J}}(J) = 1$ for $J = m$, (1.3.a)
(4) $\mu_{\tilde{J}}(J)$ is strictly decreasing on $J \in [m, a_2]$,
(5) $\mu_{\tilde{J}}(J) = 0$ for all $J \in [a_2, +\infty)$.

which will be elicited as:

$$\mu_{\tilde{J}}(J) = \begin{cases} 0, & J \leq a_1, \\ \dfrac{J-a_1}{m-a_1}, & a_1 \leq J \leq m, \\ \dfrac{a_2-J}{a_2-m}, & m \leq J \leq a_2, \\ 0, & J \geq a_2. \end{cases} \tag{1.3.b}$$

 

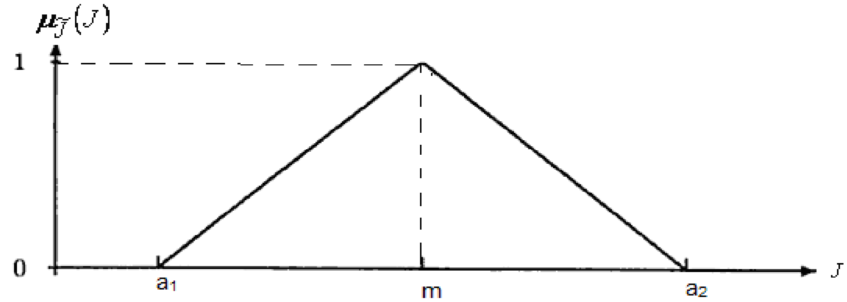


Fig.1.1 membership function of fuzzy number $J$ , $J = T, h,\ b,$ and $q$

where m is given values $a_1$ and $a_2$ denoting the lower and upper bounds. Sometime, it is more convenient to use the notation explicitly highlighting the membership function parameter's and in this case, we have the following form:

$$\mu(J; a_1, m, a_2) = \max\left\{\min\left[\frac{J-a_1}{m-a_1}, \frac{a_2-J}{a_2-m}\right], 0\right\} \tag{1.3.c}$$

In what follows, the definition of the $\alpha$-level set or $\alpha$-cut of the fuzzy number $\tilde{J}$ is introduced, for $J = T, h,\ b$ and $q$ is given.

**Definition: 1.2**

The $\alpha$-level set of the fuzzy parameters $\tilde{J}$ in problems (1.1.a)-(1.2.c) is defined as the ordinary set $L_\alpha(\tilde{J})$ for which the degree of membership function exceeds the level $\alpha$, $\alpha \in [0,1]$,

where: $$L_\alpha(\tilde{J}) = \{J \in R \mid \mu_{\tilde{J}}(J) \geq \alpha\}, \quad \text{for } J = T, h,\ b,\ q$$

For certain values $\alpha_T^*$, $\alpha_h^*$, $\alpha_b^*$, and $\alpha_q^*$ to be in the interval [0,1], the problem (FMOM) (1.1.a)-(1.2.c) can be reformulated as the following multi-objective non fuzzy optimization model for the aquifer management homogeneous system , in three-dimensions as follows:

$$(\alpha-\text{MOM}): \quad \max \left\{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} h_{ijk},\ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} q_{ijk}\right\} \tag{1.4.a}$$

$$\min \left\{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\left[\theta(h_{ij})^{\delta} + \gamma + \beta\right]\right\} \tag{1.4.b}$$

Subject to

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} q_{ijk} \geq Demand \tag{1.5.a}$$

$$A_{n\times n}(T)h_{n\times 1} \leq b_{n\times 1} + Q_{n\times 1} \tag{1.5.b}$$

$$\mu_{\tilde{T}} \geq \alpha_T \tag{1.5.c}$$

$$\mu_{\tilde{h}} \geq \alpha_h \tag{1.5.d}$$

$$\mu_{\tilde{b}} \geq \alpha_b \tag{1.5.e}$$

$$\mu_{\tilde{q}} \geq \alpha_q \tag{1.5.f}$$

$$T,\ h,\ b,\ q \geq 0$$

Problem $(\alpha - \mathrm{MOM})$ (1.4.a)-(1.5.f) can be rewritten as follows:

$$\max \left\{ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} h_{ijk},\ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} q_{ijk} \right\}, \tag{1.6.a}$$

$$\min \left\{ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} \left[\theta\left(h_{ij}\right)^{\delta} + \gamma + \beta\right] \right\} \tag{1.6.b}$$

Subject to

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} q_{ijk} \geq Demand \tag{1.7.a}$$

$$A_{n\times n}(T)h_{n\times 1} \leq b_{n\times 1} + q_{n\times 1} \tag{1.7.b}$$

$$T_l \leq T^{\alpha} \leq T_u \tag{1.7.c}$$

$$h_l \leq h^{\alpha} \leq h_u \tag{1.7.d}$$

$$b_l \leq b^{\alpha} \leq b_u \tag{1.7.e}$$

$$q_l \leq q^{\alpha} \leq q_u \tag{1.7.f}$$

$$T,\ h,\ b,\ q \geq 0 \tag{1.7.g}$$

where $T_l,\ T_u,\ h_l,\ h_u,\ b_l,\ b_u,\ q_l$ and $q_u$ are lower and upper bounds on T, h, b and q, respectively The lower bound $h_l$ and the upper bound $h_u$ can be calculate using the following nonlinear programming problems:

$$\begin{aligned} h_l^*:\quad & \text{Min}\quad h_{i,j,k}^{\alpha} \\ & \text{subject to} \\ & A\left(T^{\alpha}\right)(h)^{\alpha} = b^{\alpha} \\ & \underline{T_{i,j,k}^{\alpha}} \leq T^{\alpha} \leq \overline{T_{i,j,k}^{\alpha}} \\ & \underline{b_{i,j,k}^{\alpha}} \leq b^{\alpha} \leq \overline{b_{i,j,k}^{\alpha}} \end{aligned} \tag{1.8}$$

and

$$\begin{aligned} h_u^*:\quad & \text{Max}\quad h_{i,j,k}^{\alpha} \\ & \text{subject to} \\ & A\left(T^{\alpha}\right)(h)^{\alpha} = b^{\alpha} \\ & \underline{T_{i,j,k}^{\alpha}} \leq T^{\alpha} \leq \overline{T_{i,j,k}^{\alpha}} \\ & \underline{b_{i,j,k}^{\alpha}} \leq b^{\alpha} \leq \overline{b_{i,j,k}^{\alpha}} \end{aligned} \tag{1.9}$$

The lower bound $q_l$ and the upper bound $q_u$ can be calculated using the following nonlinear programming problems:

 

$$q_l^*: \quad \text{Min} \quad q_{i,j,k}^{\alpha}$$

$$\begin{aligned} &\text{subject to} \\ &A\left(T^{\alpha}\right)(h)^{\alpha} = b^{\alpha} \\ &\underline{T_{i,j,k}^{\alpha}} \le T^{\alpha} \le \overline{T_{i,j,k}^{\alpha}} \\ &\underline{b_{i,j,k}^{\alpha}} \le b^{\alpha} \le \overline{b_{i,j,k}^{\alpha}} \end{aligned} \tag{1.10}$$

and

$$q_u^*: \quad \text{Max} \quad q_{i,j,k}^{\alpha}$$

$$\begin{aligned} &\text{subject to} \\ &A\left(T^{\alpha}\right)(h)^{\alpha} = b^{\alpha} \\ &\underline{T_{i,j,k}^{\alpha}} \le T^{\alpha} \le \overline{T_{i,j,k}^{\alpha}} \\ &\underline{b_{i,j,k}^{\alpha}} \le b^{\alpha} \le \overline{b_{i,j,k}^{\alpha}} \end{aligned} \tag{1.11}$$

where $\underline{T_{i,j,k}^{\alpha}}$, $\overline{T_{i,j,k}^{\alpha}}$ ,$\underline{b_{i,j,k}^{\alpha}}$, $\overline{b_{i,j,k}^{\alpha}}$ , are the lower and upper bounds on $T_{i,j,k}^{\alpha}$, $b_{i,j,k}^{\alpha}$ respectively , $T^{\alpha}$ is the vector of transmissivities at the specified $\alpha$ -cut level, $A\left(T^{\alpha}\right)$ is the matrix of head coefficients which is a function of $T^{\alpha}$ ,$b^{\alpha}$ is the right hand side vector containing the boundary conditions and source/sink terms and $h^{\alpha}$ is the vector of unknown heads at the specified $\alpha$ - level cut. Thus, to calculate fuzzy head at a specific node two nonlinear programming problems are considered "the lower and upper bound of the unknown head can be calculated by optimization the tow models mathematical, and then we find the optimal solutions using any suitable software, is obtained.

## 1.3 SOLUTION ALGORITHM:

In the section, we suggest an algorithm to solve the Underground Water confined steady flow in a homogeneous system under fuzziness (1.1.a)-(1.2.c). The suggested algorithm can be summarized in the following steps:

**Step: 1** Start with initial level set $\alpha^{\circ} = \left(\alpha_T^{\circ},\ \alpha_h^{\circ},\ \alpha_b^{\circ},\ \alpha_q^{\circ}\right)$ in the interval [0, 1].

**Step: 2** Determine the points (a1, m, a2) for the transmissivity T, the boundary head condition b to elicit membership functions $\mu_{\tilde{T}}(T)$ and $\mu_{\tilde{b}}(b)$.

**Step: 3** Convert the given problem (1.1.a)-(1.2.c) into its non fuzzy version $(\alpha - \text{MOM})$ (1.6.a)-(1.7.g).

**Step: 4** Determine the lower and upper bounds of transmissivities at each node for each $\alpha$ - level cut, and the boundary head condition b for each $\alpha$ - level cut.

**Step: 5** Choose certain values for the transmissivity $T^* \in [T_l, T_u]$ and the boundary head conditions $b^* \in [b_l, b_u]$ corresponding to the $\alpha$ - level cut $\alpha = \alpha^* \in [0,1]$.

**Step: 6** Solve the nonlinear programming problems defined by (1.8),(1.9),(1.10),(1.11) using LINGO software package to determine points ($a_1$, m, $a_2$) for the head values at each node *h*, and the pumping rate q to elicit the membership functions $\mu_{\tilde{h}}(h)$, and $\mu_{\tilde{q}}(q)$ .

**Step: 7** Determine the lower and upper bounds of the head and the pumping rate for each node at a specified $\alpha$ - level cut.

**Step: 8** Choose values for the head $h^* \in [h_l, h_u]$ and the pumping rate $q^* \in [q_l, q_u]$ corresponding to the same $\alpha$ - level cut $\alpha = \alpha^* \in [0,1]$, to find (h*,q*), Then, the efficient solution (T*, h*, b*, q*) of the problem (1.1.a) –(1.2.c) is found.

**Step 9** Set $\alpha = (\alpha^\circ + \text{step}),\ \alpha \in [0,1]$.

**Step 10** Go to step (1) with a new $\alpha$ until the interval [0, 1] is fully exhausted. Then, Stop.

**1.4 APPLICATIONS AND RESULTS:**

Assuming the leakage of flux into or out of aquifer and the well diameters are to be negligible, Well losses are negligible, and the head in the well is measured from the surface of the producing layer which is considered as a horizontal datum.

Input data for the simulation model includes fuzzy transmissivity values at each node, fuzzy number head boundary conditions, transmissivity of boundary nodes, the discharge rate of the well and the basic simulation parameters.

In the following, three different case studies for optimizing the underground water aquifers of the confined steady flow in homogeneous system are discussed. The three dimensional flow in confined aquifer will be studied.

From (1.1.a) and (1.2.c), the heads on boundaries are fuzzy number values of [50-60]m. The demand is 500 m/day and the upper bound of the total water production is 2000.

$n = m = l = 2$, $\Delta x = \Delta y = \Delta z = 10\text{m}$, $\text{T} \in [200,300]\text{m}^2/\text{day}$,
$\gamma = 13.511$ (\$/well), $\beta = 3832$ (\$/pump), $\alpha \in [0,1]$

Set $\alpha = \alpha^* = 0.4$, using the given membership function we get

$220 \le \text{T} \le 280$, $52 \le \text{b} \le 58$, choose $\text{T} = 220$ and $\text{b} = 52$,

Calculate the lower and upper bound of head and pump rate for each node using (1.8)-(1.11), via the LINGO software packages, the solution of the model is given by

The lower bound of heads h:

| | | | |
|---|---|---|---|
| $h_{111} = 0$ | $h_{112} = 0$ | $h_{121} = 0$ | $h_{122} = 0$ |
| $h_{211} = 0$ | $h_{212} = 0$ | $h_{221} = 0$ | $h_{222} = 0$ |

The upper bound of heads h:

| | | | |
|---|---|---|---|
| $h_{111} = 50.49$ | $h_{112} = 50.48$ | $h_{121} = 50.48$ | $h_{122} = 50.31$ |
| $h_{211} = 51.4$ | $h_{212} = 50.76$ | $h_{221} = 50.76$ | $h_{222} = 51.06$ |

Then we get $\underline{h_{i,j,k}} \le h^{\alpha}_{i,j,k} \le \overline{h_{i,j,k}}$ for all $i, j$ and $k$.

The lower bound of pumping rate q:

| | | | |
|---|---|---|---|
| $q_{111} = 0$ | $q_{112} = 0$ | $q_{121} = 0$ | $q_{122} = 0$ |
| $q_{211} = 0$ | $q_{212} = 0$ | $q_{221} = 0$ | $q_{222} = 0$ |

The upper bound of pumping rate q:

| | | | |
|---|---|---|---|
| $q_{111} = 0$ | $q_{112} = 528.76$ | $q_{121} = 221.25$ | $q_{122} = 0$ |
| $q_{211} = 400.44$ | $q_{212} = 0$ | $q_{221} = 384.95$ | $q_{222} = 464.599$ |

 

Then we get $\underline{q_{i,j,k}} \le q^{\alpha}_{i,j,k} \le \overline{q_{i,j,k}}$ for all $i, j$ and $k$.

By using $\alpha = \alpha^* = 0.4$ at the given membership function, to get

| | |
|---|---|
| $h_{111} \in [10.10,\ 40.40]$ | $h_{112} \in [10.10,\ 40.385]$ |
| $h_{121} \in [10.10,\ 40.39]$ | $h_{122} \in [10.06,\ 40.24]$ |
| $h_{211} \in [10.28, 41.122]$ | $h_{212} \in [10.15, 40.612]$ |
| $h_{221} \in [10.15,\ 40.61]$ | $h_{222} \in [10.21,\ 40.85]$ |

and

| | |
|---|---|
| $q_{111} = 0$ | $q_{112} \in [105.76,\ 423.24]$ |
| $q_{121} \in [44.25,\ 177]$ | $q_{122} = 0$ |
| $q_{211} \in [420.35, 480]$ | $q_{212} = 0$ |
| $q_{221} \in [76.99, 307.96]$ | $q_{222} \in [92.9, 371.68]$ |

By choosing

| | | | |
|---|---|---|---|
| $h_{111} = 40.4$ | $h_{112} = 40.385$ | $h_{121} = 40.385$ | $h_{122} = 40.24$ |
| $h_{211} = 41.122$ | $h_{212} = 40.612$ | $h_{221} = 40.612$ | $h_{222} = 40.85$ |

and the well produces

| | | | |
|---|---|---|---|
| $q_{111} = 0$ | $q_{112} = 423.24$ | $q_{121} = 177$ | $q_{122} = 0$ |
| $q_{211} = 480$ | $q_{212} = 0$ | $q_{221} = 307.96$ | $q_{222} = 371.68$ |

The maximum sum of water head at each well is 324.608 $m^3$, the maximum total water production wells is 1760 m/day and the minimum capital & installation, drilling and pumping cost is $165265.8818.

Setting $\alpha = \alpha^* = 0.8$ at the proposed membership function then we get

$240 \le T \le 260$, $54 \le b \le 56$. Then we choose $T = 240$ and $b = 54$

Calculate the lower and upper bound of head and pumping rate for each node using step (1.8)-(1.11) and the suitable software packages, the solution of the model is given by

The lower bound of heads h:

| | | | |
|---|---|---|---|
| $h_{111} = 0$ | $h_{112} = 0$ | $h_{121} = 0$ | $h_{122} = 0$ |
| $h_{211} = 0$ | $h_{212} = 0$ | $h_{221} = 0$ | $h_{222} = 0$ |

The upper bound of heads h:

| | | | |
|---|---|---|---|
| $h_{111} = 52.623$ | $h_{112} = 52.61$ | $h_{121} = 52.61$ | $h_{122} = 52.45$ |
| $h_{211} = 53.45$ | $h_{212} = 52.87$ | $h_{221} = 52.87$ | $h_{222} = 53.14$ |

Then we get $\underline{h_{i,j,k}} \le h^{\alpha}_{i,j,k} \le \overline{h_{i,j,k}}$ for all $i, j$ and $k$.

 

The lower bound of pumping rate q:

| | | | |
|---|---|---|---|
| $q_{111}=0$ | $q_{112}=0$ | $q_{121}=0$ | $q_{122}=0$ |
| $q_{211}=0$ | $q_{212}=0$ | $q_{221}=0$ | $q_{222}=0$ |

The upper bound of pumping rate q:

| | | | |
|---|---|---|---|
| $q_{111}=0$ | $q_{112}=627.64$ | $q_{121}=309.54$ | $q_{122}=0$ |
| $q_{211}=0$ | $q_{212}=0$ | $q_{221}=522.22$ | $q_{222}=540.6$ |

Then we get $\underline{Q_{i,j,k}} \le Q^{\alpha}_{i,j,k} \le \overline{Q_{i,j,k}}$ for all $i, j$ and $k$

set $\alpha = \alpha^{*} = 0.8$ in the proposed membership function, to get

| | |
|---|---|
| $h_{111} \in [21.05,\ 31.57]$ | $h_{112} \in [21.04,\ 31.56]$ |
| $h_{121} \in [21.04,\ 31.56]$ | $h_{122} \in [20.98,\ 31.47]$ |
| $h_{211} \in [21.381,\ 32.07]$ | $h_{212} \in [21.15,\ 31.72]$ |
| $h_{221} \in [21.15,\ 31.72]$ | $h_{222} \in [21.26,\ 31.88]$ |

and

| | |
|---|---|
| $q_{111}=0$ | $q_{112} \in [251.1,\ 376.58]$ |
| $q_{121} \in [123.82,\ 185.73]$ | $q_{122}=0$ |
| $q_{211}=0$ | $q_{212}=0$ |
| $q_{221} \in [208.888,\ 313.33]$ | $q_{222} \in [216.24,\ 324.36]$ |

By choosing

| | | | |
|---|---|---|---|
| $h_{111}=31.57$ | $h_{112}=31.56$ | $h_{121}=31.56$ | $h_{122}=31.47$ |
| $h_{211}=32.07$ | $h_{212}=31.72$ | $h_{221}=31.72$ | $h_{222}=31.88$ |

and the well produces

| | | | |
|---|---|---|---|
| $q_{111}=0$ | $q_{112}=376.58$ | $q_{121}=185.73$ | $q_{122}=0$ |
| $q_{211}=0$ | $q_{212}=0$ | $q_{221}=313.33$ | $q_{222}=324.36$ |

The maximum sum of water head at each well is 253.55 $m^3$, the maximum total water production wells is 1200 m/day and the minimum capital & installation, drilling and pumping cost is $155711.0037.

## 1.5 CONCLUSIONS AND FUTURE WORK:

The proposed model is suitable for management planning situation for a new aquifer development in the underground water aquifers the confined steady flow in the homogeneous system where the transmissivities at all nodes are considered which are located inside the zone and represented by the same fuzzy numbers.

It can be seen that the relationship between fuzziness of the membership function of the transmissivity and the head when increasing the values of transmissivity, the head value is increased.

The model gives a wide vision in defining the best well locations, the best production regime and the minimum capital, installation, drilling and pump cost.

The proposed models can be extended for the underground water aquifers where the transmissivities at all nodes which are located inside the different zones and represented by fuzzy numbers.

 

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