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ALMOST CONTRA- ĝ -CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper, we introduce and investigate the notion of almost contra-ĝ-continuous functions which is weaker than both notions of contra -continuous functions [10] and (θ, s)-continuous functions [20] in topological spaces. We discuss the relationships with some other related functions. At the same time, we show that almost-contra- ĝ-continuity and (slc, s)-continuity are independent of each other.*

Key words and phrases: *ĝ-closed set, contra-ĝ-continuous function, almost contra-ĝ-continuous function.*

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1. INTRODUCTION:

Many topologists studied the various types of generalizations of continuity [1, 3, 5, 12, 17, 22, 25, 26, 32, 34]. In 1996, Dontchev [10] introduced the notion of contra-continuity. Recently, new types of contra-continuity, such as almost contra-continuity [3], almost contra-precontinuity [12] and contra almost β-continuity [3] have been introduced and studied. Noiri and Popa [29] have obtained a unified theory of almost contra- continuity by using the notion of minimal structures. They have also pointed out that almost contra- continuity (resp. almost contra-precontinuity, contra almost β-continuity) is equivalent to (θ, s)-continuity [20] (resp. (p, s)-continuity [18], β-quasi irresoluteness [19]). Quite recently, Ekici [13] has introduced the notion of (LC, s) - continuity and obtained some properties and relationships among the other related functions.

In this paper, we introduce the notion of almost contra ĝ-continuous functions as a generalization of both notions of contra-continuous functions and (θ, s)-continuous functions. We obtain their characterizations and properties and a new decomposition of (θ, s)-continuity.

2. PRELIMINARIES:

Throughout the present paper, (X, τ), (Y, σ) and (Z, γ) (or X, Y and Z) denote topological spaces in which no separation axiom are assumed unless explicitly stated. The closure and the interior of a subset A of a topological space (X, τ) are denoted by cl(A) and int(A), respectively.

Definition: 2.1 A subset A of a space (X, τ) is said to be
(1) regular open [40] if A = int(cl(A));
(2) α-open [28] if A⊆ int(cl(int(A)));
(3) semi-open [22] if A⊆ cl(int(A)).
(4) preopen [25] (or locally dense [7] or nearly open [17]) if A ⊆ int(cl(A));
(5) β-open [1](or semi-preopen [2]) if A⊆ cl(int(cl(A)));
(6) Locally closed [4] (or FG [41]) if A = U ∩ V, where U is open in X and V is closed in X.
(7) slc*-set [33] if A = U ∩ V, where U is semi-open in X and V is closed in X.

The δ-interior of a subset A of X is the union of all regular open sets of X contained in A and is denoted by δ-int(A). A subset A is said to be δ-open [43] if A = δ-int(A). The family of all regular open (resp. δ-open, α-open, semi-open, preopen, β-open) sets in a space (X, τ) is denoted by RO(X) (resp. δO(X), αO(X), SO(X), PO(X), βO(X)).

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The complement of a regular open (resp. δ-open, α-open, semi-open, preopen, β-open) set is said to be regular closed (resp. δ-closed [43], α-closed [26], semi-closed [8], preclosed [25], β-closed [1] or semi-preclosed [2]). The family of all clopen (resp. regular closed) subsets of X will denoted by CO(X) (resp. RC(X)). We set $CO(X, x) = \{V \in CO(X) \mid x \in V\}$.

Definition: 2.2 A subset A of a space (X, τ) is said to be g-closed [23] in X if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition: 2.3 A subset A of a space (X, τ) is said to be ĝ-closed [45] (or ω-closed [33]) in X if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.

The complement of a ĝ-closed set is said to be ĝ-open. The family of all ĝ-open sets forms a topology. The family of all ĝ-closed (resp. ĝ-open) subsets of X will denoted by ĜC(X) (resp. ĜO(X)). It is well known that every closed set is ĝ-closed set. But, the converse of this implication is not true [44].

Remark: 2.4. [33]
Closed sets imply both slc*-sets and ĝ-closed sets which are independent of each other.

Example: 2.5. [33]
(1). Let X = {a, b, c} and τ = {∅, X, {a}, {b, c}}. Let A = {a, b}. Then A is ĝ-closed set but it is not slc*-set.
(2). Let X = {a, b, c} and τ = {∅, X, {a}}. Let A = {a, c}. Then A is slc*-set but it is not ĝ-closed set.

Proposition: 2.6. [33]
A subset A is closed in a space (X, τ) if and only if it is slc*-set and ĝ-closed.

Remark: 2.7.
From the subsets mentioned above, we have the following implications.

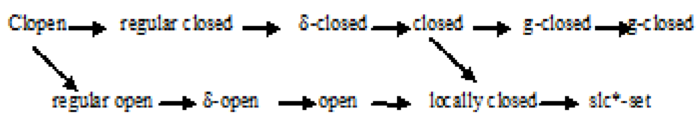

Diagram: 1

Diagram 1 is obvious. None of these implications is reversible (see, related papers).

3. CHARACTERIZATIONS OF ALMOST CONTRA ĝ -CONTINUOUS FUNCTIONS:

Definition: 3.1.
A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost contra ĝ-continuous if $f^{-1}(V)$ is ĝ-closed in X for each regular open set V of Y.

Theorem: 3.2.
For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent :
(1) f is almost contra ĝ-continuous;
(2) $f^{-1}(F) \in \hat{G}O(X)$ for every $F \in RC(Y)$;
(3) $f^{-1}(int(cl(G))) \in \hat{G}C(X)$ for every open subset G of Y;
(4) $f^{-1}(cl(int(F))) \in \hat{G}O(X)$ for every closed subset F of Y.

Proof:
(1) ⇒ (2) Let $F \in RC(Y)$. Then $(Y - F) \in RO(Y)$ and by (1), $f^{-1}(Y-F) = (X - f^{-1}(F)) \in \hat{G}C(X)$. Hence, $f^{-1}(F) \in \hat{G}O(X)$.
(2) ⇒ (1) This proof is obtained similarly to that of (1) ⇒ (2)
(1) ⇒ (3) Let G be an open subset of Y. Since int(cl(G)) is regular open, we have $f^{-1}(int(cl(G)) \in \hat{G}C(X)$ by using (1).
(3) ⇒ (1) This proof is obvious.
(2) ⇒ (4) This proof is similar as (1) ⇒ (3).
(4) ⇒ (2). This proof is similar as (3) ⇒ (1).

Theorem: 3.3
If a function $f: (X, \tau) \to (Y, \sigma)$ is almost contra ĝ-continuous, then the following equivalent properties hold:

(1) For each x ∈ X and each regular closed F in Y containing f(x), there exists a ĝ-open set U in X containing x such that f(U) ⊆ F;

(2) For each x ∈ X and each regular open V in Y non-containing f(x), there exists a ĝ-closed set K in X non-containing x such that $f^{-1}(V) \subseteq K$.

Proof: Since it is obvious that (1) and (2) are equivalent of each other, we will prove (1). Let F be any regular closed set in Y containing f(x). By Theorem 3.2(2), $f^{-1}(F) \in \hat{G}O(X)$ and $x \in f^{-1}(F)$. If we take $U = f^{-1}(F)$, we obtain immediately $f(U) \subseteq F$.

4. THE RELATED FUNCTIONS AND SOME PROPERTIES:

Definition: 4.1

A function f : (X, τ) → (Y, σ) is said to be almost continous [34] (resp. δ-continuous [30], an R-map [6], regular set-connected [11], a contra R-map[15], almost contra super continuous [14], (LC, s)-continuous [13], (θ, s)-continuous [20], (slc*, s)-continuous, almost semi-continuous [27], almost contra g-continuous [21]) if $f^{-1}(V)$ is open (resp. δ-open, regular open, clopen, regular closed, δ-closed, locally closed, closed, slc*-set, semi-open, g-closed) in X for each regular open set V of Y.

By using Diagram 1 and Proposition 2.6, we obtain Diagram 2.

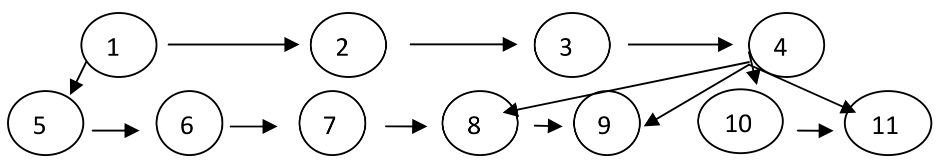

Diagram II

Where

(1) regular set-connected,	(6) δ-continuous,
(2) contra R-map,	(7) almost-continuous,
(3) almost contra super continuous,	(8) (LC, s)-continuous,
(4) (θ, s)-continuous,	(9) (slc*, s)-continuous,
(5) R-map,	(10) almost contra ĝ-continuous,
	(11). almost contra g-continuous.

Remark: 4.2

None of the implications is reversible as shown by following example.

Example: 4.3

Let X = Y = {a, b, c}, τ = {∅, X, {a}} and σ = {∅, Y, {a}, {b}, {a, b}}. Let f: (X, τ) → (Y, σ) be an identity map.

(1) Here f is (slc*, s)-continuous but it is not almost contra ĝ-continuous, since $f^{-1}(\{a\}) = \{a\}$ is not ĝ-closed in X.
(2) Here f is (slc*, s)-continuous but it is not (LC, s)-continuous, since $f^{-1}(\{b\}) = \{b\}$ is not locally closed in X.

Example: 4.4

Let X = Y = {a, b, c, d}, τ = {∅, X, {d}, {a, b, c}} and σ = {∅, Y, {c}, {a, b}, {a, b, c}}. Let f: (X, τ) → (Y, σ) be an identity map. Then f is almost contra ĝ-continuous.

(1) It is not (slc*, s)-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not slc*-set in X.
(2) It is not (θ, s)-continuous, since $f^{-1}(\{a, b\}) = \{a, b\}$ is not closed in X.

Example: 4.5

Let X = Y= {a, b, c}, τ = {∅, X, {a}} and σ = {∅, Y, {b}, {c}, {b, c}}. Let f: (X, τ) → (Y, σ) be an identity map. Then f is almost contra g-continuous but it is not almost contra ĝ-continuous, since $f^{-1}(\{b\}) = \{b\}$ is not ĝ-closed in X.

The other examples are as shown in the related papers.

Proposition: 4.6

A function f: (X, τ) → (Y, σ) is (θ, s)-continuous if and only if it is (slc*, s)-continuous and almost contra ĝ-continuous.

Proof: This obtained from Proposition 2.6.

Definition: 4.7

A space X is said to be
(1) ĝ-space (resp. locally ĝ-indiscrete) if every ĝ-open set of X is open (resp. closed) in X.
(2) In a $T_{\hat{g}}$ -space [33] if every ĝ-closed set of X is closed in X.

We obtain directly the following theorem by using Definition 4.7.

Theorem: 4.8
If a function f: (X, τ) → (Y, σ) is almost contra ĝ-continuous, then the following properties hold:

(1) If X is a ĝ-space (or $T_{\hat{g}}$ -space), then f is (θ, s)-continuous;
(2) If X is locally ĝ-indiscrete, then f is almost continuous.

Theorem: 4.9
If a function f: (X, τ) → (Y, σ) is almost contra ĝ-continuous and almost semi-continuous, then f is contra-R-map.

Proof: Let V ∈ RO(Y). Since f is almost contra ĝ-continuous and almost semi-continuous, $f^{-1}(V)$ is ĝ-closed and semi-open. So $f^{-1}(V)$ is regular closed. It turns out that f is contra R-map.

Theorem: 4.10
Let f : X → Y be a function and let g : X → X × Y be the graph function of f, defined by g(x) = (x, f(x)) for every x ∈ X. If g is almost contra -ĝ- continuous function, then f is almost contra- ĝ-continuous.

Proof: Let V ∈ RC(Y), then X × V = X × cl(int(V)) = cl(int(X)) × cl(int(V)) = cl(int(X × V)). Therefore, we have X × V ∈ RC(X × V). Since g is almost contra-ĝ-continuous, then $f^{-1}(V) = g^{-1}(X \times V) \in \hat{G}O(X)$. Thus, f is almost contra-ĝ-continuous.

Definition: 4.11
A function f: (X, τ) → (Y, σ) is said to be

(1) ĝ-continuous [32] if $f^{-1}(V)$ is ĝ-closed in X for each closed set V of Y.
(2) perfectly continuous [30] (resp. contra ĝ-continuous [32]) if $f^{-1}(V)$ is clopen (resp. ĝ-closed) in X for each open set V of Y.

Since every regular open set is open, it is obvious that every contra ĝ-continuous function is almost contra ĝ-continuous. But the converse of this implication is not true in general as shown by the following example.

Example: 4.12
Let X = Y = {a, b, c} τ = {∅, X, {b}, {c}, {a, c}, {b, c}} and σ = {∅, Y, {a}, {c}, {a, c}, {b, c}}. Let f: (X, τ) → (Y, σ) be defined by f(a) =b, f(b)= a, f(c) = (c). Then f is almost contra ĝ-continuous but it is not contra ĝ-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not ĝ-closed X.

Theorem: 4.13
For two functions f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, γ) , let g ∘ f : (X, τ) → (Z, γ) is a composition function. Then the following properties hold:

(1) If f is almost contra ĝ-continuous and g is an R-map, then g ∘ f is almost contra ĝ-continuous.
(2) If f is almost contra ĝ-continuous and g is perfectly continuous, then g ∘ f is ĝ-continuous and contra ĝ-continuous.
(3) If f is contra ĝ-continuous and g is almost continuous, then g ∘ f is almost contra ĝ-continuous.

Proof: (1) Let V be any regular open set in Z. Since g is an R-map, $g^{-1}(V)$ is regular open. Since f is almost contra ĝ-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is ĝ-closed. Therefore, g ∘ f is almost contra ĝ-continuous.
(2) and (3). These proofs are obtained similarly as the proof of (1).

To give the following two theorems, we define two functions.

Definition: 4.14
A function f : (X, τ) → (Y, σ) is said to be ĝ*-open [33] (resp. ĝ*-closed [33]) if f(U) is ĝ-open (resp. ĝ-closed) in Y for each ĝ-open (resp. ĝ-closed) set U of X.

Theorem: 4.15
If f: (X, τ) → (Y, σ) is surjective ĝ-open (or ĝ-closed) and g : Y → Z is a function such that g ∘ f : X → Z is almost contra ĝ-continuous, then ĝ is almost contra ĝ-continuous.

Proof: Let V be any regular closed (resp. regular open) set in Z. Since g ∘ f is almost contra ĝ-continuous, we have $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ĝ-open (resp. ĝ-closed). Since f is surjective and ĝ-open (or ĝ-closed) we have $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is ĝ-open (ĝ-closed). Therefore, g is almost contra ĝ-continuous.

Lemma: 4.16. [33]
If $A \subseteq B \subseteq X$ where A is ĝ-closed relative to B and B is open and ĝ-closed relative to X, then A is ĝ-closed relative to X.

Theorem: 4.17
Let f: $(X, \tau) \to (Y, \sigma)$ be a function and $x \in X$. If there exists $A \in \hat{G}O(X)$ such that $x \in A$ and the restriction of f to A is almost contra ĝ-continuous at x, then f is almost contra ĝ-continuous at x.

Proof: Suppose that $F \in RC(Y)$ containing f(x). Since $f|_A : (A, \tau|_A) \to (Y, \sigma)$ is almost contra ĝ-continuous at x, there exists $V \in \hat{G}O(A)$ containing x such that $f(V) = f|_A(V) \subset F$. Since $A \in \hat{G}O(X)$ containing x, we obtain that $V \in \hat{G}O(X)$ containing x by using Lemma 4.16.

Theorem: 4.18 [33]
A set A is ĝ-open if and only if $F \subseteq int(A)$ whenever F is semi-closed and $F \subseteq A$.

Theorem: 4.19
If f: $(X, \tau) \to (Y, \sigma)$ is an almost contra ĝ-continuous function and A is closed subset of X, then the restriction function of f to A is almost contra ĝ-continuous.

Proof: Let $F \in RO(Y)$. Since f is almost contra ĝ-continuous, then we have $f^{-1}(F) \in \hat{G}C(X)$. Since A is closed in X, we obtain that $(A \cap f^{-1}(F)) \in \hat{G}C(X)$. Since A is closed in X, $(f|_A)^{-1}(F) = (A \cap f^{-1}(F)) \in \hat{G}C(A, \tau_A)$ by using the definition of subspaces. Therefore, $f|_A$ is almost contra ĝ-continuous.

Theorem: 4.20
For a function f: $(X, \tau) \to (Y, \sigma)$, the following properties are equivalent:

(1) f is almost contra ĝ-continuous;
(2) For each $x \in X$ and each regular closed F in Y containing f(x), there exists a ĝ-open set U in X containing x such that $f(U) \subset F$;
(3) For each $x \in X$ and each regular open V in Y non-containing f(x), there exists a ĝ-closed set K in X non-containing x such that $f^{-1}(V) \subset K$.

Definition: 4.21
A filterbase B is said to be ĝ-convergent (resp. rc-convergent [14]) to a point $x \in X$ if for any $A \in \hat{G}O(X)$ containing x (resp. $A \in RC(X)$ containing x), there exists a $B \in \mathbf{B}$ such that $B \subset A$.

Theorem 4.22
If a function f : $(X, \tau) \to (Y, \sigma)$ is almost contra ĝ-continuous, then for each point $x \in X$ and each filterbase **B** in X ĝ-converging to x, the filterbase f(**B**) is rc-convergent to f(x).

Proof: Let $x \in X$ and **B** be any filterbase in X ĝ-converging to x. Since f is almost contra ĝ-continuous, then for any $V \in RC(Y)$ containing f(x), there exists $U \in \hat{G}O(X)$ containing x such that $f(U) \subset V$. Since **B** is ĝ-convergent to x, there exists a $B \in \mathbf{B}$ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filterbase f(**B**) is rc-convergent to f(x).

Recall that

(1) a space X is said to be weakly Hausdorff [37] if each element of X is an intersection of regular closed sets.
(2) a space X is said to be ĝ-T_1 [32] if for each pair of distinct points x and y of X, there exist ĝ-open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.

Theorem: 4.23.
If f : $(X, \tau) \to (Y, \sigma)$ is an almost contra ĝ-continuous injection and Y is weakly Hausdorff, then X is ĝ-T_1.

Proof: Suppose that Y is weakly Hausdorff. For any distinct points x and y in X, there exist $V, W \in RC(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(y) \in W$ and $f(x) \notin W$. Since f is almost contra ĝ-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are ĝ-open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is ĝ-T_1.

Recall that a space X is said to be ĝ-connected [32] if X cannot be expressed as the union of two distinct non-empty ĝ-open subsets of X.

Theorem: 4.24
The almost contra ĝ-continuous image of a ĝ-connected space is connected.

Proof: Let f: (X, τ) → (Y, σ) be an almost contra ĝ-continuous function of a ĝ-connected space X onto a topological space Y. Suppose that Y is not a connected space. There exist non-empty disjoint open sets V_1 and V_2 such that

$Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is almost contra ĝ-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are ĝ-open in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non-empty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not ĝ-connected. This is a contradiction and hence Y is connected.

Definition: 4.25
A topological space X is said to be ĝ-ultra-connected if every two non-empty ĝ-closed subsets of X intersect.

We recall that a topological space X is said to be hyperconnected [39] if every open set is dense.

Theorem: 4.26.
If X is ĝ-ultra-connected and f : (X, τ) → (Y, σ) is an almost contra ĝ-continuous surjection, then Y is hyperconnected.

Proof: Suppose that Y is not hyperconnected. Then, there exists an open set V such that V is not dense in Y. So, there exist non-empty regular open subsets B_1= int(cl(V)) and B_2= Y − cl(V) in Y. Since f is almost contra ĝ-continuous, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint ĝ-closed. This is contrary to the ĝ-ultra-connectedness of X. Therefore, Y is hyperconnected.

Definition: 4.27
A space X is said to be

(1) Ĝ-closed (resp. nearly compact [35]) if every ĝ-closed (resp. regular open) cover of X has a finite subcover;
(2) countably Ĝ-closed (resp. nearly countably compact [16], [36]) if every countable cover of X by ĝ-closed (resp. regular open) sets has a finite subcover;
(3) Ĝ-Lindelof (resp. nearly Lindelof [14]) if every cover of X by ĝ-closed (resp. regular open) sets has a countable subcover.

Now, we obtain the following theorem by using the definitions above.

Theorem: 4.28
Let f : (X, τ) → (Y, σ) be an almost contra ĝ-continuous surjection. Then, the following properties hold:

(1) If X is Ĝ-closed, then Y is nearly compact;
(2) If X is countably Ĝ-closed, then Y is nearly countably compact;
(3) If X is Ĝ-Lindelof, then Y is nearly Lindelof.

Proof: As the proofs of (2) and (3) are completely similar to the proof of (1), we will prove only (1). Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra ĝ-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a ĝ-closed cover of X. Since X is Ĝ-closed, there exists a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha): \alpha \in I_0\}$. Thus we have $Y = \cup \{ V_\alpha : \alpha \in I_0\}$ and Y is nearly compact.

Definition: 4.29
A space X is said to be

(1) ĜO-compact (resp. S-closed [42]) if every ĝ-open (resp. regular closed) cover of X has a finite subcover;
(2) countably-ĝ-compact (resp. countably S-closed [1]) if every countable cover of X by ĝ-open (resp. regular closed) sets of X has a finite subcover.
(3) ĝ-Lindelof [32] (resp. S-Lindelof [24]) if every ĝ-open (resp. regular closed) cover of X has a countable subcover.

Theorem: 4.30
Let f: (X, τ) → (Y, σ) be an almost contra ĝ-continuous surjection. Then, the following properties hold:

(1) If X is ĜO-compact, then Y is S-closed;
(2) If X is countably-ĝ-compact, then Y is countably S-closed;
(3) If X is ĝ-Lindelof, then Y is S-Lindelof.

Proof: Since the proofs of (2) and (3) are completely similar to the proof of (1), we will prove only (1). Let $\{V_\alpha: \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra ĝ-continuous, then $\{f^{-1}(V_\alpha): \alpha \in I\}$ is a ĝ-open cover of X and hence there exists a finite subset I_0 of I such that $X = \cup \{ f^{-1}(V_\alpha) : \alpha \in I_0\}$. So, we have $Y = \cup \{V_\alpha: \alpha \in I_0\}$ and Y is S-closed.

Recall that a space X is said to be semi-regular [40] if for any open set U of X and each point x ∈ U, there exists a regular open set V of X such that x ∈ V ⊂ U.

Definition: 4.31 [33]
Let (X, τ) be a topological space and A be any subset of X. The intersection (resp. union) of all ĝ-closed (resp. ĝ-open) sets containing (resp. contained in) a set A is called ĝ-closure (resp. ĝ- interior) of A and is denoted by ĝcl(A) (resp. ĝint(A)).

Definition: 4.32
A topological space X is said to be

(1) Ultra Hausdorff [38] if for each pair of distinct points x and y in X there exist U ∈ CO(X, x) and V ∈ CO(X, y) such that U ∩ V = ∅.
(2) ĝ- T_2 [32] if for each pair of distinct points x and y in X there exist U ∈ ĜO(X, x) and V ∈ ĜO(X, y) such that U ∩ V = ∅.

Theorem: 4.33
If f: (X, τ) → (Y, σ) is an almost contra ĝ-continuous injection and Y is Ultra Hausdorff, then X is ĝ- T_2.

Proof: Let x_1 and x_2 be any distinct points of X. Then since f is injective and Y is Ultra Hausdorff, $f(x_1) \neq f(x_2)$ and there exist V_1 and V_2 ∈ CO(Y) such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Hence V_1 and V_2 ∈ RC(Y) and then, since f is almost contra ĝ-continuous, $x_1 \in f^{-1}(V_1)$ ∈ ĜO(X), $x_2 \in f^{-1}(V_2)$ ∈ ĜO(X) and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. This shows that X is ĝ- T_2.

Definition: 4.34
Let A be a subset of a topological space X. The ĝ-frontier of A, ĝFr(A), is defined by ĝFr(A) = ĝcl(A) ∩ ĝcl(X − A) = ĝcl(A) ∩ (X − ĝint (A)).

Theorem: 4.35
The set of all points x ∈ X at which a function f: (X, τ) → (Y, σ) is not almost contra ĝ-continuous is identical with the union of the ĝ-frontier of the inverse images of regular closed sets of Y containing f(x).

Proof: Assume that f is not almost contra ĝ-continuous at x ∈ X. Then, there exists a regular closed set F of Y containing f(x) such that f(U) ∩ (Y − F) ≠ ∅ for every U ∈ ĜO(X, x). Therefore, x ∈ ĝcl(f^{-1}(Y − F)) = ĝcl(X − f^{-1}(F)). On the other hand, we obtain x ∈ f^{-1}(F) ⊂ ĝcl(f^{-1}(F)) and so x ∈ ĝFr(F).

Conversely, suppose that f is almost contra ĝ-continuous at x ∈ X and let F be any regular closed set of Y containing f(x). By Theorem 3.3, there exists U ∈ ĜO(X, x) such that x ∈ U ⊂ f^{-1}(F). Therefore, x ∈ ĝint(f^{-1}(F)). This contradicts that x ∈ ĝFr(f^{-1}(F)). Thus, f is not almost contra ĝ-continuous.

Definition: 4.36
A topological space X is said to be

(1) Ultra normal [38] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets;
(2) ĝ-normal if each pair of non-empty disjoint closed sets can be separated by disjoint ĝ-open sets.

Theorem: 4.37
If f: (X, τ) → (Y, σ) is an almost contra ĝ-continuous closed injection and Y is Ultra normal, then X is ĝ-normal.

Proof: Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, f (F_1) and f(F_2) are disjoint closed and hence they are separated by disjoint clopen sets V_1 and V_2, respectively. Since V_1, V2 ∈ RC(Y), $F_i \subset f^{-1}(V_i)$ ∈ ĜO(X) for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. This shows that X is ĝ-normal.

5. ĝ -REGULAR GRAPHS AND STRONGLY CONTRA- ĝ-CLOSED GRAPHS:

In this section, we define the notions of Ĝ-regular graphs and strongly contra- ĝ-closed graphs and investigate the relationships between the graphs and almost contra ĝ-continuous functions.

Recall that a function f: (X, τ) → (Y, σ), the subset G_f = {(x, f(x)): x ∈ X} ⊂ X × Y is said to be graph of f.

Definition: 5.1
A graph G_f of a function f : (X, τ) → (Y, σ) is said to be Ĝ -regular (resp. strongly contra- ĝ-closed) if for each (x, y) ∈ (X × Y) \ G_f, there exist a ĝ-closed (resp. ĝ-open) set U in X containing x and V ∈ RO(Y) (resp. V ∈ RC(Y)) containing y such that (U × V) ∩ G_f = ∅.

Lemma: 5.2
For a graph G_f of a function f : (X, τ) → (Y, σ), the following properties are equivalent:

(1) G_f is $\hat{G}$-regular (resp. strongly contra- $\hat{g}$-closed);
(2) For each point (x, y) ∈ (X × Y) \ G_f, there exist a $\hat{g}$-closed (resp. $\hat{g}$-open) set U in X containing x and V ∈ RO(Y) (resp. V ∈ RC(Y)) containing y such that f (U) ∩ V = ∅.

Proof: This is a direct consequence of Definition 5.1 and the fact that for any subsets A ⊂ X and B ⊂ Y, (A × B) ∩ G_f = ∅ if and only if f(A) ∩ B = ∅.

Theorem: 5.3
If f: (X, τ) → (Y, σ) is almost contra $\hat{g}$-continuous and Y is T_2, then G_f is $\hat{G}$ -regular in X × Y.

Proof: Let (x, y) ∈ (X × Y) \ G_f. It is obvious that f(x) ≠ y. Since Y is T_2, there exist V, W ∈ RO(Y) such that f(x) ∈ V, y ∈ W and V ∩ W = ∅. Since f is almost contra $\hat{g}$-continuous, $f^{-1}(V)$ is a $\hat{g}$-closed set in X containing x. If we take U = $f^{-1}(V)$. We have f(U) ⊂ V. Therefore, f(U) ∩ W = ∅ and G_f is $\hat{G}$ -regular.

Definition: 5.4 A space X is said to be $\hat{g}$-T_0 if for each pair of distinct points in X, there exists a $\hat{g}$-open set of X containing one point but not the other.

Theorem: 5.5
Let f : (X, τ) → (Y, σ) have a $\hat{G}$ -regular graph G_f. If f is injective, then X is $\hat{g}$-T_0.

Proof: Let x and y be any two distinct points of X. Then, we have (x, f(y)) ∈ (X × Y) \ G_f. Since G_f is $\hat{G}$ -regular, there exist a $\hat{g}$-closed set U of X and V ∈ RO(Y) such that (x, f(y)) ∈ (U × V) and f (U) ∩ V = ∅ by Lemma 5.2. and hence U ∩ $f^{-1}(V)$ = ∅. Therefore, we have y ∉ U. Thus, y ∈ (X−U) and x ∉ (X−U). We obtain (X−U) ∈ $\hat{G}O(X)$. This implies that X is $\hat{g}$-T_0.

Theorem: 5.6
Let f: (X, τ) → (Y, σ) have a $\hat{G}$ -regular graph G_f. If f is surjective, then Y is weakly Hausdorff.

Proof: Let y_1 and y_2 be any distinct points of Y. Since f is surjective, f(x) = y_1 for some x ∈ X and (x, y_2) ∈ (X × Y) \ G_f. By Lemma 5.2. there exist a $\hat{g}$-closed set U of X and F ∈ RO(Y) such that (x, y_2) ∈ (U × F) and f(U) ∩ F = ∅; hence y_1 ∉ F. Then y_2 ∉ (Y – F) ∈ RC(Y) and y_1 ∈ (Y − F). This implies that Y is weakly Hausdorff.

Theorem: 5.7
Let f : (X, τ) → (Y, σ) have a strongly contra $\hat{g}$-closed graph G_f. If f is an almost contra $\hat{g}$-continuous injection, then X is $\hat{g}$-T_2.

Proof: Let x and y be the two distinct points of X. Since f is injective, we have f(x) ≠ f(y). Then, we have (x, f(y)) ∈ (X × Y) \ G_f. Since G_f is strongly contra $\hat{g}$-closed, by Lemma 5.2, there exists U ∈ $\hat{G}O(X, x)$ and a regular closed set V containing f(y) such that f (U) ∩ V = ∅. Therefore, U ∩ $f^{-1}(V)$ = ∅. Since f is almost contra $\hat{g}$-continuous, $f^{-1}(V)$∈ $\hat{G}O(X, y)$. This shows that X is $\hat{g}$-T_2.

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