International Journal of Mathematical Archive-2(6), June - 2011, Page: 870-875
Available online through www.ijma.info ISSN 2229 – 5046

ON Λ_{Bg}–CLOSED SETS IN TOPOLOGICAL SPACES

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(Received on: 02-06-11; Accepted on: 11-06-11)

ABSTRACT

We introduce new classes of sets called Λ_{Bg}-closed sets and Λ_{Bg}-open sets in topological spaces. We also investigate several properties of such sets. It turns out that Λ_{Bg} - closed sets and Λ_{Bg} - open sets are weaker forms of B-closed sets and B-open sets, respectively and stronger forms of Bg -closed sets and Bg-open sets, respectively.

2000 Mathematics Subject Classification: *54B05, 54C08.*

Keywords and Phrases: *λ-open set, Bg-closed set, Λ_{Bg}–closed set, locally-B-closed set.*

1. INTRODUCTION:

In 1986, Maki [9] introduced the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel (saturated set), i.e to the intersection of all open supersets of A. Arenas et al. [2] introduced and investigated the notion of λ-closed sets by involving Λ-sets and closed sets. Caldas et al. [4] introduced the notion of the λ-closure of a set by utilizing the notion of λ-open sets defined in [2]. Levine [7] introduced the notions of simply extended topological spaces. Abd El-Monsef et al. [1] introduced the notions of B-open sets and associated interior and closure operators on simply extended topological spaces.

In this paper, we introduce new classes of sets called Λ_{Bg}-closed sets and Λ_{Bg} -open sets in topological spaces. We also establish several properties of such sets. It turns out that Λ_{Bg}-closed sets and Λ_{Bg}-open sets are weaker forms of B-closed sets and B-open sets, respectively and stronger forms of Bg-closed sets and Bg-open sets, respectively.

2. PRELIMINARIES:

Throughout this paper, by (X, τ) and (Y, σ) (or X and Y) we always mean topological spaces. Let A be a subset of X. We denote the interior, the closure and the complement of a set A by int(A), cl(A) and $X \setminus A$ or A^c, respectively.

A subset A of a space (X, τ) is called λ-closed [2] if $A = L \cap D$, where L is a Λ-set and D is a closed set. The complement of λ-closed set is called λ-open. A subset A of a space (X, τ) is called semi-open [8] if $A \subseteq cl(int(A))$. The complement of semi-open set is called semi-closed. The intersection of all semi-closed subsets of X containing A is called the semi-closure [5] of A and is denoted by scl(A).

A subset A of a space (X, τ) is called preopen [10] if $A \subseteq int(cl(A))$. The complement of preopen set is called preclosed. The intersection of all preclosed subsets of X containing A is called the preclosure of A and is denoted by pcl(A). The union of all preopen subsets of X contained in A is called the preinterior of A and is denoted by pint (A). A subset A of a space (X, τ) is called regular open [13] if $A = int(cl(A))$. The complement of regular open set is called regular closed.

Let X be a non empty set and Levine [7] defined $\tau(B) = \{ O \cup (O' \cap B) : O, O' \in \tau)$ and called it simple extension of τ by B, where $B \notin \tau$. We recall the pair $(X, \tau (B))$ a simply extended topological spaces (briefly SETS). The elements of τ (B) are called B-open [1] sets and the complements are called B-closed sets [1]. The family of B-open sets of X forms a topology. In other words, we can say, A is closed set in $(X, \tau (B))$ or A is a B-closed set in (X, τ). The B-closure of a subset S of X, denoted by Bcl(S) [1], is the intersection of B-closed sets of X containing S and the B-interior of S, denoted by Bint(S), is the union of B-open sets contained in S. A subset A of a space (X, τ) is called Bλ-closed [12] if $A = L \cap D$, where L is a Λ-set and D is a B-closed. The complement of Bλ-closed is called Bλ-open.

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The intersection of all Bλ-closed sets containing a subset A of X is called the Bλ-closure of A and is denoted by $cl_{B\lambda}(A)$. We denote the collection of all Bλ-open sets by BλO(X, τ).

Let $\tau(B_x)$ and $\tau(B_y)$ be simple extension of topologies on X and Y respectively.

Lemma: 2.1[12] Let A_i ($i \in I$) be subsets of a topological space (X, τ). The following properties hold:

(i) If A_i is Bλ-closed for each $i \in I$, then $\cap_{i \in I} A_i$ is Bλ-closed.
(ii) If A_i is Bλ-open for each $i \in I$, then $\cup_{i \in I} A_i$ is Bλ-open.

3. Λ_{Bg} - CLOSED SETS:

Definition: 3.1[1] A subset A of a topological space (X, τ) is called B-generalized closed set (briefly Bg-closed) if $Bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). B is Bg-open set of (X, τ) if and only if B^c is Bg-closed.

Definition: 3.2 A subset A of a topological space (X, τ) is called Λ_{Bg} -closed if $Bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is Bλ- open in (X, τ).

Lemma: 3.3 For subsets A and A_i ($i \in I$) of a space (X, τ), the following hold:

(1) $B \subseteq \Lambda_{Bg}(B)$,
(2) $A \subseteq B$, then $\Lambda_{Bg}(A) \subseteq \Lambda_{Bg}(B)$,
(3) $\Lambda_{Bg}(\Lambda_{Bg}(B)) = \Lambda_{Bg}(B)$,
(4) $\Lambda_{Bg}(A \cup B) = \Lambda_{Bg}(A) \cup \Lambda_{Bg}(B)$,
(5) A is B-closed ⇔ A = Bcl(A),
(6) A is B-open ⇔ A = Bint(A),
(7) $\Lambda_{Bg}(\cap \{A_i : i \in I\}) \subset \cap \{\Lambda_{Bg}(A_i) : i \in I\}$,
(8) $\Lambda_{Bg}(\cup \{A_i : i \in I\}) = \cup \{\Lambda_{Bg}(A_i) : i \in I\}$,

Remark: 3.4 Let {Ai: $i \in I$} be a family of subsets of a space X. In general $\cap \{\Lambda_{Bg}(A_i) : i \in I\} \nsubseteq \Lambda_{Bg}(\cap \{A_i : i \in I\})$ and $A_i \neq \Lambda_{Bg}(A_i)$.

Example: 3.5 Let X = {a, b, c}, τ = {∅, X, {a}, {a, b}} and B = {c}. Then $\tau(B_x)$ = {∅, X, {a}, {c}, {a, b}, {a, c}}. Let A = {a} and B = {b}. Then $\Lambda_{Bg}(A \cap B) = \Lambda_{Bg}(\emptyset) = \emptyset$. Also, we have $\Lambda_{Bg}(A) = \{a, b\}$ and $\Lambda_{Bg}(B) = \{b\}$. Thus $\Lambda_{Bg}(A) \cap \Lambda_{Bg}(B) = \{b\} \nsubseteq \Lambda_{Bg}(A \cap B) = \emptyset$ and $A = \{a\} \neq \Lambda_{Bg}(A) = \{a, b\}$.

Remark: 3.6
We have the following implications.

$$\begin{tikzcd}
\text{closed} \arrow[r] \arrow[d] & \text{B-closed} \arrow[r] \arrow[d] & \Lambda_{Bg}\text{-closed} \arrow[r] & \text{Bg - closed} \\
\lambda\text{-closed} \arrow[r] & \text{B}\lambda\text{-closed} & &
\end{tikzcd}$$

None of these implications is reversible as shown in the following example.

Example: 3.7 Let X= {a, b, c, d}, τ = {∅, X, {a, d}} and B = {c}. Then $\tau(B_x)$ = {∅, X, {c}, {a, d}, {a, c, d}}.

(1) Here A = {c} is a B-closed set but it is not closed.
(2) Here A = {a, b, c} is a Λ_{Bg}-closed set but it is not B-closed.
(3) Here A = {a, d} is a Bg-closed set but it is not Λ_{Bg} -closed.
(4) Here A = {a, c, d} is Bλ-closed set but it is not B-closed.
(5) Here A = {a, b, d} is Bλ-closed set but it is not λ-closed.
(6) λ-closed sets and B-closed sets are independent of each other. Here A = {a, d} is λ-closed set but it is not B-closed and A = {b} is B-closed set but it is not λ-closed.

Theorem: 3.8 The union of two Λ_{Bg} - closed sets is Λ_{Bg} - closed.

Proof: Let $A \cup B \subseteq U$, then $A \subseteq U$ and $B \subseteq U$ where U is Bλ-open. As A and B are Λ_{Bg} - closed $Bcl(A) \subseteq U$ and $Bcl(B) \subseteq U$. Hence $Bcl(A \cup B) = Bcl(A) \cup Bcl(B) \subseteq U$.

Remark: 3.9 The intersection of two Λ_{Bg} - closed sets need not be Λ_{Bg} - closed as can be verified from the following example.

Example: 3.10 Let X = {a, b, c}, τ = {∅, X} and B = {a}. Then τ (B_x) = {∅, X, {a}}. Here A = {a, b} and B = {a, c} are Λ_{Bg} - closed sets but A ∩ B = {a} is not Λ_{Bg} - closed set.

Theorem: 3.11 If a subset A of (X, τ) is Λ_{Bg} -closed, then Bcl(A) \ A contains no non empty B-closed subset of (X, τ).

Proof: Let F be a B-closed subset contained in Bcl(A) \ A. Clearly A ⊆ F^c where A is Λ_{Bg} -closed and F^c is an B-open set of X. Thus Bcl (A) ⊆ F^c or F ⊆ $[Bcl(A)]^c$. Then F ⊆ $[Bcl(A)]^c$ ∩ (Bcl(A) \ A) ⊆ $[Bcl(A)]^c$ ∩ Bcl(A)= ∅. This shows that F= ∅.

The converse of the above theorem is not true in general as it is shown in the following example.

Example: 3.12 Let A = {d} from Example 3.7. Then Bcl(A) \ A={a, b} does not contain nonempty B -closed set . But A is not Λ_{Bg} -closed in (X, τ).

Definition: 3.13 A topological space (X, τ) is called a BT_1 -space if to each pair of distinct points x, y of (X, τ) there exist a B-open set U containing x but not y and a B-open set V containing y but not x.

Theorem: 3.14 A topological space (X, τ) is a BT_1-space if and only if every subset of X consisting of exactly one point is B-closed.

Proof: Let (X, τ) be a BT1-space and x be an arbitrary point of X. Then, we must show that {x} is B-closed or equivalently that $(\{x\})^c$ is B-open. If $(\{x\})^c$ = ∅, then it is clearly B-open. So, let $(\{x\})^c$ ≠ ∅ and let y ∈ $(\{x\})^c$. Then y ≠ x. But, (X, τ) being a BT_1-space there exist a B-open set G containing y but not x. Consequently, y ∈ G ⊆ $(\{x\})^c$. This shows that $(\{x\})^c$ is neighbourhood of each of its points and therefore, B-open. Hence, {x} is B-closed.

Conversely, let (X, τ) be a topological space such that every subset of X consisting of exactly one point is B-closed. Let x and y be any two distinct points of X. Then, by hypothesis, {x} as well as {y} is B-closed. Consequently, G = $(\{x\})^c$ and H = $(\{y\})^c$ are B-open sets such that y ∈ G but x ∉ G and x ∈ H but y ∉ H. Hence (X, τ) is a BT_1-space.

Corollary: 3.15 In a BT_1 - space, every Λ_{Bg} -closed set is B-closed.

Proof: Let A be a Λ_{Bg} -closed set in a BT_1- space (X, τ). Let x ∈ Bcl(A) \ A. Since (X, τ) is BT_1, {x} is a B-closed set in (X, τ). By Theorem 3.11. there exists no nonempty B -closed set in Bcl(A) \ A and so Bcl(A) \ A= ∅. Therefore Bcl (A) = A, i.e., A is B-closed.

Theorem: 3.16 A set A is Λ_{Bg} - closed if and only if Bcl (A) \ A contains no nonempty Bλ -closed sets.

Proof: Necessity. Suppose that A is Λ_{Bg} -closed. Let S be a Bλ -closed subset of Bcl (A) \ A. Then A ⊆ S^c. Since A is Λ_{Bg}-closed, we have Bcl(A) ⊆ S^c. Consequently S ⊆ $[Bcl(A)]^c$. Hence S ⊆ Bcl(A) ∩$[Bcl(A)]^c$= ∅. Therefore S is empty.

Sufficiency. Suppose that Bcl(A) \ A contains no nonempty Bλ -closed sets. Let A ⊆ G and G be Bλ -open. If Bcl(A) ⊈ G, then Bcl(A) ∩ G^c is a nonempty Bλ-closed subset of Bcl(A) \ A. Therefore, A is Λ_{Bg}-closed.

Theorem: 3.17 If A is a Λ_{Bg} -closed set of (X, τ) and A ⊆ B ⊆ Bcl(A), then B is a Λ_{Bg} -closed set of (X, τ).

Proof: Since B ⊆ Bcl(A), we have Bcl(B) ⊆ Bcl(A). Hence (Bcl(B) \ B) ⊆ (Bcl(A) \ A). But by Theorem 3.16 Bcl(A) \ A contains no nonempty Bλ-closed subsets of X and hence Bcl(B) \ B does not contain Bλ-closed sets. Again by Theorem 3.16, B is Λ_{Bg} - closed.

Theorem: 3.18 If A is a Bλ -open and Λ_{Bg}-closed set in (X, τ), then A is B-closed in (X, τ).

Proof: Since A is Bλ-open and Λ_{Bg} -closed, Bcl(A) ⊆ A and hence A is B-closed in (X, τ).

Theorem: 3.19 For each x ∈ X, either {x} is Bλ -closed or $\{x\}^c$ is Λ_{Bg}-closed in (X, τ).

Proof: Suppose {x} is not Bλ -closed in (X, τ). Then $\{x\}^c$ is not Bλ -open and the only Bλ -open set containing $\{x\}^c$ is the space X itself. Therefore Bcl ($\{x\}^c$) ⊆ X and so $\{x\}^c$ is Λ_{Bg} -closed in (X, τ).

Theorem: 3.20 Let A be a Λ_{Bg}-closed set in (X, τ). Then

(1) If A is regular open, then pint (A) and scl(A) are also Λ_{Bg}-closed.
(2) If A is regular closed, then pcl (A) is also Λ_{Bg}-closed.

Proof:
(1) Since A is regular open in (X, τ), we have $scl(A)=A \cup int(cl(A)) = A$ and $pint(A)=A \cap int(cl(A)) = A$. Thus scl(A) and pint(A) are Λ_{Bg}-closed in (X, τ).
(2) Let A be regular closed in (X, τ). Then $pcl(A)=A \cup cl(int(A)) = A$. Thus pcl(A) is Λ_{Bg}-closed in (X, τ).

Definition: 3.21 A space X is said to be a B-normal space if for every pair of disjoint B-closed subsets A and B of X there exist B-open sets U, V such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$.

Remark: 3.22 If (X, τ) is a B-normal space and suppose that Y is a Bg-closed subset of X. Then $(Y, Y \cap \tau)$ is B-normal.

Proof: Let E and F be B-closed in X and suppose that $(Y \cap E) \cap (Y \cap F) = \emptyset$. Then $Y \subseteq (E \cap F)^c \in \tau$ and hence $Bcl(Y) \subseteq (E \cap F)^c$. Thus $(Bcl(Y) \cap E) \cap (Bcl(Y) \cap F) = \emptyset$. Since (X, τ) is B-normal, there exists disjoint B-open sets U and V such that $Bcl(Y) \cap E \subseteq U$ and $Bcl(Y) \cap F \subseteq V$. It follows then that $Y \cap E \subseteq U \cap Y$ and $Y \cap F \subseteq V \cap Y$.

Remark: 3.23 By Remark 3.6. every Λ_{Bg}- closed set of a B-normal space is B-normal.

Definition: 3.24. A subset S of X is said to be locally-B-closed if $S =U \cap F$, where U is B-open and F is B-closed in (X, τ).

Theorem: 3.25. For a subset S of (X, τ), the following are equivalent.

(1) S is locally-B-closed.
(2) $S = P \cap Bcl(S)$ for some B-open set P.
(3) $Bcl(S) – S$ is B-closed.
(4) $S \cup (X– Bcl(S))$ is B-open.
(5) $S \subseteq Bint(S \cup (X – Bcl(S)))$.

Proof:
(1) → (2) $S = P \cap Q$ where P is B-open and Q is B-closed. $S \subset Q$ implies $Bcl(S) \subset Q$. So $S = P \cap Q \supset P \cap Bcl(S)$. And $S \subset P$ and $S \subset Bcl(S)$ implies $S\subset P \cap Bcl(S)$. Hence $S = P \cap Bcl(S)$.
(2) → (3) $Bcl(S) – S = Bcl(S) \cap (X – P)$ which is B-closed.
(3) → (4) $S \cup (X – Bcl(S)) = X – (Bcl(S) – S)$. Hence $S \cup (X – (Bcl(S)))$ is B- open.
(4) → (5) Since $S \cup (X – (Bcl(S)))$ is B-open, $S \subseteq Bint(S \cup (X – (Bcl(S))))$.
(5) → (6) $S \subseteq Bint (S \cup (X – (Bcl(S))))$ implies $S = Bint(S \cup (X – (Bcl(S)))) \cap Bcl(S)$.

Theorem: 3.26 Let A be locally-B-closed subset of (X, τ). For the set A, the following properties are equivalent:

(1) A is B-closed;
(2) A is Λ_{Bg}-closed;
(3) A is Bg -closed.

Proof: By Remark 3.6, it suffices to prove that (3) implies (1). By Theorem 3.25 $A \cup (Bcl(A))^c$ is B-open in (X, τ) since A is locally-B-closed. Now $A \cup (Bcl(A))^c$ is an B-open set of (X, τ) such that $A \subseteq A\cup (Bcl(A))^c$. Since A is Bg-closed, then $Bcl (A) \subseteq A \cup (Bcl (A))^c$. But $Bcl(A) \cap(Bcl(A))^c=\emptyset$. Thus we have $Bcl(A) \subseteq A$ and hence A is B-closed.

Definition: 3.27 A subset A in (X, τ) is said to be Λ_{Bg}-open in (X, τ) if and only if A^c is Λ_{Bg}-closed in (X, τ).

Every B-open set in (X, τ) is Λ_{Bg}-open in (X, τ) but not conversely. It can be verified from the following example.

Example: 3.28 Let $A = \{a\}$ from Example 3.7. Then A is Λ_{Bg}-open set but it is not B-open in (X, τ).

Theorem: 3.29 The intersection of two Λ_{Bg}- open sets is Λ_{Bg}- open.

Proof: This is obvious by Theorem 3.8.

Theorem: 3.30 A set A is Λ_{Bg}-open in (X, τ) if and only if $F \subseteq Bint(A)$ whenever F is $B\lambda$-closed in (X, τ) and $F \subseteq A$.

Proof: Suppose that $F \subseteq Bint(A)$ whenever F is Bλ -closed and $F \subseteq A$. Let $A^c \subseteq G$, where G is Bλ-open. Hence $G^c \subseteq A$. By assumption $G^c \subseteq Bint(A)$ which implies that $(Bint(A))^c \subseteq G$, so $Bcl(A^c) \subseteq G$. Hence A^c is Λ_{Bg} -closed i,e., A is Λ_{Bg} - open.

Conversely, let A be Λ_{Bg} -open. Then A^c is Λ_{Bg} -closed. Also let F be a Bλ-closed set contained in A. Then F^c is Bλ-open. Therefore whenever $A^c \subseteq F^c$, $Bcl(A^c) \subseteq F^c$. This implies that $F \subseteq (Bcl(A^c))^c = Bint(A)$. Thus $F \subseteq Bint(A)$.

Theorem: 3.31 A set A is Λ_{Bg} -open in (X, τ) if and only if G = X whenever G is Bλ -open and $Bint(A) \cup A^c \subseteq G$.

Proof: Let A be Λ_{Bg} -open, G Bλ-open and $Bint(A) \cup A^c \subseteq G$. This gives $G^c \subseteq (Bint(A))^c \cap (A^c)^c = (Bint(A))^c \setminus A^c = Bcl(A^c) \setminus A^c$. Since A^c is Λ_{Bg} -closed and G^c is Bλ -closed, by Theorem 3.16 it follows that $G^c = \emptyset$. Therefore X = G. Conversely, suppose that F is Bλ - closed and $F \subseteq A$. Then $Bint(A) \cup A^c \subseteq Bint(A) \cup F^c$. It follows that $Bint(A) \cup F^c = X$ and hence $F \subseteq Bint(A)$. Therefore A is Λ_{Bg} - open.

Theorem: 3.32 If $Bint(A) \subseteq B \subseteq A$ and A is Λ_{Bg} -open in (X, τ), then B is Λ_{Bg} - open in (X, τ).

Proof: Suppose $Bint(A) \subseteq B \subseteq A$ and A is Λ_{Bg}-open in (X, τ). Then $A^c \subseteq Bc \subseteq Bcl(A^c)$ and A^c is Λ_{Bg} -closed. By Theorem 3.17, B is Λ_{Bg} -open in (X, τ).

Theorem: 3.33. A set A is Λ_{Bg} -closed in (X, τ) if and only if $Bcl(A) \setminus A$ is Λ_{Bg} -open in (X, τ).

Proof: Necessity. Suppose that A is Λ_{Bg} -closed in (X, τ). Let $F \subseteq Bcl(A) \setminus A$, where F is Bλ - closed. By Theorem 3.16, $F = \emptyset$. Therefore $F \subseteq Bint(Bcl(A) \setminus (A))$ and by Theorem 3.30, $Bcl(A) \setminus A$ is Λ_{Bg} -open in (X, τ).

Sufficiency. Let $A \subseteq G$ where G is Bλ - open. Then $Bcl(A) \cap G^c \subseteq Bcl(A) \cap A^c = Bcl(A) \setminus A$. Since $Bcl(A) \cap G^c$ is Bλ - closed and $Bcl(A) \setminus A$ is Λ_{Bg} -open, by Theorem 3.30, we have $Bcl(A) \cap G^c \subseteq Bint(Bcl(A) \setminus A) = \emptyset$. Hence A is Λ_{Bg} - -closed in (X, τ).

Theorem: 3.34 For a subset $A \subseteq X$, the following properties are equivalent.

(1) A is Λ_{Bg}-closed.
(2) $Bcl(A) \setminus A$ contains no nonempty Bλ - closed set.
(3) $Bcl(A) \setminus A$ is Λ_{Bg} -open.

Proof: This follows from Theorems 3.16 and 3.33.

Theorem: 3.35 A subset A in (X, τ) is Λ_{Bg} -closed if and only if $cl_{B\lambda}(\{x\}) \cap A \neq \emptyset$ for every $x \in Bcl(A)$.

Proof: Necessity. Suppose that $cl_{B\lambda}(\{x\}) \cap A = \emptyset$ for some $x \in Bcl(A)$. Then $X - cl_{\lambda}(\{x\})$ is a Bλ-open set containing A. Furthermore, $x \in Bcl(A) - (X - cl_{B\lambda}(\{x\}))$ and hence $Bcl(A) \not\subset X - cl_{B\lambda}(\{x\})$. This shows that A is not Λ_{Bg} -closed.

Sufficiency. Suppose that A is not Λ_{Bg}-closed. There exists a Bλ-open set U containing A such that $Bcl(A) - U \neq \emptyset$. There exists $x \in Bcl(A)$ such that $x \notin U$; hence $cl_{B\lambda}(\{x\}) \cap U = \emptyset$. Therefore, $cl_{B\lambda}(\{x\}) \cap A = \emptyset$ for some $x \in Bcl(A)$.

4. FUNCTIONS:

Definition: 4.1 A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

(1) Bλ -irresolute if $f^{-1}(V)$ is Bλ-open in X for every Bλ-open set V of Y,
(2) Bλ-closed if f(F) is Bλ-closed in Y for every Bλ-closed set F of X,
(3) B-continuous if $f^{-1}(V)$ is B-closed in X for every B-closed set V of Y.

Definition: 4.2 A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be B-closed if the image of every B-closed set in (X, τ) is B-closed set in (Y, σ).

Theorem: 4.3 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be Bλ-irresolute B-closed function. If A is Λ_{Bg} -closed in X, then f (A) is Λ_{Bg} -closed in Y.

Proof: Let A be a Λ_{Bg} -closed set of X and V a Bλ-open set of Y containing f(A). Since f is Bλ-irresolute, $f^{-1}(V)$ is Bλ-open in X and $A \subset f^{-1}(V)$. Since A is Λ_{Bg}-closed, $Bcl(A) \subset f^{-1}(V)$ and $f(A) \subset f(Bcl(A)) \subset V$. Since f is B-closed, we obtain $Bcl(f(A)) \subset V$. This shows that f (A) is Λ_{Bg}-closed in Y.

Lemma: 4.4 A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is Bλ-closed if and only if for each subset B of Y and each $U \in B\lambda O(X, \tau)$ containing $f^{-1}(B)$, there exists $V \in B\lambda O(Y, \sigma)$ such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof: Necessity. Suppose that f is Bλ-closed. Let $B \subset Y$ and $U \in B\lambda O(X, \tau)$ containing $f^{-1}(B)$. Put $V = Y - f(X - U)$. Then we obtain $V \in B\lambda O(Y, \sigma)$, $B \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency. Let F be any Bλ-closed set of (X, τ). Set $f(F) = B$, then $F \subset f^{-1}(B)$ and $f^{-1}(Y - B) \subset X - F \in B\lambda O(X, \tau)$. By hypothesis, there exists $V \in B\lambda O(Y, \sigma)$ such that $Y - B \subset V$ and $f^{-1}(V) \subset X - F$. Therefore we obtain $Y - V \subset B = f(F) \subset Y - V$. Hence $f(F) = Y - V$ and $f(F)$ is Bλ-closed in (Y, σ). Therefore, f is Bλ-closed.

Theorem: 4.5 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a B-continuous Bλ-closed function. If B is a Λ_{Bg}-closed set of (Y, σ), then $f^{-1}(B)$ is Λ_{Bg}-closed in (X, τ).

Proof: Let B be a Λ_{Bg}-closed in (Y, σ) and U a Bλ-open set of (X, τ) containing $f^{-1}(B)$. Since f is Bλ-closed, by Lemma 4.4 there exists a Bλ-open set V of (Y, σ) such that $B \subset V$ and $f^{-1}(V) \subset U$. Since B is Λ_{Bg}-closed in (Y, σ), $Bcl(B) \subset V$ and hence $f^{-1}(B) \subset f^{-1}(Bcl(B)) \subset f^{-1}(V) \subset U$. Since f is B-continuous, $f^{-1}(Bcl(B))$ is B-closed and hence $Bcl(f^{-1}(B)) \subset U$. This shows that $f^{-1}(B)$ is Λ_{Bg}-closed in (X, τ).

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