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DEVELOPMENT OF SOME SUMMATION FORMULAE BASED ON HALF ARGUMENT USING GAUSS SECOND SUMMATION THEOREM

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ABSTRACT

The main object of this paper is to develop some summation formulae based on half argument by the help of Gauss second summation theorem. The results derived in this paper are of general character.

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A. INTRODUCTION:

Generalized Gaussian Hypergeometric function of one variable:

$${}_AF_B(a_1,a_2,\ldots,a_A;b_1,b_2,\ldots b_B;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \ldots (a_A)_k\, z^k}{(b_1)_k (b_2)_k \ldots (b_B)_k\, k!} \quad (1)$$

or

$${}_AF_B((a_A);(b_B);z) \equiv {}_AF_B((a_j)^A_{j=1};(bj)^B_{j=1};z) = \sum_{k=0}^{\infty} \frac{((a_A))_k\, z^k}{((b_B))_k\, k!} \quad (2)$$

where the parameters b_1 , b_2 , ….,b_B are neither zero nor negative integers and A , B are non negative integers.

Contiguous Relations:

[Andrews p.363 (9.16), E.D. p.51 (10), H.T.F.I. p.103(32)]

$$(a-b)\ {}_2F_1(a, b ; c; z) = a\ {}_2F_1(a+1,b; c; z) - b\ {}_2F_1(a,b+1; c; z) \quad (3)$$

[Abramowitz p.558 (15.2.19)]

$$(a-b)(1-z)\ {}_2F_1(a, b ; c; z) = (c-b)\ {}_2F_1(a,b-1; c; z) + (a-c)\ {}_2F_1(a-1,b; c; z) \quad (4)$$

Gauss second summation theorem is defined as [Prud .,491(7.3,7.5)]

$${}_2F_1\left(a, b ; \frac{a+b+1}{2}; \frac{1}{2}\right) = 2^{b-1} \frac{\Gamma(\frac{b}{2})\,\Gamma(\frac{a+b+1}{2})}{\Gamma(b)\,\Gamma(\frac{a+1}{2})} \quad (5)$$

Recurrence relation:

$$\Gamma(\zeta+1) = \zeta\,\Gamma(\zeta) \quad (6)$$

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B. MAIN RESULT OF SUMMATION FORMULAE:

$$ {}_2F_1\left(a, b ; \frac{a+b+15}{2}; \frac{1}{2}\right) = 2^b \frac{\Gamma\left(\frac{a+b+15}{2}\right)}{(a-b)\Gamma(b)}\left[\frac{\Gamma\left(\frac{b}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)}\left\{ \frac{64a(10395-19524a+12139a^2-3480a^3+505a^4-36a^5+a^6)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]} \right.\right. $$

$$ + \frac{64a(-4056b+58526ab-12896a^2b+8788a^3b-520a^4b+78a^5b+33111b^2-1144ab^2+25454a^2b^2-1144a^3b^2)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]} $$

$$ + \frac{64a(715a^4b^2+3952b^3+17732ab^3+1716a^3b^3+2561b^4+572ab^4+1287a^2b^4+104b^5+286ab^5+13b^6)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]} $$

$$ + \frac{64b(10395-4056a+33111a^2+3952a^3+2561a^4+104a^5+13a^6-19524b+58526ab-1144a^2b+17732a^3b)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]} $$

$$ + \frac{64b(572a^4b+286a^5b+12139b^2-12896ab^2+25454a^2b^2+1287a^4b^2-3480b^3+8788ab^3-1144a^2b^3)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]} $$

$$ \left. + \frac{64b(1716a^3b^3+505b^4-520ab^4+715a^2b^4-36b^5+78ab^5+b^6)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]} \right\} $$

$$ - \frac{\Gamma\left(\frac{b+1}{2}\right)}{\Gamma\left(\frac{a}{2}\right)}\left\{ \frac{128(10395+4056a+33111a^2-3952a^3+2561a^4-104a^5+13a^6+19524b+58526ab+1144a^2b+17732a^3b)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]} \right. $$

$$ + \frac{128(-572a^4b+286a^5b+12139b^2+12896ab^2+25454a^2b^2+1287a^4b^2+3480b^3+8788ab^3+1144a^2b^3)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]} $$

$$ + \frac{128(1716a^3b^3+505b^4+520ab^4+715a^2b^4+36b^5+78ab^5+b^6)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]} + \frac{128(10395+19524a+12139a^2+3480a^3)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]} $$

$$ + \frac{128(505a^4+36a^5+a^6+4056b+58526ab+12896a^2b+8788a^3b+520a^4b+78a^5b+33111b^2+1144ab^2)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]} $$

$$ + \frac{128(25454a^2b^2+1144a^3b^2+715a^4b^2-3952b^3+17732ab^3+1716a^3b^3+2561b^4-572ab^4+1287a^2b^4)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]} $$

$$ \left.\left. + \frac{128(-104b^5+286ab^5+13b^6)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]} \right\}\right] \tag{7} $$

$$ {}_2F_1\left(a, b ; \frac{a+b+16}{2}; \frac{1}{2}\right) = 2^b \frac{\Gamma\left(\frac{a+b+16}{2}\right)}{(a-b)\Gamma(b)}\left[\frac{\Gamma\left(\frac{b}{2}\right)}{\Gamma\left(\frac{a}{2}\right)}\left\{ \frac{128(46080a-56448a^2+25984a^3-5880a^4+700a^5-42a^6)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]} \right.\right. $$

$$ + \frac{128(a^7+46080b+174720a^2b-27664a^3b+14924a^4b-728a^5b+91a^6b+56448b^2+174720ab^2+56056a^3b^2)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]} $$

$$ + \frac{128(-2002a^4b^2+1001a^5b^2+25984b^3+27664ab^3+56056a^2b^3+3003a^4b^3+5880b^4+14924ab^4+2002a^2b^4)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]} $$

$$ + \frac{128(3003a^3b^4+700b^5+728ab^5+1001a^2b^5+42b^6+91ab^6+b^7)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]} + \frac{256b(46080+14208a+4377a^2+4808a^3)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]} $$

$$ + \frac{256b(1988a^4+70a^5+7a^6-14208b+113152ab+7592a^2b+17264a^3b+546a^4b+182a^5b+43776b^2-7592ab^2)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]} $$

$$+\frac{256b\left(33176a^2b^2+572a^3b^2+1001a^4b^2-4808b^3+17264ab^3-572a^2b^3+1716a^3b^3+1988b^4-546ab^4\right)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]}$$

$$+\frac{256b\left(1001a^2b^4-70b^5+182ab^5+7b^6\right)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]}\Big\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Big\{\frac{256a\left(46080-14208a+43776a^2-4808a^3+1988a^4\right)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]}$$

$$+\frac{256a\left(-70a^5+7a^6+14208b+113152ab-7592a^2b+17264a^3b-546a^4b+182a^5b+43776b^2+7592ab^2\right)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]}$$

$$+\frac{256a\left(33176a^2b^2-572a^3b^2+1001a^4b^2+4808b^3+17264ab^3+572a^2b^3+1716a^3b^3+1988b^4+546ab^4\right)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]}$$

$$+\frac{256a\left(1001a^2b^4+70b^5+182ab^5+7b^6\right)}{[\prod_{\lambda=0}^{6}\{a-b-2\lambda\}]\,[\prod_{\mu=1}^{7}\{a-b-2\mu\}]}+\frac{128\left(46080a+56448a^2+25984a^3+5880a^4+700a^5+42a^6+a^7\right)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]}$$

$$+\frac{128\left(46080b+174720a^2b+27664a^3b+14924a^4b+728a^5b+91a^6b-56448b^2+174720ab^2+56056a^3b^2\right)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]}+$$

$$+\frac{128\left(2002a^4b^2+1001a^5b^2+25984b^3-27664ab^3+56056a^2b^3+3003a^4b^3-5880b^4+14924ab^4\right)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]}$$

$$+\frac{128\left(-2002a^2b^4+3003a^3b^4+700b^5-728ab^5+1001a^2b^5-42b^6+91ab^6+b^7\right)}{[\prod_{\sigma=0}^{7}\{a-b-2\sigma\}]\,[\prod_{\tau=1}^{6}\{a-b-2\tau\}]}\Big\}\Big] \qquad (8)$$

C. DERIVATIONS OF SUMMATION FORMULAE (7) TO (8):

Derivation of (7): Replacing $c=\frac{a+b+15}{2}$ and $z=\frac{1}{2}$ in equation (3) , we get

$$(a-b)\,{}_2F_1(a,b;\tfrac{a+b+15}{2};\tfrac{1}{2}) = a\,{}_2F_1(a+1,b;\tfrac{a+b+15}{2};\tfrac{1}{2}) - b\,{}_2F_1(a,b+1;\tfrac{a+b+15}{2};\tfrac{1}{2})$$

Now with the help of the derived result from Gauss second summation theorem , we get

$$\text{L.H.S} = a\,2^b\,\frac{\Gamma(\frac{a-b+15}{2})}{\Gamma(b)}\Big[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Big\{\frac{64(10395-19524a+12139a^2-3480a^3+505a^4-36a^5+a^6-4056b+58526ab)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{\varepsilon-b-(2\omega-1)\}]}$$

$$+\frac{64(-12896a^2b+8788a^3b-520a^4b+78a^5b+33111b^2-1144ab^2+25454a^2b^2-1144a^3b^2+715a^4b^2+3952b^3)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]}$$

$$+\frac{64(17732ab^3+1716a^3b^3+2561b^4+572ab^4+1287a^2b^4+104b^5+286ab^5+13b^6)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]}\Big\}-$$

$$\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+2}{2})}\Big\{\frac{64(10395+4056a+33111a^2-3952a^3+2561a^4-104a^5+13a^6+19524b+58526ab+1144a^2b)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]}$$

$$+\frac{64(17732a^3b-572a^4b+286a^5b+12139b^2+12896ab^2+25454a^2b^2+1287a^4b^2+3480b^3+8788ab^3)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]}$$

$$+\frac{64(1144a^2b^3+1716a^3b^3+505b^4+520ab^4+715a^2b^4+36b^5+78ab^5+b^6)}{[\prod_{\varepsilon=1}^{6}\{a-b-(2\varepsilon-1)\}]\,[\prod_{\omega=1}^{7}\{a-b-(2\omega-1)\}]}\Big\}\Big]$$

$$-\ b\,2^{b+1}\,\frac{\Gamma(\frac{a+b+15}{2})}{\Gamma(b+1)}\Big[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{1}{2})}\Big\{\frac{64(10395+19524a+12139a^2+3480a^3+505a^4+36a^5+a^6+4056b+58526ab)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]}$$

$$+\frac{64(12896a^2b+8788a^3b+520a^4b+78a^5b+33111b^2+1144ab^2+25454a^2b^2+1144a^3b^2+715a^4b^2-3952b^3)}{[\prod_{\alpha=1}^{7}\{a\ \ b\ \ (2\alpha\ \ 1)\}]\,[\prod_{\beta=1}^{6}\{a\ \ b\ \ (2\beta\ \ 1)\}]}$$

$$+\frac{64(17732ab^3+1716a^3b^3+2561b^4-572ab^4+1287a^2b^4-104b^5+286ab^5+13b^6)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]}\}-$$

$$\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\{\frac{64(10395-4056a+33111a^2+3952a^3+2561a^4+104a^5+13a^6-19524b+58526ab-1144a^2b)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]}+$$

$$+\frac{64(17732a^3b+572a^4b+286a^5b+12139b^2-12896ab^2+25454a^2b^2+1287a^4b^2-3480b^3+8788ab^3)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]}$$

$$+\frac{64(-1144a^2b^3+1716a^3b^3+505b^4-520ab^4+715a^2b^4-36b^5+78ab^5+b^6)}{[\prod_{\alpha=1}^{7}\{a-b-(2\alpha-1)\}]\,[\prod_{\beta=1}^{6}\{a-b-(2\beta-1)\}]}\}]$$

On simplification, we get the formula (7)

Similarly, we can prove the formula (8).

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