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SOLUTIONS FOR HIGH LEVEL DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS WITH METHOD TO LOWER THE RANK

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ABSTRACT

In this peaper will be describing an elementary method for finding general differencial linear equation of higher levels with constant coefficients while using method to lower the level, then passing on to another lower differencial linear equation than the given differencial equation.

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1. INTRUDUCTION:

Definition: 1.1 Differential linear equations of first rank we would call it:

$$y' + f(x)\,y = g(x) \tag{1}$$

where

$$y = F(x, c_1, \ldots c_{n-1}, c_n)$$

In C. Corduneanu can find this result:

Theorem: 1.2 General solution of equation (1) is:

$$y = \left(c + \int g(x)\, e^{\int f(x)dx}\, dx\right) e^{-\int f(x)dx} \tag{2}$$

Definition: 1.3 Differential linear equation non-homogeny of level n with constant coefficient we would call the form of equation:

$$y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \ldots + a_{n-1} y' + a_n y = h(x) \tag{3}$$

where $a_1, a_2, \ldots, a_{n-1}, a_n$ are the elements of union of real numbers.

Definition: 1.4 Differential linear equation homogeny of n rank with constant equation of the form.:

$$y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \ldots + a_{n-1} y' + a_n y = 0 \tag{4}$$

where $a_1, a_2, \ldots, a_{n-1}, a_n$ are the elements of union of real numbers.

Definition: 1.5 General solution of differential equation (3) we call the function:

$$y = F(x, c_1, \ldots, c_n) \tag{5}$$

Definition: 1.6 General solution of differential equation (4) we call the function:

$$y = G(x, c_1, \ldots, c_n) \tag{6}$$

2. MAIN RESULTS:

Theorem: 2.1 Differential equations (3) at particular (4) with transformation method:

$$y = e^{-\frac{a_1}{n}x} z \tag{7}$$

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Brought to differential equation with constant coefficient of rank n according to z, where it misses margin with power $n-1$.

Proof: At differential equations (3) or (4) we replace:

$$y = \alpha z \tag{8}$$

where $\alpha = \alpha(x)$ and $z = z(x)$. While derivation of y (from (8)) to the end rank n, we gain as a result:

$y' = \alpha' z + \alpha' z$ and $y'' = \alpha'' z + 2\alpha z'' + \alpha' z'$, $y''' = \alpha''' z + 3\alpha'' z' + 3\alpha' z'' + \alpha z''$,..

and when we utilize the substitution of one of the above equations (example (3)) and we order it according to z we will gain:

$$\alpha z^{(n)} + (n\alpha' + a_1 \alpha) z^{(n-1)} + \omega_2 z^{(n-2)} + ... + \omega_n z = h(x) \tag{9}$$

where $\omega_2, \omega_3, ..., \omega_n$ are coefficients that are dependents from α and it's derivates.

At equation (9) we set α so coefficient of margin $n-1$ to be zero.Then $n\alpha' + a_1 \alpha = 0$ from the equation we gain:

$$\alpha = e^{-\frac{a_1}{n}x} \tag{10}$$

where α from (10) we substitute at (9) and re-order, we will gain the following equation:

$$z^{(n)} + \beta_2 z^{(n-2)} + \beta_3 z^{(n-3)} + ... + \beta_n = e^{\frac{a_1}{n}x} h(x) \tag{11}$$

where $\beta_2, \beta_3, ..., \beta_n$ are real numbers.

Theorem: 2.2 Every differential equation of the form:

$$y^{(n)} + \omega_2 y^{(n-2)} + \omega_3 y^{(n-3)} + ... + \omega_n y = H(x) \tag{12}$$

where $\omega_2, \omega_3, ..., \omega_n$ are real numbers, we can bring linear equation of the rank $n-1$ according to y.

Proof: To proof the theorem, we start from differential equation with constant coefficients:

$$y^{(n-1)} + A_2 y^{(n-2)} + A_3 y^{(n-3)} + ... + A_n y = \Phi(x) \tag{13}$$

where $A_2, A_3, ..., A_n$ are real numbers. We derivate according to x equation (13)
and we gain:

$$y^{(n)} + A_2 y^{(n-1)} + A_3 y^{(n-2)} + ... + A y_n' = \Phi'(x), \tag{14}$$

If $y^{(n-1)}$ to find from equation (13), we substitute equation (14) , after re-ordering we gain:

$$y^{(n)} + (-A_2^2 + A_3) y^{(n-2)} + (-A_2 A_3 + A_4) y^{(n-3)} + ... + + (-A_2 A_n) y = \Phi'(x) - A_2 \Phi(x) \tag{15}$$

When we compare equation (15) with equation (12) , we gain :

$$\begin{aligned} A_3 - A_2^2 &= \omega_2 \\ A_4 - A_2 A_3 &= \omega_3 \\ &\dots\dots \end{aligned} \tag{16}$$

$$-A_2 A_n = \omega_n \tag{17}$$

and of course::

$$\Phi'(x) - A_2 \Phi(x) = H(x) \tag{18}$$

While solving system (16) , we gain polynomial equation of row n, according to A_2, which presents particular differential equation (12) , and differential equation (17), Which presents differential equation of the form (1) according to Φ (x), when we set Coefficients A_2 , A_3 , ... , A_n from (16) and $\Phi(x)$ from(18), and we substitute at (13), we gain one differential equations of a lower rank than equation (12). This way theorem (8) has been proved.

While utilizing theorem (7) and (8) in equation (13), we gain one equation of a lower rank for one, and we continue until we reach first rank of differential linear equation, after it is solved we gain a solution for general differential equation (12)

Example: Solve differential equations:

$$y'' - y = 0 \tag{19}$$

Solution: Start from differential form:

$$y' + A y = \Phi(x) \tag{20}$$

We derivate side to side according to x and we gain::

$$y'' + A y' = \Phi'(x) \tag{21}$$

We find y' from (19) and we replace with (20) therefore we gain::

$$y'' - A^2 y = \Phi'(x) - A\Phi(x) \tag{22}$$

while comparing equations (22) with (19) , therefore we gain :

$A^2 = 1$

$$\Phi'(x) - A\Phi(x) = 0 \tag{23}$$

From first system equations (23) we gain A = 1 and A = -1. We take one of these two values example: A = 1 and we replace at second equation of the system (23) . In this case as a result we gain:

$$\Phi'(x) - \Phi(x) = 0 \tag{24}$$

While solving equations (24) according $\Phi(x)$, therefore we gain:

$$\Phi(x) = C_1 e^x \tag{25}$$

When substituted A = 1 and $\Phi(x)$ nga from (25) to (20) , therefore we gain:

$$y' + y = C_1 e^x \tag{26}$$

While using formula (2) , solution of equation (26) is:

$$y = C_1 e^x + C_2 e^{-x} \tag{27}$$

That presents the general solution of the differential equation (19) .

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