



EXISTENCE OF SEMI-PSEUDO RICCI SYMMETRIC MANIFOLD

Musa A. A. Jawarneh*

AL-Balqa' Applied University, AL-Huson University College,
Department of Basic Sciences, P. O. Box 50, AL-Huson 21510

E-mail: musa_jawarneh@yahoo.com

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ABSTRACT

Conharmonic and concircular curvature tensors on Semi-Pseudo Ricci Symmetric Manifolds has been studied. Also Kaehlerian and Sasakian Semi-Pseudo Ricci Symmetric Manifolds, and anon trivial example of Semi-Pseudo Ricci Symmetric Manifold is obtained.

Keywords: Riemannian manifold, Semi-Pseudo Ricci Symmetric manifold, Kaehlerian and Sasakian manifolds, Pseudo Ricci symmetric manifolds.

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1. INTRODUCTION:

The notion of Semi-Pseudo Ricci Symmetric Manifolds was introduced by M. Tarafdar and the author in 1993 [1], which is defined as follows:

A non-flat n-dimensional Riemannian manifold (M_n, g) ($n > 3$) is called Semi-Pseudo Ricci Symmetric Manifold if the Ricci tensor S satisfies:

$$(1.1) (\nabla_X S)(Y, Z) = \pi(Y)S(X, Z) + \pi(Z)S(X, Y), \text{ where } \pi \text{ is a non zero 1-form,}$$

$$(1.2) g(X, \rho) = \pi(X),$$

for every vector field X and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such a manifold is denoted by $(SPRS)_n$.

It is known [1] that on $(SPRS)_n$ the scalar curvature $r = 0$, and,

$$(1.3) \pi(QX) = 0, (1.4) S(X, \rho) = 0,$$

where Q is the symmetric endomorphism of the tangent space at each point of (M_n, g) corresponding to the Ricci tensor.

The object of this paper is to further study Semi-Pseudo Ricci Symmetric Manifold. It is shown in section2 that $(SPRS)_n$ cannot be neither conharmonically (concircularly) flat nor conharmonically (concircularly) symmetric.

In section3&4 it is proved that $(SPRS)_n$ cannot be neither Kaehlerian nor Sasakian. Finally a non trivial example introduced in section5.

2. CONHARMONIC AND CONCIRCULAR CURVATURE TENSORS ON $(SPRS)_N$.

Conharmonic and concircular curvature tensors are given by [2],

$$(2.1) L(X, Y, Z) = R(X, Y, Z) - \frac{1}{n-2}[S(Y, Z)X - S(X, Z)Y - g(X, Z)QY + g(Y, Z)QX],$$

$$(2.2) Z(X, Y, Z) = R(X, Y, Z) - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y].$$

Corresponding author: Musa A. A. Jawarneh, *E-mail: musa_jawarneh@yahoo.com

$$(2.3) (\nabla_W R)(X, Y, Z) - \frac{1}{n-2} [(\nabla_W S)(Y, Z)X - (\nabla_W S)(X, Z)Y - g(X, Z)(\nabla_W Q)(Y) + g(Y, Z)(\nabla_W Q)(X)].$$

Contracting with respect to X, taking in account $r = 0$, we get,

$$(2.4) \frac{1}{n-2} (\nabla_W S)(Y, Z) = 0.$$

By virtue of (1.1) we have,

$$(2.5) \pi(Y)S(X, Z) + \pi(Z)S(X, Y) = 0.$$

Let $Y = \rho$, then by virtue of (1.4) we get $\pi(\rho)S(X, Z) = 0$. Since $\pi(\rho) \neq 0$, we have $S(X, Z) = 0$, which contradicts the hypothesis of $(SPRS)_n$. Thus we can state,

Theorem: 2.1 $(SPRS)_n$ cannot be conharmonically flat.

Further if the manifold is conharmonically symmetric then we have (2.3). Thus we can state,

Theorem: 2.2 $(SPRS)_n$ cannot be conharmonically symmetric.

Similarly we can prove,

Theorem: 2.3 $(SPRS)_n$ cannot be neither concircularly flat nor concircularly symmetric.

3. KAEHLERIAN $(SPRS)_n$.

An even dimensional manifold M_{2n} with a Riemannian metric g is a Kaehler manifold if there exist a mixed tensor F such that [2],

$$(3.1) F^2 = I_n,$$

$$(3.2) g(\bar{X}, \bar{Y}) = g(X, Y), \text{ where } F(X) = \bar{X},$$

$$(3.3) (\nabla_W F)Y = 0,$$

Where ∇ as stated above. It is known [2] that on a Kaehler manifold,

$$(3.4) S(\bar{X}, Y) + S(X, \bar{Y}) = 0.$$

If we define,

$$(3.5) H'(X, Y) = -\frac{1}{2} C_3^1 \overline{R(X, Y)},$$

where C denotes the operator of contraction, then we have [2],

$$(3.6) H'(X, Y) = S(X, \bar{Y}),$$

$$(3.7) dH' = 0.$$

Let us assume that the manifold be Kaehlerian. Then from (3.6) and (3.7) by virtue of (3.4) we get,

$$(3.8) (\nabla_X S)(Y, \bar{Z}) + (\nabla_Y S)(\bar{Z}, X) + (\nabla_Z S)(X, \bar{Y}) = 0.$$

By virtue of (1.10) and (3.4) we have,

$$(3.9) \pi(Y)S(X, \bar{Z}) + \pi(\bar{Y})S(X, Z) = 0.$$

Now let $\rho = Y$, we get,

$$(3.10) S(X, \bar{Z})\rho - S(X, Z)\bar{\rho} = 0.$$

But ρ and $\bar{\rho}$ are linearly independent, and therefore we have $S(X, Z) = 0$, which contradicts the hypothesis of $(SPRS)_n$. Thus we can state,

Theorem: 3.1 (SPRS)_n cannot be Kaehler manifold.

4. SASAKIAN (SPRS)_n.

Let (Mⁿ, g) be n-dimensional (n=2m+1, m>1) differentiable manifold with contact form η, associated vector field ξ, and (1-1) tensor field Φ. Then M_n is called Sasakian manifold [3] if ξ is a killing vector field, and,

$$(4.1) (\nabla_X \Phi)Y = g(X, Y)\xi + \eta(Y)X,$$

where ∇ as stated above. On Sasakian manifold we have [3],

$$(4.2) \Phi\xi = 0,$$

$$(4.3) \eta(\xi) = 1,$$

$$(4.4) S(X, \xi) = (n-1)\eta(X),$$

$$(4.5) (\nabla_X S)(Y, \xi) = (n-1)g(\Phi X, Y) + S(Y, \Phi X).$$

Assume the manifold be Sasakian, and let Z = ξ in (1.1) we get,

$$(4.6) (\nabla_X S)(Y, \xi) = \pi(Y)S(X, \xi) + \pi(\xi)S(X, Y).$$

By virtue of (4.4) and (4.5) we get,

$$(4.7) (n-1)g(\Phi X, Y) + S(Y, \Phi X) = (n-1)\eta(X)\pi(Y) + \pi(\xi)S(X, Y).$$

Now if X = ξ, we can have by virtue of (4.2) and (4.3),

$$(4.8) (n-1)\pi(Y) + (n-1)\eta(Y)\pi(\xi) = 0.$$

Next if Y = ξ we get π(ξ) = 0, which contradicts the hypothesis of (SPRS)_n. Thus we can state,

Theorem4.1 (SPRS)_n cannot be Sasakian manifold.

5- (SPRS)₄:

Let us consider M⁴ = {(x¹, x², x³, x⁴) ∈ R⁴} be an open subsets of R⁴ endowed with the metric,

$$(5.1) d^2 = g_{ij}dx^i dx^j = f(x^1, x^3)d(x^1)^2 + 2dx^1 dx^2 + d(x^3)^2 d(x^4)^2,$$

where i, j = 1,2,3,4, and f is continuously differentiable function of x¹ and x³ such that f_{.33} ≠ 0, and f_{.331} ≠ 0, where (.) denote the partial differentiation with respect to coordinates. Therefore the only non vanishing Ricci tensors and their covariant derivatives[4] are,

$$(5.2) S_{11} = \frac{1}{2}f_{.33} ; S_{11,1} = \frac{1}{2}f_{.331}, S_{11,3} = \frac{1}{2}f_{.333},$$

where (,) denotes the covariant differentiation with respect to the metric tensor g. Hence under consideration is a Riemannian manifold which is neither Ricci symmetric non Ricci recurrent and it is of vanishing scalar curvature tensor.

If we now define the 1-form,

$$(5.3) \pi_i(x) = \begin{cases} \frac{f_{.331}}{2f_{.33}}, & i = 1 \\ 0, & otherwise \end{cases}$$

for any point x ∈ M. Then (1.1) reduces to the form,

$$(5.4) S_{11,1} = \pi_1 S_{11} + \pi_1 S_{11}$$

and all other cases vanishes identically. Therefore (5.4) holds because we have,

$$\frac{1}{2}f_{.331} = S_{11,1} = 2\pi_1 S_{11} = \frac{1}{2}f_{.331}$$

Theorem3.1) A Riemannian manifold (M^4, g) endowed with the metric (5.1) is a semi-pseudo Ricci symmetric manifold with vanishing scalar curvature tensor which is neither Ricci symmetric nor Ricci recurrent.

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