

On Λ_r -compact and Λ_r -connected spaces

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ABSTRACT

The aim of this paper is to introduce and study two new classes of spaces, called Λ_r -compact and Λ_r -connected spaces. Some basic properties of these spaces are studied by utilizing Λ_r -open set.

Key words and phrases: Λ_r -open set, Λ_r -compact space, Λ_r -connected space.

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1. INTRODUCTION:

Λ_r -open sets is recently introduced by the authors [4] and studied Λ_r - T_0 , Λ_r - T_1 and Λ_r - T_2 spaces, Λ_r -homeomorphisms and variants of continuity related to this concept in [4,5,6]. In this paper, we introduce two new classes of topological spaces called Λ_r -compact and Λ_r -connected spaces. To define and investigate these spaces, we use Λ_r -open sets.

Throughout the paper, (X, τ) (or simply X) will always denote a topological space. For a subset S of a topological space X , S is called regular-open if $S = \text{Int } cl S$. Then the complement $S^c (= X-S)$ of a regular-open set S is called the regular-closed set. The family of all regular-open sets (resp. regular-closed sets) in (X, τ) will be denoted by $RO(X, \tau)$ (resp. $RC(X, \tau)$). A subset S of a topological space (X, τ) is called Λ_r -set [4] if $S = \Lambda_r(S)$ where $\Lambda_r(S) = \bigcap \{G / G \in RO(X, \tau) \text{ and } S \subseteq G\}$. The collection of all Λ_r -sets in (X, τ) is denoted by $\Lambda_r(X, \tau)$.

2. PRELIMINARIES:

Through this paper, we adopt the notations and terminology of [4]. Let A be a subset of a space (X, τ) . Then A is called a Λ_r -closed set if $A = S \cap C$ where S is a Λ_r -set and C is a closed set. The complement of a Λ_r -closed set is called Λ_r -open. The collection of all Λ_r -open (resp. Λ_r -closed) sets in (X, τ) is denoted by $\Lambda_r O(X, \tau)$ (resp. $\Lambda_r C(X, \tau)$). Also note that every open set is Λ_r -open; arbitrary union of Λ_r -open sets is Λ_r -open and arbitrary intersection of Λ_r -closed sets is Λ_r -closed. A point $x \in X$ is called a Λ_r -cluster point of A if for every Λ_r -open set U containing x , $A \cap U \neq \emptyset$. The set of all Λ_r -cluster points of A is called the Λ_r -closure of A and it is denoted by $\Lambda_r-cl(A)$. Then $\Lambda_r-cl(A)$ is the intersection of Λ_r -closed sets containing A and it is the smallest Λ_r -closed set containing A . Also A is Λ_r -closed if and only if $A = \Lambda_r-cl(A)$. The union of Λ_r -open sets contained in A is called Λ_r -interior of A and it is denoted by $\Lambda_r-int(A)$. Also A is Λ_r -open if and only if $A = \Lambda_r-int(A)$ and $\Lambda_r-cl(X-A) = X-(\Lambda_r-int(A))$.

Before we entering into our work, we recall the following definitions.

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Definition: 2.1 A function $f: X \rightarrow Y$ is called

- (i) Λ_r -continuous [5] if $f^{-1}(V)$ is a Λ_r -open set in X for each open set V in Y .
- (ii) Λ_r -irresolute [5] if $f^{-1}(V)$ is a Λ_r -open set in X for each Λ_r -open set V in Y .
- (iii) Λ_r -open [6] if the image of each open set in X is a Λ_r -open set in Y .
- (iv) Λ_r^* -open [6] if the image of each Λ_r -open set in X is a Λ_r -open set in Y .
- (v) Λ_r -closed [6] if the image of each closed set in X is a Λ_r -closed set in Y .
- (vi) Λ_r^* -closed [6] if the image of each Λ_r -closed set in X is a Λ_r -closed set in Y .

3. Λ_r -COMPACT SPACES:

In this section, we introduce and study a class of compact space called Λ_r -compact space in topological spaces.

Definition: 3.1 A collection $\{A_i: i \in I\}$ of Λ_r -open sets in a topological space (X, τ) is called a Λ_r -open cover of a subset B of X if $B \subseteq \cup \{A_i: i \in I\}$.

Definition: 3.2 A topological space (X, τ) is called Λ_r -compact if every Λ_r -open cover of X has a finite subcover.

Definition: 3.3 A subset B of a topological space (X, τ) is said to be Λ_r -compact relative to X if for every collection $\{A_i: i \in I\}$ of Λ_r -open subsets of X such that $B \subseteq \cup \{A_i: i \in I\}$, \exists a finite subset I_0 of I such that $B \subseteq \cup \{A_i: i \in I_0\}$.

Definition: 3.4 A subset B of a topological space (X, τ) is said to be Λ_r -compact if B is Λ_r -compact as the subspace of X .

Theorem: 3.5 Every Λ_r -closed subset of a Λ_r -compact space X is Λ_r -compact relative to X .

Proof: Let A be a Λ_r -closed subset of a Λ_r -compact space X . Then A^c is Λ_r -open in X . Let $\{G_\alpha: \alpha \in I\}$ be any Λ_r -open cover of A . Then $\{G_\alpha: \alpha \in I\} \cup A^c$ is a Λ_r -open cover of X . Since X is Λ_r -compact, it has a finite subcover, say $\{G_1, G_2, \dots, G_n\}$. If this subcover contains A^c , we discard it. Otherwise leave the subcover as it is. Thus we have obtained a finite Λ_r -open subcover of A and so A is Λ_r -compact relative to X .

Theorem: 3.6 Let f be an Λ_r -irresolute surjective function from a topological space X onto a topological space Y . If X is Λ_r -compact, then Y is Λ_r -compact.

Proof: Let $\{A_i: i \in I\}$ be a Λ_r -open cover of Y . Then $\{f^{-1}(A_i): i \in I\}$ is a Λ_r -open cover of X since f is Λ_r -irresolute. Since X is Λ_r -compact, X has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$. Since f is onto, $\{A_1, A_2, \dots, A_n\}$ is a finite subcover of Y and hence Y is Λ_r -compact.

Theorem: 3.7 Let f be an Λ_r -irresolute bijective function from a topological space X onto a topological space Y . If f is Λ_r^* -open and Y is Λ_r -compact, then X is Λ_r -compact.

Proof: Let $\{A_i: i \in I\}$ be a Λ_r -open cover of X . Since f is Λ_r^* -open, $\{f(A_i): i \in I\}$ is a Λ_r -open cover of Y . Since Y is Λ_r -compact, Y has a finite subcover, say $\{f(A_1), f(A_2), \dots, f(A_n)\}$. Since f is Λ_r -irresolute, $\{f^{-1}(f(A_1)), f^{-1}(f(A_2)), \dots, f^{-1}(f(A_n))\}$ is a finite subcover of X . That is, $\{A_1, A_2, \dots, A_n\}$ is a finite subcover of X . Hence X is Λ_r -compact.

Theorem: 3.8 A topological space X is Λ_r -compact if and only if for every family $\{F_\alpha: \alpha \in I\}$ of Λ_r -closed sets with finite intersection property, $\bigcap_{\alpha \in I} F_\alpha \neq \emptyset$.

Proof: Necessity: Let X be a Λ_r -compact space and suppose that $\{F_\alpha: \alpha \in I\}$ be a family of Λ_r -closed subsets of X with finite intersection property such that $\bigcap_{\alpha \in I} F_\alpha = \emptyset$.

Let us consider the Λ_r -open sets $U_\alpha = X - F_\alpha$.

Then $\cup \{U_\alpha: \alpha \in I\} = \cup \{X - F_\alpha: \alpha \in I\} = X - \bigcap \{F_\alpha: \alpha \in I\} = X - \emptyset = X$.

Hence $\{U_\alpha: \alpha \in I\}$ is a Λ_r -open cover of X .

Since X is Λ_r -compact, it has a finite subcover $\{U_{\alpha_i} : \alpha_i \in I_0\}$ where $I_0 = \{1, 2, \dots, n\}$.

Then $X = \cup \{U_{\alpha_i} : \alpha_i \in I_0\} = \cup \{X - F_{\alpha_i} : \alpha_i \in I_0\} = X - \cap \{F_{\alpha_i} : \alpha_i \in I_0\}$.

Hence $\cap \{F_{\alpha_i} : \alpha_i \in I_0\} = \emptyset$, which is a contradiction. Thus, if the family $\{F_{\alpha} : \alpha \in I\}$ of Λ_r -closed sets with finite intersection property, then $\cap_{\alpha \in I} F_{\alpha} \neq \emptyset$.

Sufficiency: Suppose that for every family $\{F_{\alpha} : \alpha \in I\}$ of Λ_r -closed sets with finite intersection property, $\cap_{\alpha \in I} F_{\alpha} \neq \emptyset$. Let $\{U_{\alpha} : \alpha \in I\}$ be a Λ_r -open cover of X .

Then $X = \cup \{U_{\alpha} : \alpha \in I\}$. Now, $\emptyset = X - \cup \{U_{\alpha} : \alpha \in I\} = \cap \{X - U_{\alpha} : \alpha \in I\}$.

Hence $\{X - U_{\alpha} : \alpha \in I\}$ is a family of Λ_r -closed sets with an empty intersection.

By the hypothesis, \exists a finite subset $\{X - U_{\alpha_i} : \alpha_i \in I_0\}$ such that $\cap \{X - U_{\alpha_i} : \alpha_i \in I_0\} = \emptyset$. Hence $X = X - \cap \{X - U_{\alpha_i} : \alpha_i \in I_0\} = \cup \{X - (X - U_{\alpha_i}) : \alpha_i \in I_0\} = \cup \{U_{\alpha_i} : \alpha_i \in I_0\}$.

This shows that $\{U_{\alpha_i} : \alpha_i \in I_0\}$ is a finite subcover of X and hence X is Λ_r -compact.

Theorem: 3.9 For any topological space X , the following properties are equivalent:

- (i) X is Λ_r -compact
- (ii) Every proper Λ_r -closed set is Λ_r -compact relative to X .

Proof: (i) \rightarrow (ii) By Theorem 3.5, it is obvious.

(ii) \rightarrow (i) Let $\{V_{\alpha} : \alpha \in I\}$ be a Λ_r -open cover of X . Then $X \subseteq \cup \{V_{\alpha} : \alpha \in I\}$.

We choose and fix one $\alpha_0 \in I$.

Then $X - V_{\alpha_0}$ is a proper Λ_r -closed subset of X and $X - V_{\alpha_0} \subseteq \cup \{V_{\alpha} : \alpha \in I - \{\alpha_0\}\}$.

Therefore $\{V_{\alpha} : \alpha \in I - \{\alpha_0\}\}$ is a Λ_r -open cover of $X - V_{\alpha_0}$.

By the hypothesis, \exists a finite subset I_0 of $I - \{\alpha_0\}$ such that $X - V_{\alpha_0} \subseteq \cup \{V_{\alpha} : \alpha \in I_0\}$.

Therefore $X \subseteq \cup \{V_{\alpha} : \alpha \in I_0 \cup \{\alpha_0\}\}$. Hence X is Λ_r -compact.

Theorem: 3.10 If a function $f : X \rightarrow Y$ is Λ_r -irresolute and a subset B of X is Λ_r -compact relative to X , then the image $f(B)$ is Λ_r -compact relative to Y .

Proof: Let $\{A_i : i \in I\}$ be a family of Λ_r -open subsets of Y such that $f(B) \subseteq \cup \{A_i : i \in I\}$. Then $B \subseteq \cup \{f^{-1}(A_i) : i \in I\}$. By using assumption, \exists a finite subset I_0 of I such that

$B \subseteq \cup \{f^{-1}(A_i) : i \in I_0\}$. Then we have $f(B) \subseteq \cup \{A_i : i \in I_0\}$.

This shows that $f(B)$ is Λ_r -compact relative to Y .

Theorem: 3.11 Every Λ_r -continuous image of a Λ_r -compact space is compact.

Proof: Let $f : X \rightarrow Y$ be a Λ_r -continuous function from a Λ_r -compact space X onto a topological space Y . Let $\{A_i : i \in I\}$ be an open cover of Y . Since f is Λ_r -continuous, $\{f^{-1}(A_i) : i \in I\}$ is a Λ_r -open cover of X . Since X is Λ_r -compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$. Since f is onto, $\{A_1, A_2, \dots, A_n\}$ is an open cover of Y and hence Y is compact.

Theorem: 3.12 If $f : X \rightarrow Y$ is a Λ_r^* -closed surjective function such that for each $y \in Y$, $f^{-1}(y)$ is Λ_r -compact relative to X and Y is Λ_r -compact, then X is Λ_r -compact.

Proof: Let $\{U_\alpha : \alpha \in I\}$ be a Λ_r -open cover of X . Then $X = \bigcup_{\alpha \in I} U_\alpha$.

For each $y \in Y$, $\{f^{-1}(y)\}$ is Λ_r -compact relative to X .

Then \exists a finite subset I_0 of I such that $\{f^{-1}(y)\} \subseteq \bigcup \{U_\alpha : \alpha \in I_0\}$.

Take $U_y = \bigcup \{U_\alpha : \alpha \in I_0\}$ and $V_y = Y - f(X-U_y)$.

Then U_y is Λ_r -open and so $X-U_y$ is Λ_r -closed. Since f is Λ_r^* -closed, $f(X-U_y)$ is Λ_r -closed in Y .

Hence $Y - f(X-U_y)$ is Λ_r -open and hence V_y is Λ_r -open in Y . Now, $f^{-1}(y) \in U_y$.

That is, $f^{-1}(y) \not\subseteq X-U_y$. That implies $f(f^{-1}(y)) \not\subseteq f(X-U_y)$. Since f is onto, $y \notin f(X-U_y)$ and hence $y \in Y - f(X-U_y) = V_y$. Also $V_y = Y - f(X-U_y)$.

That implies $f^{-1}(V_y) = f^{-1}(Y - f(X-U_y)) = f^{-1}(Y) - f^{-1}(f(X-U_y)) = X - f^{-1}(f(X-U_y))$ (since f is onto) $\subseteq X - (X-U_y) = U_y$.

Thus V_y is a Λ_r -open set in Y containing y such that $f^{-1}(V_y) \subseteq U_y$. Hence $\{V_y : y \in Y\}$ is a Λ_r -open cover of Y . Since Y

is Λ_r -compact, it has a finite subcover, say $\{V_{y_1}, V_{y_2}, \dots, V_{y_n}\}$. That is, $Y = \bigcup \{V_{y_i} : i = 1, 2, \dots, n\}$. Therefore $X = f^{-1}(Y) = f^{-1}(\bigcup \{V_{y_i} : i = 1, 2, \dots, n\}) = \bigcup \{f^{-1}(V_{y_i}) : i = 1, 2, \dots, n\} \subseteq \bigcup \{U_{y_i} : i = 1, 2, \dots, n\} = \bigcup \{U_\alpha : \alpha \in I_i, i = 1, 2, \dots, n\}$. This shows that X is Λ_r -compact.

4. Λ_r -CONNECTED SPACES:

In this section, we introduce a new class of connected space called Λ_r -connected space and prove some of its properties.

Definition: 4.1 A topological space X is said to be Λ_r -connected if there does not exist a pair A, B of non-empty disjoint Λ_r -open subsets of X such that $X = A \cup B$, otherwise X is called Λ_r -disconnected. In this case, the pair (A, B) is called a Λ_r -disconnection of X . A subset A of a space X is Λ_r -connected if it is Λ_r -connected as a subspace.

Theorem: 4.2 For a topological space X , the following are equivalent:

- (i) X is Λ_r -connected
- (ii) The only subsets of X which are both Λ_r -open and Λ_r -closed are the sets X and \emptyset
- (iii) Each Λ_r -continuous function of X into a discrete space Y with atleast two points is a constant function

Proof: (1) \rightarrow (2) Let U be a both Λ_r -open and Λ_r -closed subset of X . Then $X-U$ is both Λ_r -open and Λ_r -closed. Since X is Λ_r -connected and X is the disjoint union of Λ_r -open sets U and $X-U$, one of these must be empty. Hence either $U = \emptyset$ or $U = X$. (2) \rightarrow (1) Suppose that X is Λ_r -disconnected. Then $X = A \cup B$ where A and B are nonempty Λ_r -open sets such that $A \cap B = \emptyset$. Since $B = X-A$ is Λ_r -open, A is both Λ_r -open and Λ_r -closed. Hence by (2), $A = \emptyset$ or X . That is, either $A = \emptyset$ or $B = \emptyset$, which is a contradiction. Therefore X is Λ_r -connected. (2) \rightarrow (3) Let $f: X \rightarrow Y$ be a Λ_r -continuous function from a topological space X into a discrete topological space Y . For each $y \in Y$, $\{y\}$ is both open and closed in Y since Y is a discrete topological space. Since f is Λ_r -continuous, $f^{-1}(y)$ is both Λ_r -open and Λ_r -closed in X . Hence X is covered by Λ_r -open and Λ_r -closed covering $\{f^{-1}(y) : y \in Y\}$. By (2), $f^{-1}(y) = \emptyset$ or X for each $y \in Y$. If $f^{-1}(y) = \emptyset$ for each $y \in Y$, then f fails to be a map. Hence \exists only one point $y \in Y$ such that $f^{-1}(y) = X$ which shows that f is a constant function. (3) \rightarrow (2) Let U be both Λ_r -open and Λ_r -closed in X . Suppose $U \neq \emptyset$. Let $f: X \rightarrow Y$ be a Λ_r -continuous function from a topological space X into a discrete topological space Y defined by $f(U) = \{y\}$ and $f(X-U) = \{w\}$ where $y, w \in Y$ and $y \neq w$. By (3), f is constant. Hence $U = X$.

Theorem: 4.3 Every Λ_r -connected space is connected.

Proof: Let X be a Λ_r -connected space. If possible, let X be not connected. Then X can be written as $X = A \cup B$ where A and B are disjoint nonempty open sets in X . Since every open set is Λ_r -open, $X = A \cup B$ where A and B are disjoint nonempty Λ_r -open sets in X . Therefore X is Λ_r -disconnected. This is a contradiction. Hence X is connected.

Remark: 4.4 The following example shows that the converse of the above theorem need not be true.

Example: 4.5 Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then $\Lambda_r O(X, \tau) = \{X, \emptyset, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Here (X, τ) is connected but Λ_r -disconnected since $X = \{a, d\} \cup \{b, c\}$ where $\{a, d\}$ and $\{b, c\}$ are two disjoint nonempty Λ_r -open sets in X .

Definition: 4.6 Let (X, τ) be a topological space and A be a subset of X . Then we define the Λ_r -frontier of A by $\Lambda_r Fr(A) = \Lambda_r cl(A) \setminus \Lambda_r int(A) = \Lambda_r cl(A) \cap \Lambda_r cl(X-A)$.

Next, we characterize Λ_r -connectedness interms of Λ_r -frontier.

Theorem: 4.7 A space X is Λ_r -connected if and only if every non-empty proper subset of X has a non-empty Λ_r -frontier.

Proof: Suppose that X is Λ_r -connected and A be a proper non-empty subset of X . If possible, let $\Lambda_r\text{-Fr}(A) = \emptyset$. Then $\Lambda_r\text{-cl}(A) \cap \Lambda_r\text{-cl}(X-A) = \emptyset$. This implies that $\Lambda_r\text{-cl}(A) \subseteq X - (\Lambda_r\text{-cl}(X-A)) = X - (X - \Lambda_r\text{-int}(A)) = \Lambda_r\text{-int}(A)$. This shows that $\Lambda_r\text{-int}(A) = A = \Lambda_r\text{-cl}(A)$ and hence A is both Λ_r -open and Λ_r -closed. Then by Theorem 4.2, X is Λ_r -disconnected. This contradiction proves that A has a non-empty Λ_r -frontier.

Conversely, suppose that X is Λ_r -disconnected. Then X can be written as $X = A \cup B$ where A and B are two non-empty disjoint Λ_r -open subsets of X . Since $B = X - A$ is Λ_r -open, A is both Λ_r -open and Λ_r -closed. Hence $A = \Lambda_r\text{-int}(A)$ and $A = \Lambda_r\text{-cl}(A)$. Now we have

$\Lambda_r\text{-Fr}(A) = \Lambda_r\text{-cl}(A) \cap \Lambda_r\text{-cl}(X-A) = A \cap (X - \Lambda_r\text{-int}(A)) = A \cap (X - A) = \emptyset$. So A has empty Λ_r -frontier. By this contradiction, X is Λ_r -connected.

Definition: 4.8 Let (X, τ) be a topological space and A be a subspace of X . Then the class of Λ_r -open sets in A is defined in a natural way as: $\Lambda_r\text{O}(A, \tau_A) = \{ A \cap O : O \in \Lambda_r\text{O}(X, \tau) \}$ where $\Lambda_r\text{O}(X, \tau)$ is the class of Λ_r -open sets in X . That is, G is Λ_r -open in A if and only if $G = A \cap O$, where O is Λ_r -open in X .

Theorem: 4.9 Let (A, B) be a Λ_r -disconnection of a space X and C be a Λ_r -connected subspace of X . Then C is contained in A or in B .

Proof: Suppose that C is neither contained in A nor in B . Since (A, B) is a Λ_r -disconnection of a space X , $X = A \cup B$ where A and B are non-empty disjoint Λ_r -open subsets of X . Then $C \cap A$ and $C \cap B$ are both non-empty Λ_r -open sets of C such that $(C \cap A) \cap (C \cap B) = \emptyset$ and $(C \cap A) \cup (C \cap B) = C$. This gives that $(C \cap A, C \cap B)$ is a Λ_r -disconnection of C . Hence C is Λ_r -disconnected. This contradiction proves the theorem.

Theorem: 4.10 Let $X = \bigcup_{\alpha \in I} \{X_\alpha\}$, where each X_α is Λ_r -connected and $\bigcap_{\alpha \in I} \{X_\alpha\} \neq \emptyset$. Then X is Λ_r -connected.

Proof: Suppose that X is Λ_r -disconnected and (A, B) is a Λ_r -disconnection of X . Since each X_α is Λ_r -connected, by Theorem 4.9, $X_\alpha \subseteq A$ or $X_\alpha \subseteq B$. Since $\bigcap_{\alpha \in I} \{X_\alpha\} \neq \emptyset$, all X_α are contained in A or in B . This gives that $X \subseteq A$ or $X \subseteq B$. If $X \subseteq A$, then $B = \emptyset$ or if $X \subseteq B$, then $A = \emptyset$. This contradiction proves that X is Λ_r -connected.

Theorem: 4.11 A space X is Λ_r -connected if and only if for every pair of points x, y in X , there is a Λ_r -connected subset of X which contains both x and y .

Proof: Necessity: Since the Λ_r -connected space X itself contain these two points, it is obvious. Sufficiency: Suppose that for any two points x and y of X , there is a Λ_r -connected subspace $C_{x,y}$ of X such that $x, y \in C_{x,y}$. Let $a \in X$ be a fixed point and $\{C_{a,x} : x \in X\}$ be a class of all Λ_r -connected subspace of X which contain the points a and x . Then $X = \bigcup_{x \in X} \{C_{a,x}\}$ and $\bigcap_{x \in X} \{C_{a,x}\} \neq \emptyset$. Therefore by Theorem 4.10, X is Λ_r -connected.

Theorem: 4.12 If $f : X \rightarrow Y$ is a Λ_r -continuous surjective function and X is Λ_r -connected, then Y is connected.

Proof: Suppose that Y is not connected. Then Y can be written as $Y = A \cup B$ where A and B are disjoint nonempty open sets in Y . Since f is Λ_r -continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are Λ_r -open in X . Since f is onto, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$. Thus, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty Λ_r -open sets in X such that $f^{-1}(A) \cap f^{-1}(B) = \emptyset$. Therefore X

is Λ_r -disconnected, which is a contradiction. Hence Y is connected.

Theorem: 4.13 If $f : X \rightarrow Y$ is a Λ_r -irresolute surjective function and X is Λ_r -connected, then Y is Λ_r -connected.

Proof: Suppose that Y is Λ_r -disconnected. Then Y can be written as $Y = A \cup B$ where A and B are disjoint nonempty Λ_r -open sets in Y . Since f is Λ_r -irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are Λ_r -open in X . Since f is onto, $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty subsets of X and $X = f^{-1}(Y)$. Thus, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty Λ_r -open sets in X . Hence X is Λ_r -disconnected, which is a contradiction. Therefore Y is Λ_r -connected.

Theorem: 4.14 If $f : X \rightarrow Y$ is a Λ_r^* -open bijective function and Y is Λ_r -connected, then X is Λ_r -connected.

Proof: Suppose that X is Λ_r -disconnected. Then X can be written as $X = A \cup B$ where A and B are disjoint nonempty Λ_r -open subsets of X . Since f is Λ_r^* -open, $f(A)$ and $f(B)$ are Λ_r -open in Y . Also $f(A)$ and $f(B)$ are nonempty subsets of Y . Since f is injective,

$$f(A) \cap f(B) = f(A \cap B) = f(\emptyset) = \emptyset. \text{ Since } f \text{ is onto, } f(X) = Y. \text{ Thus, } Y = f(X) = f(A \cup B) = f(A) \cup f(B)$$

where $f(A)$ and $f(B)$ are disjoint nonempty Λ_r -open sets in Y and so Y is Λ_r -disconnected. This is a contradiction.

Hence X is Λ_r -connected.

Theorem: 4.15 If $f : X \rightarrow Y$ is a Λ_r -open bijective function and Y is Λ_r -connected, then X is connected.

Proof: Suppose that X is not connected. Then $X = A \cup B$ where A and B are disjoint nonempty open sets in X . Since f is Λ_r -open, $f(A)$ and $f(B)$ are Λ_r -open in Y . Since f is bijective, $f(A) \cap f(B) = \emptyset$ and $f(X) = Y$. Therefore $Y = f(A) \cup f(B)$ where $f(A)$ and $f(B)$ are disjoint nonempty Λ_r -open sets in Y . Hence Y is Λ_r -disconnected, which is a contradiction.

Hence X is connected.

REFERENCES:

- [1] Ahmad Al-Omari and Mohd. Salmi Md. Noorani – New Characterization of compact spaces – Proceedings of the 5th Asian Mathematical Conference, Malaysia 2009.
- [2] K. Balachandran, P. Sundaram and H. Maki – On generalized continuous maps in topological spaces – *Mem. Fac. Sci. Kochi. Univ. (Math)*12 (1991), 5-13.
- [3] S. Eswaran and A. Pushpalatha - τ^* -generalized compact spaces and τ^* -generalized connected spaces in topological spaces – *International Journal of Engineering Science and Technology* – Vol. 2(5), 2010, 2466-2469.
- [4] M. J. Jeyanthi, Adem Kilicman, S. Pious Missier and P. Thangavelu, Λ_r -sets and Separation Axioms, *Malaysian Journal of Mathematical Sciences*, 5 (1) 2011 : 45-60
- [5] Jeyanthi.M. J, Pious Missier.S and Thangavelu. P, Λ_r -homeomorphisms and Λ_r^* -homeomorphisms, *Journal of Mathematical Sciences and Computer Applications*. (To appear).
- [6] M. J. Jeyanthi, S. Pious Missier and P. Thangavelu, On Λ_r -open sets and functions. (Submitted)
- [7] Miguel Galdas Cueva – Semi-generalized continuous maps in topological spaces – *Portugaliae Mathematica* – Vol. 52 Fasc.4 – 1995.
