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International Journal of Mathematical Archive 1(1), Oct.-2010, 10-15

MIXED CONVECTION FLOW OF A VISCOELASTIC FLUID THROUGH A POROUS MEDIUM IN A VERTICAL CHANNEL WITH PERMEABLE WALLS

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(Accepted on: 08-10-2010)

ABSTRACT

In this paper, we investigated the fully developed mixed convection flow of a viscoelastic fluid through a porous medium in a vertical channel with permeable walls. The governing non – linear equations are solved for the velocity field and temperature field using the traditional perturbation technique. The effects of viscoelastic parameter Reynolds number Re, Grashof number Gr, cross flow parameter R and Prandtl number Pr temperature on the velocity and temperature are discussed in detail with the help of graphs.

1. INTRODUCTION:

The Problem of convective flow in fluid saturated porous medium has been the subject of several resent papers. Interest in understanding the convective transport processes in porous material is increasing owing to the development of geothermal energy technology, high performance insulation for building and cold storage, renewed interest in the energy efficient drying processes and many other areas. It is also interest in the nuclear industry, particularly in the evaluation of heat removal from a hypothetical accident in a nuclear reactor and to provide effective insulation. Compressive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop [1], Nield and Bejan [2], Vafai [3], Pop and Ingham [4] and Bejan and Kraus [5].

In the literature there are a fairly large number of flows of Newtonian fluids for which a closed form analytical solution is possible. Due to the complexity of the governing equations for non- Newtonian fluids, to find closed form analytical solution is not easy. So, Perturbation technique are widely used to resolve this difficult by assuming the non – Newtonian fluid parameter small, the classical paper being by Beard and Walters [6], who considered the two dimensional stagnation point flow of the Walters' B fluid. Ariel [7] has presented perturbation solutions of laminar forced convection of a second – grade fluid through two parallel porous walls. Chamka et al. [8] have studied the fully developed free Connective flow of - micropular fluid between two vertical parallel plates analytically. Recently Hayat and Abbas [9] have investigated the two dimensional boundary layer flow of an upper – convected maxwell fluid in a channel with chemical reaction, the walls of two channels being permeable (porous). Sajid et al. [10] have obtained analytical solution for the problem of fully developed mixed convention flow of viscoleastic fluid between two permeable parallel vertical walls no one studied the fully developed mixed convection flow of a viscoelastic fluid through a porous medium in a vertical channel with permeable walls.

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In the present paper, we studied the fully developed mixed convection flow of a viscoelastic fluid through a porous medium in a vertical channel with permeable walls. The governing non – linear equations are solved for the velocity field and temperature field using the traditional perturbation technique. The effects of viscoelastic parameter Reynolds number Re, Grashof number Gr, cross flow parameter R and Prandtl number Pr temperature on the velocity and temperature are discussed in detail through graphs.

2. MATHEMATICAL FORMULATION:

We consider the laminar mixed convection flow of a viscoelastic fluid through a porous medium in a vertical permeable channel, the space between the plates being h, as shown in Fig. 1. It is assumed that the rate of injection at one wall is equal to the rate of suction at the other wall. A rectangular coordinate system (x, y) is chosen such that the x

- axis is parallel to the gravitational acceleration vector g , but with opposite direction and the y - axis is transverse to the channel walls. The left wall (i.e, at $y=0$) is maintained at constant temperature T_1 and the right wall (i.e, at $y=h$) is maintained at constant temperature T_2, where $T_1 > T_2$. The flow assumed steady and fully developed, i.e. the transverse velocity is zero. Then, the continuity equation drops to $\partial u / \partial x = 0$.

Viscoelastic fluids can be modeled by Rivlin – Ericksen constitutive equation

$$S = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{1}$$

where S is the Cauchy stress tensor, p is the scalar pressure, μ, α_1 and α_2 are the material constants, customarily known as the coefficients of viscosity, elasticity and cross - viscosity, respectively. These material constants can be determined from viscometric flows for any real fluid. A_1 and A_2 are Rivlin-Ericksen tensors and they denote, respectively, the rate of strain and acceleration. A_1 and A_2 are defined by

$$A_1 = \nabla V + (\nabla V)^T \tag{2}$$

and $$A_2 = \frac{dA_1}{dt} + A_1(\nabla V) + (\nabla V)^T A_1 \tag{3}$$

where d/dt is the material time derivative and ∇ gradient operator and $(\ )^T$ transpose operator. The viscoelastic fluids when modeled by Rivlin-Ericksen constitutive equation are termed as second-grade fluids. A detailed account of the characteristics of second - grade fluids is well documented by Dunn and Rajagopal [11]. Rajagopal and Gupta [12] have studied the thermodynamics in the form of dissipative inequality (Clausius –Duhem) and commonly accepted the idea that the specific Helmholtz free energy should be a minimum in equilibrium. From the thermodynamics consideration they assumed

$$\mu \geq 0,\ \alpha_1 > 0,\ \alpha_1 + \alpha_2 = 0 \tag{4}$$

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are

$$\rho v_0 \frac{du}{dy} = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + \alpha_1 v_0 \frac{d^3u}{dy^3} - \frac{\mu}{k_0} u + \rho g \beta (T - T_0) \tag{5}$$

$$v_0 \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} \tag{6}$$

where p is the pressure, ρ is the density, μ is the dynamic viscosity of the fluid, g acceleration due to gravity , β coefficient of thermal expansion, α_1 is the viscoelastic parameter, k_0 is the permeability of the porous medium and v_0 is the transpiration cross flow velocity . Further, here dp/dx is a constant.

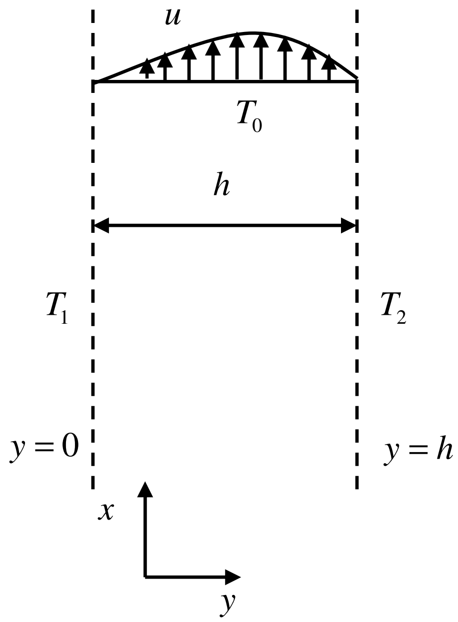

Fig. 1 The physical model

The boundary conditions are given by

$$u(0) = u(h) = 0, \quad T(0) = T_1, \quad T(h) = T_2 \tag{7}$$

Introducing the following non-dimensional variables

$$\bar{y} = \frac{y}{h}, \quad \bar{u} = \frac{u}{h^2}, \quad \theta = \frac{T - T_0}{T_2 - T_0}$$

in to the equations (5) and (6), we obtain

$$kR\frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} - R\frac{du}{dy} - \frac{1}{Da}u + \frac{Gr}{Re}\theta + A = 0 \tag{8}$$

$$\frac{d^2\theta}{dy^2} - R\,\mathrm{Pr}\frac{d\theta}{dy} = 0 \tag{9}$$

where $k = \frac{\alpha_1}{\rho h^2}$ is the viscoelastic parameter, $R = \frac{\rho v_0 h}{\mu}$ is the cross flow Reynolds number, $Gr = \frac{g\beta(T_2 - T_1)h^3}{\nu^2}$ is the Grashof number, $\mathrm{Re} = \frac{\rho U_0 h}{\mu}$ is the Reynolds number, $\mathrm{Pr} = \frac{\nu}{\alpha}$ is the Prandtl number, $r_T = \frac{T_1 - T_0}{T_2 - T_0}$ is the wall temperature parameter and $A = -\left(\frac{dp}{dx}\right)\frac{U_0 \nu}{h^2}$ is the constant pressure gradient.

The corresponding dimensionless boundary conditions are given by

$$u(0)=u(1)=0, \quad \theta(0)=r_T, \quad \theta(1)=1 \tag{10}$$

3. SOLUTION:

We consider the first - order perturbation solution of the BVP (4) - (6) for small k. Since the constitute equation (1) has been derived up to only the first - order of smallness of k, therefore, the perturbation solution obtained by retaining the terms up to the same order of smallness of k must be quite logical and reasonable. We write

$$u=u_0+ku_1 \tag{11}$$

and $$\theta=\theta_0+k\theta_1 \tag{12}$$

Substituting equations (11) and (12) into equations (8) and (9) and boundary conditions (10) and then equating the like powers of k, we obtain

3.1 Zeroth-order system $\left(k^0\right)$:

$$\frac{d^2u_0}{dy^2}-R\frac{du_0}{dy}-\frac{1}{Da}u_0=-\frac{Gr}{\mathrm{Re}}\theta_0-A \tag{13}$$

$$\frac{d^2\theta_0}{dy^2}-R\Pr\frac{d\theta_0}{dy}=0 \tag{14}$$

Together with boundary conditions

$$u_0(0)=u_0(1)=0, \quad \theta_0(0)=r_T, \quad \theta_0(1)=1 \tag{15}$$

3.2 First-order system $\left(k^1\right)$

$$\frac{d^2u_1}{dy^2}-R\frac{du_1}{dy}-\frac{1}{Da}u_0=-R\frac{d^3u_0}{dy^3}-\frac{Gr}{\mathrm{Re}}\theta_1 \tag{16}$$

$$\frac{d^2\theta_1}{dy^2}-R\Pr\frac{d\theta_1}{dy}=0 \tag{17}$$

Together with boundary conditions

$$u_1(0)=u_1(1)=0, \quad \theta_1(0)=0, \quad \theta_1(1)=0 \tag{18}$$

3.3 Zeroth-order solution (or Solution for a Newtonian fluid):

Solving equations (13) and (14) using the boundary conditions (15), we get

$$\theta_0=\frac{\left(1-r_Te^{R\Pr}\right)+\left(r_T-1\right)e^{R\Pr y}}{\left(1-e^{R\Pr}\right)} \tag{19}$$

$$u_0=c_1e^{ay}+c_2e^{by}+\frac{Gr}{\mathrm{Re}}\left(f_1-f_2e^{R\Pr y}\right)+ADa \tag{20}$$

where $a=\dfrac{R+\sqrt{R^2+4/Da}}{2}$, $b=\dfrac{R-\sqrt{R^2+4/Da}}{2}$,

$$f_1=\frac{\left(1-r_Te^{R\Pr}\right)Da}{\left(1-e^{R\Pr}\right)},\quad f_2=\frac{\left(r_T-1\right)}{\left(1-e^{R\Pr}\right)\left(R^2\Pr^2-R^2\Pr-\dfrac{1}{Da}\right)},$$

$$f_3=\frac{Gr}{\mathrm{Re}}\left(f_1-f_2\right)+ADa,\quad f_4=\frac{Gr}{\mathrm{Re}}\left(f_1-f_2e^{R\Pr}\right)+ADa,$$

$$c_1=\frac{f_4-f_3e^b}{e^b-e^a},\qquad c_2=\frac{f_3e^a-f_4}{e^b-e^a}.$$

3.4 First-order solution (or Solution for a second - grade fluid):

Solving Eq. (17) with corresponding boundary conditions, we obtain

$$\theta_1=0 \tag{21}$$

Substituting the equations (20) and (21) into the Eq. (16) and then solving the resulting equation with the corresponding conditions, we get

$$u_1=c_3e^{ay}+c_4e^{by}-f_6ye^{ay}-f_7ye^{by}+f_5e^{R\Pr y}$$

where $f_5=\dfrac{Gr}{\mathrm{Re}}f_2\dfrac{R^4\Pr^3}{\left(R^2\Pr^2-R^2\Pr-\dfrac{1}{Da}\right)}$,

$$f_6=\frac{Rc_1a^3}{2a-R},\qquad f_7=\frac{Rc_2b^3}{2b-R},$$

$$f_8=f_5e^{R\Pr}-f_6e^a-f_7e^b,\quad c_3=\frac{f_8-f_5e^b}{e^b-e^a},$$

$$c_4=\frac{e^af_5-f_8}{e^b-e^a}.$$

Note that when $k=0$, $R=0$ and $Da\to\infty$ our results reduces to those given by Aung and Worku (1986).

4. DISCUSSION OF THE RESULTS:

In order to see the effects of $Da, k, R, \Pr, Gr, \mathrm{Re}$ and r_T on the velocity u, we have plotted Figs. 2-8.

The effect of Darcy number Da on u for $k=0.02, r_T=0.5, R=5, A=1$, $Gr=1, \Pr=2$ and $\mathrm{Re}=1$ is depicted in Fig. 2. It is observed that, the velocity u is increases with increasing Da.

Fig. 3 shows the effect of viscoelastic parameter k on u for $Da = 0.1,$ $r_T = 0.5,$ $R = 5,$ $A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$. It is found that, the velocity u decreases with an increase in k. The point where the maximum velocity occurs is shifted away from the upper wall as the value of the viscoelastic parameter is increased.

The influence of R on u for $Da = 0.1, r_T = 0.5, k = 0.02, A = 1$ $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$ is presented in Fig. 4. It is observed that, the velocity u decreases with increasing R.

Fig. 5 depicts the effect of Prandtl number $\Pr$ on u for $Da = 0.1,$ $r_T = 0.5,$ $R = 5, A = 1$, $Gr = 1, k = 0.02$ and $\mathrm{Re} = 1$. It is found that, the velocity u increases on increasing Prandtl number $\Pr$.

The effect of Grashof number Gr on u for $Da = 0.1, r_T = 0.5, R = 5, A = 1$ $k = 0.02, \Pr = 2$ and $\mathrm{Re} = 1$ is shown in Fig. 6. It is observed that, the velocity u increases with increasing Grashof number Gr.

Fig. 7 shows the effect of Reynolds number Re on u for $Da = 0.1, r_T = 0.5,$ $R = 5, A = 1$, $Gr = 1, \Pr = 2, k = 0.02$ and $\mathrm{Re} = 1$. It is found that, the velocity u decreases with increasing Reynolds number Re.

The effect of wall temperature parameter r_T on u for $Da = 0.1,$ $k = 0.1,$ $R = 5,$ $A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$ is depicted in Fig. 8. It is observed that, the velocity u increases with increasing r_T.

Fig. 9 shows the effect of R on the temperature θ for $r_T = 0.5$ and $\Pr = 2$. It is found that, the temperature θ decreases on increasing R.

The effect of Prandtl number $\Pr$ on the temperature θ for $r_T = 0.5$ and $R = 5$ is presented in Fig. 10. It is observed that, the temperature θ is decreases with increasing Prandtl number $\Pr$.

Fig. 11 depicts the effect of r_T on temperature θ for $R = 5$ and $\Pr = 2$. It is found that, the temperature θ increases with an increase in r_T

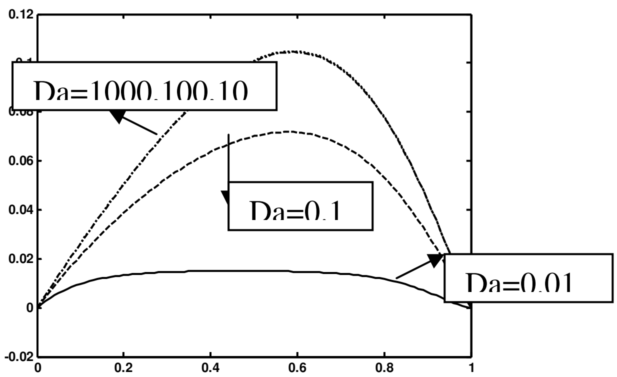

Fig. 2. Effect of Darcy number Da on u for $k = 0.02, r_T = 0.5, R = 5, A = 1$ $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

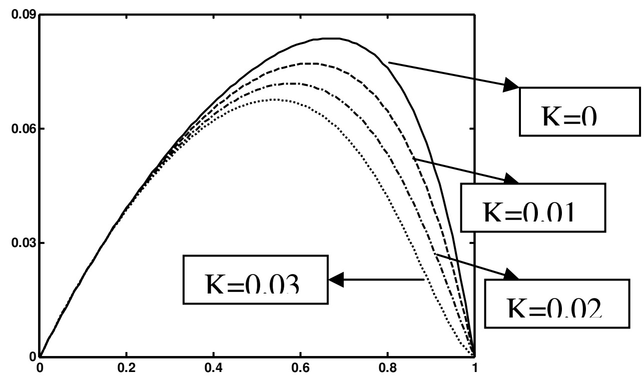

Fig. 3. Effect of viscoelastic parameter k on u for $Da = 0.1, r_T = 0.5, R = 5,$ $A = 1$, $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

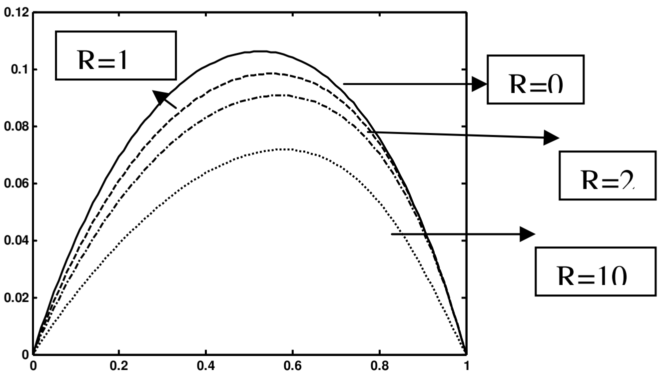

Fig. 4. Effect of R on u for $Da = 0.1, r_T = 0.5, k = 0.02, A = 1$ $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

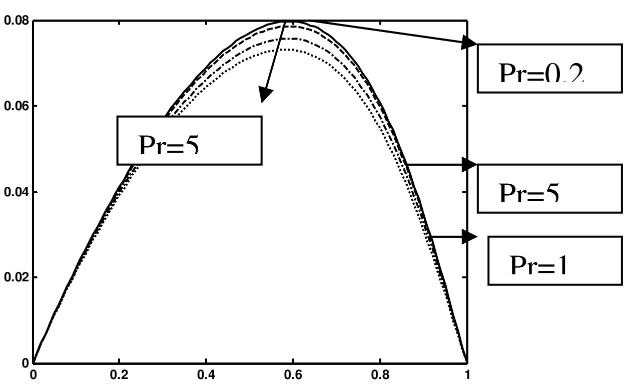

Fig. 5. Effect of Prandtl number $\Pr$ on u for $Da = 0.1, r_T = 0.5, R = 5, A = 1$ $Gr = 1, k = 0.02$ and $\mathrm{Re} = 1$.

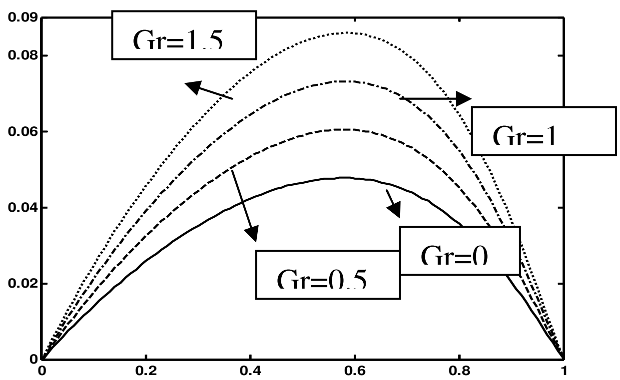

Fig. 6. Effect of Grashof number Gr on u for $Da = 0.1, r_T = 0.5, R = 5, A = 1$ $k = 0.1, \Pr = 2$ and $\mathrm{Re} = 1$.

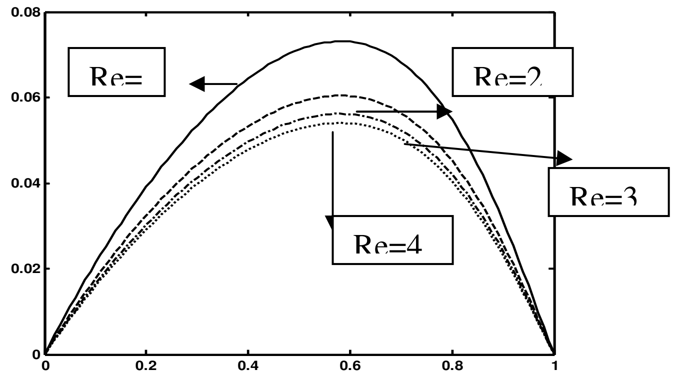

Fig. 7. Effect of Reynolds number Re on u for $Da = 0.1, r_T = 0.5, R = 5, A = 1$ $Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

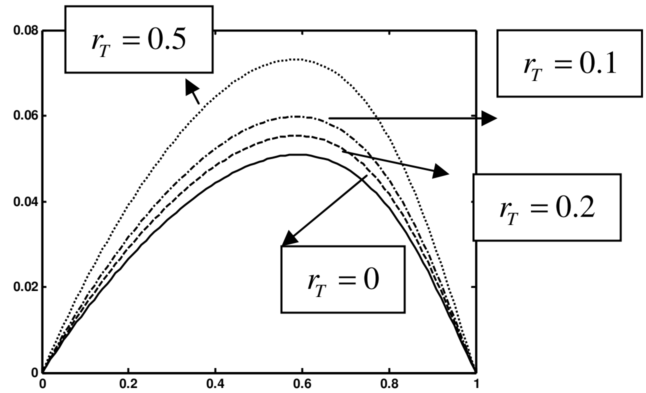

Fig. 8. Effect of wall temperature parameter k on u for

$Da = 0.1, k = 0.1,$
$R = 5,\ A = 1, Gr = 1, \Pr = 2$ and $\mathrm{Re} = 1$.

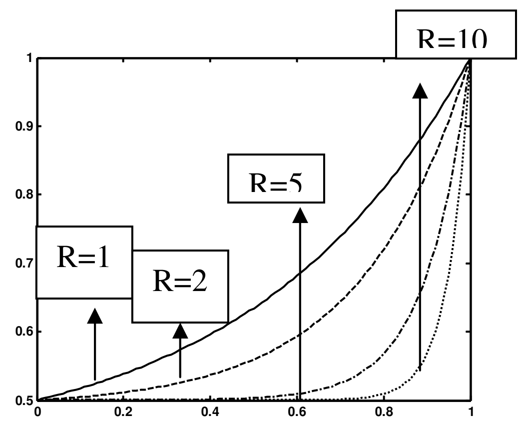

Fig. 9. Effect of R on θ for $r_T = 0.5$ and $\Pr = 2$.

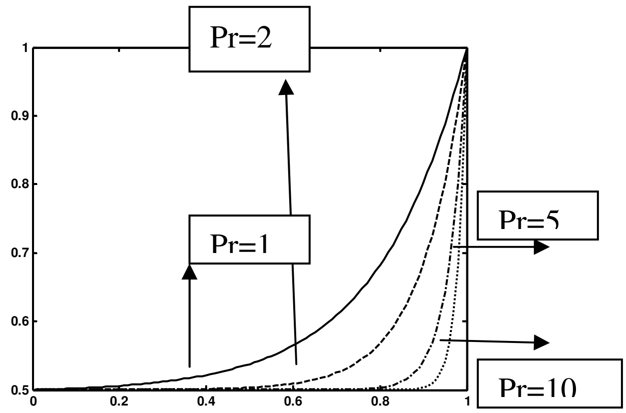

Fig. 10. Effect of $\Pr$ on θ for $r_T = 0.5$ and $R = 5$.

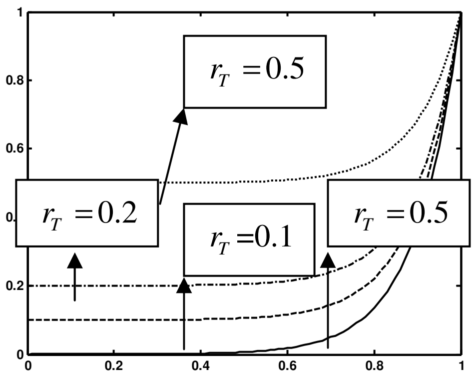

Fig. 11. Effect of r_T on θ for $R = 5$ and $\Pr = 2$

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