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GEOMETRIC MEAN LABELING ON DOUBLE TRIANGULAR SNAKES

[1]S. S. Sandhya* and [2]S. Somasundaram

[1]*Department of Mathematics, Sree Ayyappa College for Women, Chunkankadai-629003, India.*

[2]*Department Mathematics, M.S. University, Tirunelveli – 627 012, India.*

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ABSTRACT

A Graph G = (V, E) with p vertices and q edges is called a Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1, 2………q+1 in such a way that when each edge e=uv is labeled with f(e = vu) = $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ or $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$ then the edge labels are distinct. In this case, f is called Geometric mean labeling of G. In this paper we prove that Double Triangle snake and Alternate Double Triangular snake graphs are Geometric mean graphs.

Keywords: *Graph, Geometric mean graph, Double Triangular snake, Alternate Double Triangular snake.*

1. INTRODUCTION

All graph in this paper are finite and undirected graph G= (V, E) with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. We will provide brief summary of definitions and other informations which are required for the present investigation.

Definition 1.1: A graph G = (V,E) with p vertices and q edges is called a Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…….q+1 in such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ or $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$ then the edge labels are distinct.

In this case f is called Geometric mean labeling of G.

Definition 1.2: A Triangular snake T_n is obtained from a path $v_1, v_2, \ldots v_n$ by joining v_i to v_{i+1} to a new vertex w_i for $1 \le i \le n-1$.

That is, every edge of path is replaced by a Triangle C_3.

Definition 1.3: An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.

Definition 1.4: A Double triangular snake $D(T_n)$ is the graph obtained from the path $u_1, u_2, \ldots u_n$ by joining u_i u_{i+1} to two new vertices v_i, w_i, $1 \le i \le n-1$.

Definition 1.5: Alternate Double triangular snake $A(D(T_n))$ is the graph obtained from the path $u_1, u_2 \ldots u_n$ by joining u_i, u_{i+1} (Alternatively) with two new vertices v_i and w_i, $1 \le i \le n-1$.

S.Somasundaram and S.S.Sandhya introduced Harmonic mean labeling of a Graphs [3] and studied their behaviour in [4]. S. Somasundaram, P. Vidhyarani and S.S.Sandhya introduced the concept of Geometric mean labeling of Graphs.

In this paper we prove that Double Triangular snakes and Alternate Double Triangular snakes are Geometric mean graphs.

*Corresponding author: [1]S. S. Sandhya**
[1]Department of Mathematics, Sree Ayyappa College for Women, Chunkankadai-629003, India.

2. MAIN RESULTS

Theorem 2.1: A Double Triangular Snake $D(T_n)$ is a Geometric mean graph

Proof: Consider a path $u_1, u_2 \ldots\ldots\ldots u_n$ Join u_i, u_{i+1} with two new vertices $v_i, w_i,\ 1 \le i \le n-1$.

Define a function $f: V(D(T_n)) \rightarrow \{1,2\ldots\ldots q+1\}$by

$f(u_1) = 2$

$f(u_i) = 5i-4,\ 2 \le i \le n$

$f(v_1) = 1$

$f(v_i) = 5i-3,\ 2 \le i \le n-1$

$f(w_i) = 5i-1,\ 1 \le i \le n-1.$

The edges are labeled with

$f(u_1u_2) = 4$

$f(u_iu_{i+1}) = 5i-2,\ 2 \le i \le n-1.$

$f(u_iv_i) = 5i-4,\ 1 \le i \le n-1$

$f(u_1v_1) = 2$

$f(u_{i+1}v_{i)}) = 5i-1,\ 2 \le i \le n-1$

$f(u_1w_1) = 3$

$f(u_iw_{i)}) = 5i-3,\ 2 \le i \le n-1.$

$f(u_{i+1}w_i) = 5i,\ 1 \le i \le n-1.$

This makes $D(T_n)$ a Geometric mean graph

Example 2.2: The Geometric mean labeling of $D(T_4)$ is given below

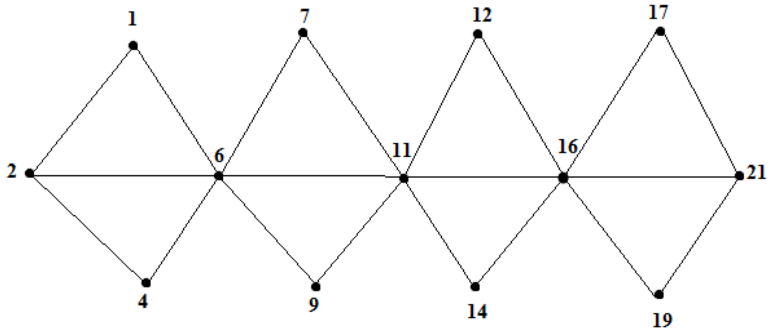

Figure: 1

Next we have the following

Theorem 2.3: Alternate Double Triangular snake $A(D(T_n))$ is a Geometric mean graph.

Proof: Let G be the graph $A(D(T_n))$. Let P_n be the path $u_1\ u_2\ldots\ldots\ldots\ldots u_{n.}$ To construct G, join u_i, u_{i+1} (alternatively) with two new vertices $v_i, w_i\ 1 \le i \le n-1$.

Here we consider two different cases

Case (1): If the Double Triangular Snake $A(D(T_n))$ starts from u_1 then we need to considered two subcases.

Sub case (1) (a): If n is odd, then

Define a function f: $V(G) \rightarrow \{1,2\ldots.q+1\}$by $f(u_1)=2$

$f(u_i)=3i-1,\ 2 \le i \le n-1$ $f(u_n) = 3n-1$

$f(v_1) = 1$

$f(v_i) = 6(i-1),\ 2 \le i \le \frac{n-1}{2}$

$f(w_i) = 6i-2,\ 1 \le i \le \frac{n-1}{2}$

The edges are labeled with

$f(u_iu_{i+1}) = 3i+1,\ \forall i = 1,3\ldots\ldots\ldots..n-2$

$f(u_i\, u_{i+1}) = 3i,\ \forall i = 2,4\ldots\ldots\ldots,n-1.$

$f(u_{2i-1}v_i) = 6i-5\ \forall i = 1,2\ldots\ldots\frac{n-1}{2}$

$f(u_{2i}\, v_i) = 6i-4,\ \forall i = 1,2\ldots\ldots\ldots\frac{n-1}{2}$

$f(u_{2i-1}\, w_i) = 6i-3,\ \forall i = 1,2\ldots\ldots\ldots..\frac{n-1}{2}$

$f(u_{2i}\, w_i) = 6i-1,\ \forall i = 1,2\ldots\ldots\ldots\frac{n-1}{2}$

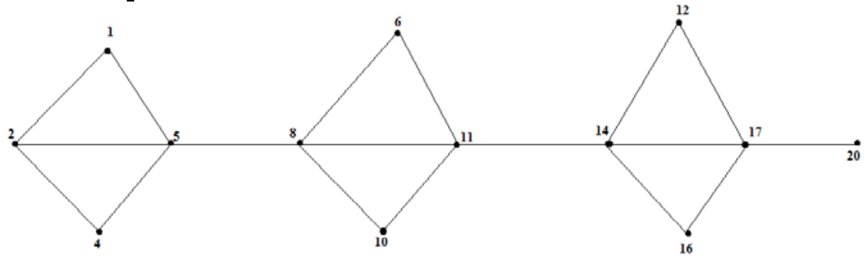

Figure: 2

In this case, f provides a Geometric mean labeling

Sub case (1) (b): If n is even, then define a function $f:V(G) \rightarrow \{1,2\ldots.q+\}$by

$f(u_1) = 2$

$f(u_i) = 3i-1,\ 2 \le i \le \frac{n}{2}$

$f(v_1) = 1$

$f(v_i) = 6(i-1),\ 2 \le i \le \frac{n-1}{2}$

$f(w_i) = 6i-2,\ 1 \le i \le \frac{n-1}{2}$

The edges are labeled with

$f(u_iu_{i+1}) = 3i+1,\ \forall i = 1,3\ldots\ldots\ldots,n-1$

$f(u_iu_{i+1}) = 3i,\ \forall i = 2,4,6\ldots\ldots..n-2$

$f(u_{2i-1}\, v_i) = 6i-5,\ \forall i = 1,2\ldots\ldots\frac{n}{2}$

$f(u_{2i}\, v_i) = 6i-4,\ \forall i = 1,2\ldots\ldots\ldots\frac{n}{2}$

$f(u_{2i-1}w_i) = 6i-3, \forall i = 1,2\ldots\ldots\frac{n}{2}$

$f(u_{2i}\ w_i) = 6i-1, \forall i = 1,2\ldots\frac{n}{2}$

The labeling pattern is shown below

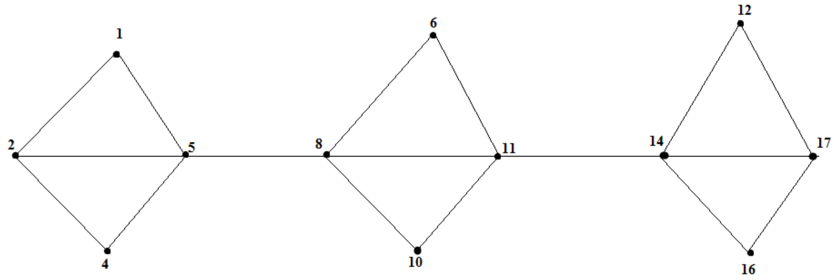

Figure: 3

In the labeling pattern f is a Geometric mean labeling of G.

Case (ii): If the Double Triangular snake $A(D(T_n))$ starts from u_2, then we have to consider two subscases.

Sub case (ii) (a): If n is odd then define a function f: $V(G) \rightarrow \{1,2\ldots\ldots q+1\}$by

$f(u_1) = 1$

$f(u_i) = 3i-3,\ 2 \leq i \leq n$

$f(v_1) = 2$

$f(v_i) = 6i-5,\ 2 \leq i \leq \frac{n-2}{2}$

$f(w_i) = 6i-1,\ 1 \leq i \leq \frac{n-2}{2}$

Then the edges are labeled with

$f(u_iu_{i+1}) = 3i-2,\ \forall\ i = 1,3,5,\ldots\ldots n-1$

$f(u_iu_{i+1}) = 3i-1,\ \forall i=2,4,6\ldots\ldots n$

$f(u_{2i}v_i) = 6i-4,\ \forall i= 1,2\ldots\ldots\frac{n-2}{2}$

$f(u_{2i+1}, v_i) = 6i-3,\ \forall i = 1,2\ldots\ldots\frac{n-2}{2}$

$f(u_{2i}\ w_i) = 6i-2,\ \forall i = 1,2\ldots\ldots, \frac{n-2}{2}$

$f(_{2i+1}w_i) = 6i,\ \forall i = 1,2\ldots\ldots\frac{n-2}{2}$

The labeling pattern is

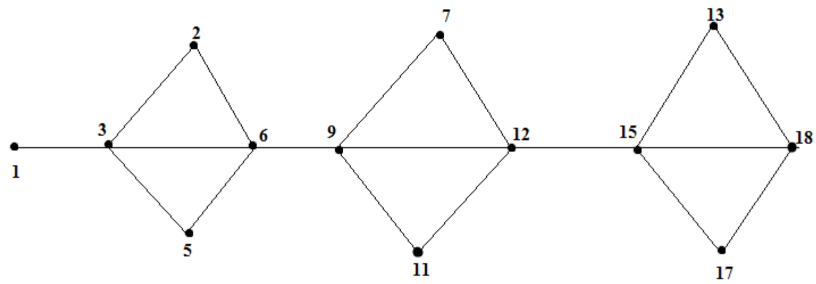

Figure: 4

In this case, f provides a Geometric mean labeling for G.

Sub case (ii) (b): If n is even then define a function f: V(G) → {1,2….q+1} by $f(u_1) = 1$

$f(u_i) = 3i-3,\ 2 \le i \le n$

$f(v_1) = 2$

$f(v_i) = 6i-5,\ 2 \le i \le \frac{n-2}{2}$

$f(w_i) = 6i-1,\ 1 \le i \le \frac{n-2}{2}$

Then the edges are labeled with

$f(u_iu_{i+1}) = 3i-2,\ \forall i = 1,3,5 \ldots n-1$

$f(u_iu_{i+1}) = 3i-1,\ \forall i = 2,4 \ldots n-2$

$f(u_{2i}\, v_1) = 6i-4,\ \forall i = 1, 2, \ldots \frac{n-2}{2}$

$f(u_{2i+1}\, v_i) = 6i-3,\ \forall i = 1,2 \ldots \frac{n-2}{2}$

$f(u_{2i}\, w_i) = 6i-2,\ \forall i = 1,2 \ldots \frac{n-2}{2}$

$f(u_{2i+1}w_i) = 6i,\ \forall i = 1,2 \ldots \frac{n-2}{2}$

The labeling pattern is

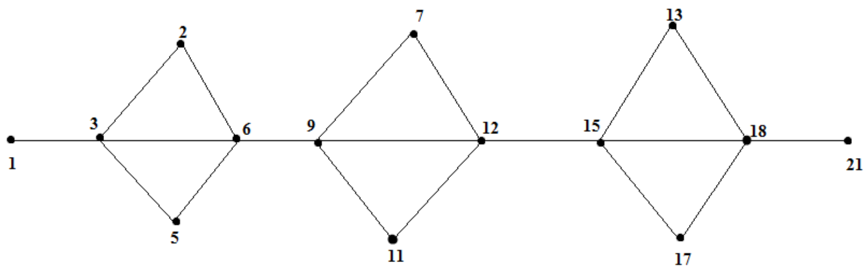

Figure: 5

From all the above cases, we conclude that Alternate Double Triangular snakes $A(D(T_n))$ is a Geometric mean graph.

REFERENCES

[1] J.A.Gallian, 2010, A dynamic survey of graph labeling, The electronic Journal of Combinatories 17 # DS6.

[2] Harary.F 1988, Graph theory, Narosa publishing House Reading, New Delhi.

[3] Somasundaram.S and Ponraj.R 2003, Mean labeling of graphs, National Academy of Science Letters Vol.26, p.210-213.

[4] Somasundaram.S and Ponraj.R and Sandhya S.S, Harmonic mean labeling of Graphs, Communicated.

[5] C. Jayasekharan, S.S.Sandhya and C.DavidRaj, Harmonic Mean Labeling on Double Triangular snakes, "International Journal of Mathematics Research Vol.5 No.2 (2013), pp.251-256.

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