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T-FUZZY BI-IDEALS IN Γ-NEAR-RINGS WITH RESPECT TO t-NORM

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ABSTRACT

In this paper we introduce the concept of T-fuzzy bi-ideals using t-norm in zero-symmetric Γ-near-rings and investigate some of their properties.

Key words: *Γ-near-rings, T-fuzzy Bi-ideals, t-norm.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [7] in his classic paper in 1965. Nobuaki kurokic is the pioneer of fuzzy ideal theory of semigroups. Gamma near-rings were defined by Bh.Satyanarayana [5] and G.L.Booth [2]. Fuzzy ideals in gamma near-rings were introduced by Y. B. Jun, K. H. Kim and M. AOzturk [3]. Fuzzy bi-ideals in gamma near-rings were introduced by N. Meenakumari and T.Tamizh chelvam [4]. The notion of fuzzy ideals of a Γ-near-ring with respect to t-norm was introduced by T.Srinivas and T.Nagaiah [6]. In this paper we introduce the concept of T-fuzzy bi-ideals using t-norm in zero-symmetric Γ-near-rings and investigate some of their properties.

2. PRELIMINARIES

We first recall some basic concepts for the sake of completeness.

Definition 2.1: A Γ-near-ring [5] is a triple (M, +, Γ) where
(i) (M, +) is a group.
(ii) Γ is non-empty set of binary operations on M such that for each α ∈ Γ, (M, +, α) is a near-ring.
(iii) $x\,\alpha\,(y\,\beta\,z) = (x\alpha y)\,\beta\,z$ for all x, y, z ∈ M and for all α, β ∈ Γ.

Definition 2.2: A Γ- near-ring M is said to be zero-symmetric if $m\,\gamma\,0 = 0$ for all m ∈ M and for all $\gamma \in \Gamma$.

Throughout this paper, we assume that M is a zero-symmetric Γ- near-ring.

Definition 2.3: A subgroup B of M is said to be a bi-ideal if $B\Gamma M\Gamma B \subseteq B$.

Definition 2.4: A fuzzy set on M is a function $\mu: M \to [0, 1]$.

Definition 2.5: A fuzzy set μ in M is called a fuzzy bi-ideal of M if
(i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$ for all x, y ∈ M
(ii) $\mu(x\,\alpha\,y\,\beta\,z) \geq \min\{\mu(x), \mu(z)\}$ for all x, y, z ∈ M and α, β ∈ Γ.

Definition 2.6: ([1]) A t-norm is a function T: [0, 1] x [0, 1] → [0, 1] that satisfies the following conditions:
(i) $T(x, 1) = x$
(ii) $T(x, y) = T(y, x)$
(iii) $T(x, T(y, z)) = T(T(x, y), z)$
(iv) $T(x, y) \leq T(x, z)$ whenever $y \leq z$

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Definition 2.7: Let A and B be fuzzy subsets of a non-empty set X. A fuzzy subset $A \wedge B$ is defined by $(A \wedge B)(x) = T(A(x), B(x))$ for all $x \in X$.

Definition 2.8: Let M and M′ be Γ-near-rings. A mapping $f: M \to M'$ is called a Γ-near-ring homomorphism if
(i) $f(x + y) = f(x) + f(y)$
(ii) $f(x \gamma y) = f(x) \gamma f(y)$

Definition 2.9: Let μ be a fuzzy set defined on M and f be a function defined on M then the fuzzy set μ_f in f(M) is defined by $\mu_f(y) = \sup_{x \in f^{-1}(y)} \mu(x)$ for all $y \in f(M)$ and is called the image of μ under f. Similarly if ν is a fuzzy set in f(M), then $\mu = \nu \circ f$ in M is defined as $\mu(x) = \nu(f(x))$ for all $x \in M$ and is called the pre-image of ν under f.

Definition 2.10: A fuzzy set μ of M has the Sup property if for any subset N of M, there exists $a_0 \in N$ such that $\mu(a_0) = \sup_{a \in N} \mu(a)$

3. T-FUZZY BI-IDEALS IN Γ-NEAR-RINGS

Definition 3.1: A fuzzy set μ in a Γ-near-ring M is called a T-fuzzy bi-ideal of M if
(i) $\mu(x-y) \geq T(\mu(x), \mu(y))$ for all $x, y \in M$
(ii) $\mu(x \alpha y \beta z) \geq T(\mu(x), \mu(z))$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Proposition 3.2: If μ and λ are T-fuzzy bi-ideals of M then $\mu \wedge \lambda$ is a T-fuzzy bi-ideal of M.

Proof: Let μ and λ be T-fuzzy bi-ideals of M. Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Then

(i)
$$\begin{aligned} \mu \wedge \lambda(x - y) &= T(\mu(x-y), \lambda(x-y)) \\ &\geq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\ &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\ &= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y)) \\ &= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y))) \\ &= T(\mu \wedge \lambda(x), \mu \wedge \lambda(y)) \end{aligned}$$

(ii)
$$\begin{aligned} \mu \wedge \lambda(x \alpha y \beta z) &= T(\mu(x \alpha y \beta z), \lambda(x \alpha y \beta z)) \\ &\geq T(T(\mu(x), \mu(z)), T(\lambda(x), \lambda(z))) \\ &= T(T(T(\mu(x), \mu(z)), \lambda(x)), \lambda(z)) \\ &= T(T(T(\mu(x), \lambda(x))), \mu(z)), \lambda(z)) \\ &= T(T(\mu(x), \lambda(x)), T(\mu(z)), \lambda(z)) \\ &= T(\mu \wedge \lambda(x), \mu \wedge \lambda(z)) \end{aligned}$$

Thus $\mu \wedge \lambda$ is a T-fuzzy bi-ideal of M.

Proposition 3.3: A fuzzy set μ in a Γ-near-ring M is a T-fuzzy bi-ideal then the level set $U(\mu, t) = \{x \in M / \mu(x) \geq t\}$ is a bi-ideal of M when it is non-empty.

Proof: Let μ be a T-fuzzy bi-ideal of M. Let $x, y \in U(\mu, t)$. Then $\mu(x) \geq t$ & $\mu(y) \geq t$.

Consider $\mu(x-y) \geq T(\mu(x), \mu(y)) \geq T(t, t) = t$ which implies $x - y \in U(\mu, t)$.

Now $\mu(x \alpha y \beta z) \geq T(\mu(x), \mu(z)) \geq T(t, t) = t$ which implies $x \alpha y \beta z \in U(\mu, t)$.

Hence $U(\mu, t)$ is a bi-ideal of M.

Theorem 3.4: Let $f: M \to M'$ be an onto homomorphism of Γ-near-rings. If μ is a T-fuzzy bi-ideal of M then f(μ) is a T-fuzzy bi-ideal of M′.

Proof: Let μ be a T-fuzzy bi-ideal of M. Then $\{x / x \in f^{-1}(y_1 - y_2)\} \supseteq \{x_1 - x_2 / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$.

$$\begin{aligned} f(\mu)(y_1 - y_2) &= \sup\{\mu(x) / x \in f^{-1}(y_1 - y_2)\} \\ &\geq \sup\{\mu(x_1 - x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{T(\mu(x_1), \mu(x_2)) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\} \\ &= T(\sup\{\mu(x_1) / x_1 \in f^{-1}(y_1)\}, \sup\{\mu(x_2) / x_2 \in f^{-1}(y_2)\} \\ &= T(f(\mu)(y_1), f(\mu)(y_2)) \end{aligned}$$

$$\begin{aligned} f(\mu)\,(y_1\,\alpha\,y_2\,\beta y_3) &= \sup\{\mu(x)\,/\,x \in f^{-1}(y_1\,\alpha\,y_2\,\beta y_3)\} \\ &\geq \sup\{\mu\,(x_1\,\alpha\,x_2\,\beta\,x_3)\,/\,x_1 \in f^{-1}(y_1),\ x_2 \in f^{-1}(y_2), x_3 \in f^{-1}(y_3)\} \\ &= \sup\{T(\mu(x_1), \mu(x_3))\,/\,x_1 \in f^{-1}(y_1),\ x_3 \in f^{-1}(y_3)\} \\ &= T(\sup\{\mu(x_1)\,/\,x_1 \in f^{-1}(y_1), \sup\{\mu(x_3)\,/\,x_3 \in f^{-1}(y_3)\}) \end{aligned}$$

Hence $f(\mu)$ is a T-fuzzy bi-ideal of M'.

Theorem 3.5: An onto homomorphic image of a T-fuzzy bi-ideal with Sup property is a T-fuzzy bi-ideal.

Proof: Let M and M′ be Γ-near-rings. Let f: M→ M′ be an epimorphism and μ be a T-fuzzy bi-ideal of M with Sup property. Let $x, y \in M'$, $x_0 \in f^{-1}(x)$, $y_0 \in f^{-1}(y)$ and $z_0 \in f^{-1}(z)$ be such that $\mu(x_0) = \sup_{t \in f^{-1}(x)} \mu(t)$; $\mu(y_0) = \sup_{t \in f^{-1}(y)} \mu(t)$ and $\mu(z_0) = \sup_{t \in f^{-1}(z)} \mu(t)$ respectively. Then we have

$$\begin{aligned} \mu_f(x - y) &= \sup_{z \in f^{-1}(x-y)} \mu(z) \\ &\geq \mu(x_0 - y_0) \\ &\geq T(\mu(x_0), \mu(y_0)) \\ &= T(\sup_{t \in f^{-1}(x)} \mu(t), \sup_{t \in f^{-1}(y)} \mu(t)) \\ &= T(\mu_f(x), \mu_f(y)) \end{aligned}$$

Let $x, y, z \in M'$ and $\alpha, \beta \in \Gamma$.

$$\begin{aligned} \mu_f(x\,\alpha\,y\,\beta\,z) &= \sup_{t \in f^{-1}(x\,\alpha\,y\,\beta\,z)} \mu(t) \\ &\geq \mu(x_0\,\alpha\,y_0\,\beta\,z_0) \\ &\geq T(\mu(x_0), \mu(z_0)) \\ &= T(\sup s \in f^{-1}(x)\,\mu(s), \sup s \in f^{-1}(z) \\ &= T(\mu_f(x), \mu_f(z)) \end{aligned}$$

Theorem 3.6: An epimorphic preimage of a T-fuzzy bi-ideal of a Γ-near-ring is a T-fuzzy bi-ideal.

Proof: Let M and M′ be Γ-near-rings. Let f: M → M′ be an epimorphism. Then we have

$$\begin{aligned} \mu\,(x-y) &= (\nu \circ f)\,(x - y) \\ &= \nu(f(x - y)) \\ &= \nu\,(f(x) - f(y)) \\ &\geq T(\nu\,(f(x)), \nu\,(f(y))) \\ &= T((\nu \circ f)\,(x), (\nu \circ f)\,(y)) \\ &= T(\mu\,(x), \mu\,(y)\,) \end{aligned}$$

$$\begin{aligned} \mu(x\,\alpha\,y\,\beta\,z) &= (\nu \circ f)\,(x\,\alpha\,y\,\beta\,z) \\ &= \nu\,(f(x\,\alpha\,y\,\beta\,z)) \\ &= \nu\,(f(x)\,\alpha\,f(y)\,\beta\,f(z)) \\ &\geq T(\nu\,(f(x)), \nu\,(f(z))) \\ &= T((\nu \circ f)\,(x), (\nu \circ f)\,(z)) \\ &= T(\mu\,(x), \mu\,(z)\,) \end{aligned}$$

Hence μ is a T-fuzzy bi-ideal of M.

Definition 3.7: Let μ and γ be T-fuzzy bi-ideals of a Γ-near-ring M. Then the direct product of T-fuzzy bi-ideals is defined by $(\mu \times \gamma)\,(x, y) = T(\mu\,(x), \gamma(y))$ for all $x, y \in M$.

Theorem 3.8 Let M and M′ be Γ-near-rings. If μ and γ are T-fuzzy bi-ideals of M and M′ respectively then $\mu \times \gamma$ is a T-fuzzy bi-ideal of the direct product M x M′.

Proof: Let μ and γ be T-fuzzy bi-ideals of M and M′ respectively. Let $(x_1, y_1), (x_2, y_2) \in M \times M'$.

$$\begin{aligned} (\mu \times \gamma)((x_1, y_1) - (x_2, y_2)) &= (\mu \times \gamma)\,((x_1 - x_2, y_1 - y_2)) \\ &= T(\mu(x_1 - x_2), \gamma(y_1 - y_2)) \\ &\geq T(T(\mu\,(x_1), \mu\,(x_2)\,), T(\gamma(y_1), \gamma\,(y_2))) \\ &= T(T(T(\mu(x_1), \mu(x_2)), \gamma(y_1), \gamma\,(y_2)) \\ &= T(T(T(\mu(x_1),\gamma(y_1)), \mu\,(x_2)), \gamma\,(y_2)) \\ &= T(T(\mu(x_1), \gamma(y_1)), T(\mu(x_2), \gamma\,(y_2)) \\ &= T((\mu \times \gamma)((x_1, y_1), (\mu \times \gamma)\,(x_2, y_2)) \end{aligned}$$

Let (x_1, y_1), (x_2, y_2) & $(x_3, y_3) \in M \times M'$ and $\alpha, \beta \in \Gamma$.

$$
\begin{aligned}
(\mu \times \gamma)((x_1, y_1)\alpha (x_2, y_2)\ \beta (x_3, y_3)) &= (\mu \times \gamma) (x_1 \alpha x_2 \beta x_3, y_1 \alpha y_2 \beta y_3) \\
&= T(\mu(x_1 \alpha x_2 \beta x_3), \gamma(y_1 \alpha y_2 \beta y_3)) \\
&\geq T (T (\mu(x_1), \mu (x_3)), T(\gamma(y_1)), \gamma(y_3)) \\
&= T(T(T(\mu(x_1), \mu (x_3)), \gamma(y_1)), \gamma (y_3)) \\
&= T(T(T(\mu(x_1), \gamma(y_1)), \mu (x_3)), \gamma (y_3)) \\
&= T(T(\mu(x_1), \gamma(y_1)),\ T(\mu(x_3), \gamma (y_3)) \\
&= T((\mu \times \gamma) (x_1, y_1), (\mu \times \gamma) (x_3, y_3))
\end{aligned}
$$

Hence $\mu \times \gamma$ is a T-fuzzy bi-ideal of the direct product $M \times M'$.

Theorem 3.9: Let μ be a T-fuzzy bi-ideal of M. Then the set M/μ of all fuzzy cosets of μ is a Γ- near-ring w.r.to the operations defined by $(x+\mu) +(y+\mu) = x + y + \mu$ and $(x+\mu)\ \alpha\ (y+\mu) = x\ \alpha\ y + \mu$ for all $x, y \in M$ and $\alpha \in \Gamma$

Theorem 3.10: Let I be a bi-ideal of M. If μ is a T-fuzzy bi-ideal of M then the fuzzy set $\bar{\mu}$ of M / I defined by $\bar{\mu}(a + I) = \sup_{x \in I} \mu (a + x)$ is a T-fuzzy bi-ideal of M / I

Proof: Let M be a Γ-near-ring and μ be a T-fuzzy bi-ideal of M. Let $x, y \in M$ such that $x + I = y + I$. Then $y = x + z$ for some $z \in I$.

Thus

$\bar{\mu} (y + I) = \text{Sup}_{a \in I}\ \mu (y + a) = \sup_{a \in I}\ \mu (x + z + a) = \text{Sup}_{z+a = t \in I}\ \mu (x + t) = \bar{\mu} (x + I)$

which implies that $\bar{\mu}$ is well defined.

$$
\begin{aligned}
\text{Now } \bar{\mu} ((x + I) - (y + I)) &= \bar{\mu} (x - y + I) \\
&= \sup_{u - v \in I} \mu((x - y) + (u - v)) \\
&= \sup_{u, v \in I} \mu((x + u) - (y + v)) \\
&\geq \sup_{u, v \in I} T(\mu(x + u), \mu(y + v)) \\
&= T(\sup_{u \in I} \mu(x + u),\ \sup_{v \in I} \mu(y + v)) \\
&= T(\bar{\mu} ((x + I), \bar{\mu} ((y + I))
\end{aligned}
$$

$$
\begin{aligned}
\bar{\mu} ((x + I)\ \alpha\ (y + I)\ \beta\ (z + I)) &= \bar{\mu} (x\ \alpha\ y\ \beta\ z + I) \\
&= \sup_{i \in I} \mu(x\ \alpha\ y\ \beta\ z + i) \\
&= \sup_{i \in I} \mu((x + i)\ \alpha\ (y + i)\ \beta\ (z + i)) \\
&= \sup_{i \in I} T(\mu(x + i), \mu(z + i)) \\
&= T(\sup_{i \in I} \mu(x + i),\ \sup_{i \in I} \mu(z + i)) \\
&= T(\bar{\mu} (x + I), \bar{\mu} (z + I))
\end{aligned}
$$

Hence $\bar{\mu}$ is a T-fuzzy bi-ideal of M / I.

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