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A MATHEMATICAL MODEL FOR THE EFFECT OF TRH ON ACETYLCHOLINE USING FUZZY TRUNCATED GAMMA DISTRIBUTION

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ABSTRACT

The theoretical study of the effect of Thyrotropin Releasing Hormone administration upon acetylcholine release from the cortex was determined. A mathematical model using fuzzy truncated Gamma distribution was developed and used this model to calculate the mean values of Acetylcholine in the given time interval. The result shows that Thyrotropin Releasing Hormone exerts a strong stimulant action on cortical areas.

Key Words: *Fuzzy Truncated gamma distribution, TRH, Upper Incomplete Gamma Function.*

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1. INTRODUCTION

Gamma Distribution is used in many of the real world Problems. In practical life we cannot record all the values in an experiment or trials so that the recorded values are truncated. The truncated distributions would reflect the real world problems better than the original distributions. If we are dealing with life time data, we cannot able to record all the value. Due to some practical problems, the values of a random variable become observable only when they lies between some interval [minimum, maximum] even the random variable is unrestricted. All values of such a random variable that fall outside the interval are never observable, and consequently their existence is not known to us. This property of a statistical distribution is called truncation. Truncation can be observed not only in lifetime data, but also in real life data. Estimation of parameters and properties of Truncated Gamma distribution was developed by D.G. Chapman [6], G. Baikunth Nath[7], L.M Hegdge and R.C. Dahiya[8], A. Philipe[1], C.S. Coffey and K.E. Muller[5]. In many methods and models we assume that all parameters of lifetime density function are precise. But in the real world randomness and fuzziness are mixed up in the lifetime of the systems. In 1965, Zadeh [11] introduced fuzzy set theory. Subsequently, the theory and the mathematics of fuzzy sets were developed and applied in many research fields. The theory of fuzzy probability was developed by Buckley. J.J [4].

Thyrotropin releasing hormone [TRH] and its receptors are widely distributed throughout the brain and spinal cord. Thyrotropin releasing hormone was first characterized by its thyrotropic actions, but it also promotes the release of prolactin and growth hormone. Recent neurochemical studies suggest that TRH and TRH analogs increase the release of acetylcholine. TRH receptors have been found in many areas of the central nervous system, the highest densities being presented in limbic structures and the low densities were presented in the brainstem and cerebellum. TRH is not only a hormone and it also works as a neurotransmitter or modulator of other neurotransmitter agents. The study of TRH was carried out by Boler. J, Enzmann. F. M [2], Brownstein [3] and Simasko and Horita[10].

In this paper, we developed a mathematical model using fuzzy truncated gamma distribution for the effect of TRH administration upon acetylcholine release from the cortex. We determined the alpha cuts of the scale and shape parameter of truncated gamma distribution and calculated the mean values of various alpha level using incomplete gamma functions described by M. Abramowitz and I. Stegun [9].

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2. NOTATIONS

T	-	random variable
a	-	Lower truncation value
b	-	Upper truncation value
λ	-	Scale parameter of gamma distribution
r	-	Shape parameter of gamma distribution
$\Gamma(x, y)$	-	Upper Incomplete Gamma Function
$\bar{\lambda}[\alpha]$	-	α cut of scale parameter
$\bar{r}[\alpha]$	-	α cut of shape parameter
E[t]	-	Mean values

3. MATHEMATICAL MODEL USING FUZZY TRUNCATED GAMMA DISTRIBUTION

Let T be the random variable taking values 't' in the interval [a, b]. If λ and r are unknowns then we must estimate them from a random sample and we obtain a fuzzy estimate $\bar{\lambda}$ for λ and $\bar{r}$ for r. Now consider the probability density function of fuzzy truncated gamma distribution for the fuzzy numbers $\bar{\lambda}$ and $\bar{r}$.

$$\bar{f}(t, \lambda, r, a, b) = \bar{k}[\alpha]\left(\frac{t}{\lambda}\right)^{r-1} e^{-\frac{t}{\lambda}}, \quad t \geq 0, \quad \lambda \in \bar{\lambda}[\alpha], \quad r \in \bar{r}[\alpha], k \in \bar{k}[\alpha]$$

where

$$\bar{k}[\alpha] = \frac{r}{\bar{\lambda}[\alpha]\Gamma\left((1+\bar{r}[\alpha]), \frac{a}{\bar{\lambda}[\alpha]}\right) - \bar{\lambda}[\alpha]\Gamma\left((1+\bar{r}[\alpha]), \frac{b}{\bar{\lambda}[\alpha]}\right) + e^{-\frac{b}{\bar{\lambda}[\alpha]}}(\bar{\lambda}[\alpha])^{1-\bar{r}[\alpha]}(b)^{\bar{r}[\alpha]} - e^{-\frac{a}{\bar{\lambda}[\alpha]}}(\bar{\lambda}[\alpha])^{1-\bar{r}[\alpha]}(a)^{\bar{r}[\alpha]}}$$ and

the upper incomplete Gamma function is defined by

$$\Gamma(x, y) = \int_{y}^{\infty} t^{x-1} e^{-t} dt$$

Its mean value is given by

$$\bar{E}[\lambda, r, a, b] = -\bar{\lambda}[\alpha]^2 \bar{k}[\alpha]\left(-\Gamma\left((1+\bar{r}[\alpha]), \frac{a}{\bar{\lambda}[\alpha]}\right) + \Gamma\left((1+\bar{r}[\alpha]), \frac{b}{\bar{\lambda}[\alpha]}\right)\right)$$

4. APPLICATION

The experimental subjects were housed in groups of three with free access to food and water and kept on a 12 hr light/dark cycle. The subjects were anesthetized with chloral hydrate and placed in a stereotaxic frame. Micro dialysis tubes were inserted transversely in the parietal cortex. After 1 hr settling period the perfusate was collected at 20 minute intervals and directly assayed for Acetylcholine. After collecting the first three samples to measure the basal release of acetylcholine, TRH was injected subcutaneously (1-10 mg/kg) into the group of subjects. Then the release of Acetylcholine was measured from the cortex area. It shows that the effect of perfusing 1-10 mg/ml TRH through the dialysis tubing on Acetylcholine release from the cortex, TRH perfusion stated 60 min after the beginning of the collection period and lasted throughout the experiment. TRH perfusion was accompanied by a significant increase in Acetylcholine release from the cortex. The Maximal increase of 300% and the minimum of 90% in the cortex are shown in the Fig. 4.1.

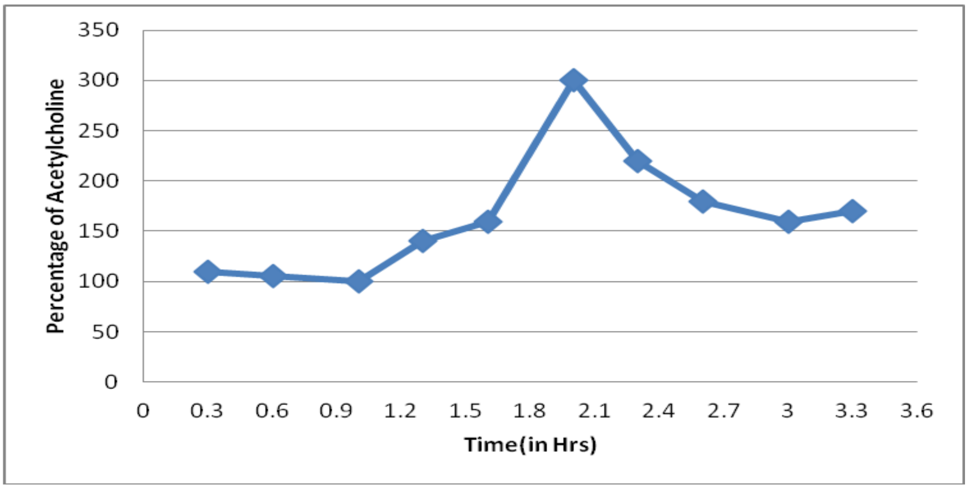

Fig 4.1: *Effects of administration of TRH perfusion on Acetylcholine release from the cortex*

5. SOLUTION BY FUZZY TRUNCATED GAMMA DISTRIBUTION

From the Fig 4.1, the scale and shape parameters of gamma distributions are $\lambda = 0.045$ and $r = 7.375$ respectively. The lower and upper truncation values are assumed to be a = 0.33 and b = 3.33 respectively.
The triangular fuzzy numbers for the scale and shape parameters are

$$\bar{\lambda} = [0.039, \quad 0.045, \quad 0.050]$$

$$\bar{r} = [7.25, \; 7.375, \; 7.430]$$

and the corresponding α cuts are

$$\bar{\lambda}[\alpha] = [0.039 + 0.006\alpha, \; 0.050 - .0.005\alpha]$$

$$\bar{r}[\alpha] = [7.25 + 0.13\alpha, \; 7.430 - .0.050\alpha]$$

Let $\alpha = 0$ then

$$\bar{\lambda}[\alpha] = [0.040, \; 0.054]$$

$$\bar{r}[\alpha] = [2.045, \; 2.070]$$

The mean values are calculated by using

$$\bar{E}[\lambda, r, a, b] \; = \{E_1[\alpha], \; E_2[\alpha]\}$$

Where $E_1[\alpha]$ = Minimum of $\bar{E}[\lambda, r, a, b]$

$E_2[\alpha]$ = Maximum of $\bar{E}[\lambda, r, a, b]$

Table 5.1: *Mean values for the lower alpha levels*

α levels	$\bar{\lambda}[\alpha]$	$\bar{r}[\alpha]$	$\bar{k}\,[\alpha]$	$E_1[\alpha]$
0	0.039	7.25	0.076171386	0.412506545
0.1	0.0396	7.263	0.069140994	0.414646444
0.2	0.0402	7.276	0.063081095	0.417120448
0.3	0.0408	7.289	0.057662708	0.419538981
0.4	0.0414	7.302	0.049294988	0.413960361
0.5	0.042	7.315	0.048516054	0.424430961
0.6	0.0426	7.328	0.044651689	0.426924907
0.7	0.0432	7.341	0.041184205	0.429461141
0.8	0.0438	7.354	0.034807902	0.432082532
0.9	0.0444	7.367	0.035267453	0.434715917
1	0.045	7.38	0.032728106	0.437403843

Table 5.2: *Mean values for the upper alpha levels*

α levels	$\bar{\lambda}[\alpha]$	$\bar{r}[\alpha]$	$\bar{k}\,[\alpha]$	$E_2[\alpha]$
0	0.05	7.43	0.021240594	0.460210399
0.1	0.0495	7.425	0.022051931	0.457670302
0.2	0.049	7.42	0.022984183	0.455434273
0.3	0.0485	7.415	0.023945419	0.453107435
0.4	0.048	7.41	0.02496493	0.450790615
0.5	0.0475	7.405	0.026049116	0.448488618
0.6	0.047	7.4	0.027140784	0.44597376
0.7	0.0465	7.395	0.028445931	0.443967062
0.8	0.046	7.39	0.029771906	0.441744996
0.9	0.0455	7.385	0.030733477	0.438021606
1	0.045	7.38	0.032696981	0.437303699

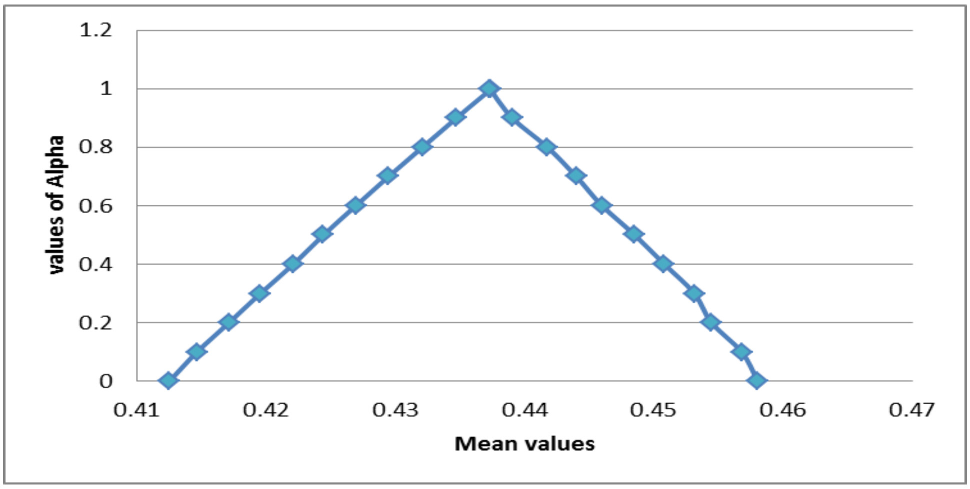

Fig. 5.1: *Membership function for the mean values*

5. CONCLUSION

In this paper, we study the increasing level of Acetylcholine after the administration of TRH in the cortex area and the mean values of the given interval are tabulated using fuzzy truncated gamma distribution. From the membership function of the mean values, it was observed that, after TRH treatment the releasing level of Acetylcholine is increased. This shows that TRH exerts a strong stimulant action on cortical areas and the action of TRH on cholinergic neurons was equally present in various brain areas and its characteristics in terms of dose dependence, time course and reproducibility.

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