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FIRST ORDER TPS METHOD FOR MULTI-OBJECTIVE QUADRATIC OPTIMIZATION PROBLEM

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ABSTRACT

In this paper, we have proposed a solution method for multi-objective quadratic optimization problem (MOQOP) by using the first order Taylor polynomial series (TPS). We observed that there is a change in the method of Pramanik S. et al. [9] in selecting the objective function and constraints in the last stage of the proposed methodology. For this, we find out a new problem in which the objective function and constraints depend on membership functions. The effectiveness of the proposed methodology is illustrated by numerical example in order to support the solution method.

Keyword: *QOP, Excel Solver, Tolerance Limits, LP.*

1. INTRODUCTION

The TPS approximation method is one of the most widely used methods for linearization of quadratic objective functions. Most of the real-life decision problems are in the form of multi-objective functions and conflict to each other regarding optimization of these functions. To resolve the conflict, the GP approach has been introduced by Charnes A and Cooper W.W. [1]. The main problem of using GP is that a precise aspiration level needs to be assigned for each of the objective functions. But, in a real-life decision situation, it is difficult to set precise objective values to target due to imprecise in nature of human judgments. In recent research, the fuzzy sets theory is very active in fuzzy optimization and can be characterized by a membership function which assigns to each object of a domain its grade of membership function and these are defined on the basis of assigned aspiration levels and tolerance limits for the fuzzy goals. But, it is difficult to define tolerance limits in a highly sensitive decision situation. In the real life, multi-objective problems may be faced up frequently with the decisions to optimize like corporate planning, financial planning, production planning, marketing planning, and hospital planning. There exist several techniques to solve these problems in the literature such as Durga Prasad Dash P. *et al.* [4] presented a method in which a fuzzy multi-objective NLP is reduced to crisp using ranking function and then the crisp problem is solved by FGP technique. Pramanik S. *et al.* [9] suggested a FGP approach for solving MOQOP and deals with a decision-making unit with multiple objective functions. Osama E. Emam [8] gives a parametric study on multi-objective integer QOPs under uncertainty. Sugiyarto S. *et al.* [12] proposed a computational procedure by using fuzzy approach to find the optimal solution of QOPs and divide the calculation of the optimal solution into two stages. Toksari M.D. [13] suggested Taylor series method to fuzzy multi-objective LFP problem. Durga Prasad Dash P. *et al.* [3] presented a method to solve the multi-objective fuzzy LFP. Singh P. *et al.* [10] developed an algorithm for solving a fuzzy multi-objective linear plus LFP problem and proposed approach membership functions associated with each objective function that are transformed into linear functions by using Taylor's theorem, then the multi-objective LFP changed into equivalent multi-objective LP problem and then it can be solved as equal weighted multi-objective LP problem. Singh P. *et al.* [11] proposed an approximation method for the solution of a multi-objective linear plus LFP problem in which the membership functions are defined for each fuzzy goal and then a method of variable change on the under and over deviational variables of the membership functions associated with the fuzzy goals of the model is introduced. Dangwal R. *et al.* [2] suggested a method of multi-objective LFP problem with the help of first order TPS method. In order to extend this work for MOQOP, we proposed a solution method at the optimal point of each quadratic objective function in feasible region. This problem has transformed into a more simplified multi-objective LP problem by using TPS and find out a very strong efficient solution.

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2. FORMULATION OF THE PROBLEM

The general mathematical format of the MOQOP can be defined as:

$$\text{Max } \psi(x) = \{\psi_1, \psi_2, \psi_3 \ldots \psi_p\}, \quad \text{subject to } x \in S = \{x \in R^n: Ax \le b, x \ge 0\} \tag{1}$$

where $\psi_i = \frac{1}{2} x^T Q_i x + c_i^T x$ and $x, c^T \in R^n$, $Q_i \in R^{nxn}$ and $A \in R^{mxn}$, for i= 1, 2, ….p

3. METHODOLOGY

In many real life, the optimization problems are in the form of quadratic multi-objectives and optimized subject to a common set of linear constraints. Solving the problem (1) by considering one objective function at a time and ignoring all others. Repeat this process p time for p objective functions. Let us assume that the maximum value of the i^{th} objective function $\psi_i = \psi_i^*$ at $x_i^* = (x_{i1}^*, x_{i2}^*, \ldots x_{in}^*)$ on the feasible region S. Suppose that $\psi(x)$ and all of its partial derivatives of order less than and equal to p + 1 are continuous on the feasible region S. By using first order TPS method, the linear objective function $\varphi_i(x)$ is obtained as follows:

$$\varphi_i(x) \cong \psi_i^*(x_i) + \sum_{j=1}^{n}.\{(x_j - x_{ij}^*)\, \partial\psi_i(x^*) / \partial x_j\} + o(h^2) \text{ for all i}$$

where $o(h^2)$ is the order of the maximum error. This series gives accurate approximation of φ_i and x is close to x^*. We observe that all the quadratic objective functions are transformed into linear objectives functions and then MOQOP reduces into the following multi-objective LP problem as:

$$\text{Max } \varphi(x) = \{\varphi_1, \varphi_2, \varphi_3 \ldots \varphi p\}, \text{ subject to } x \in S = \{x \in R^n: Ax \le b, x \ge 0\} \tag{2}$$

where $\varphi_i(x) \cong \psi_i(x) = \sum_{j=1}^{p}.a_{ij}x_j + d_i$ for all i, x, $c^T \in R^n$, $A \in R^{mxn}$. Now it is possible to gives aspirant for both goals as: $\psi_i \succcurlyeq \psi_i^U$, where ψ_i^U is upper tolerance limit for the i^{th} objective function. Let us considering ψ_i^L is lower tolerance limit for the i^{th} objective function then membership function μ_i for each i^{th} fuzzy goal of the $\psi_i(x) \succcurlyeq \psi_i^U$ can be defined as:

$$\mu_i(x) = \begin{cases} 1, & \text{if } \varphi_i(x) \succcurlyeq \psi_i^U \\ (\varphi_i(x) - \psi_i^L)/(\psi_i^U - \psi_i^L), & \text{if } \psi_i^L \preccurlyeq \varphi_i(x) \preccurlyeq \psi_i^U \\ 0, & \text{if } \varphi_i(x) \preccurlyeq \psi_i^L \end{cases} \tag{3}$$

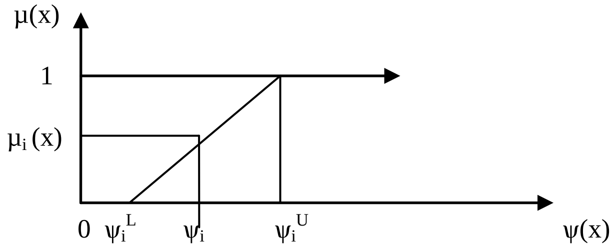

Figure1. Membership function $\mu_i(x)$

Then LP problem reduce as follows:

$$\begin{cases} \text{Max } \theta(\mu) = \sum_{i=1}^{p} \mu_i \\ \text{subject to } \mu_i(\psi_i^U - \psi_i^L) - \varphi_i(x) = -\psi_i^L \\ Ax \le b, \mu_i \le 1, x, \mu_i \ge 0 \text{ for all i} \end{cases} \tag{4}$$

The optimum solution of LP problem (4) gives the efficient solution of (1). The values of membership functions at the optimal points are also determined and give the satisfaction percentage of the objective function to the solution. This reduces the computational complexity and yields an efficient solution.

The Method

The proposed method proceeds as:

Step - 1: Solve MOQOP as a single- objective QOP for p time by taking one of the objective functions at a time with subject to given constraints.

Step - 2: From the above results, determine the corresponding values for every objective at each point. According to each solution and value of every objective, find out a pay-off matrix and then define upper and lower tolerance limits.

Step - 3: Formulate the membership function $\mu_p(x)$ for p^{th} objective function and transform these functions into linear membership functions by using first order TPS.

Step - 4: Formulate the LP model (4) by changing in the rule of selecting objective function and constraints with the help of membership function and solve

4. ILLUSTRATIVE EXAMPLE [Pramanik S. *et al.* (2011)**]**

$$\begin{cases} \text{Max } \psi(x) = \{\psi_1(x), \psi_2(x), \psi_3(x)\}, \\ \text{subject to } x_1 + 4x_2 \le 4,\ x_1 + x_2 \le 2,\ x_1, x_2 \ge 0 \end{cases} \tag{5}$$

Where Max $\psi_1(x) = 4x_1 + 6x_2 - 3x_1^2 - 2x_2^2$, Max $\psi_2(x) = 7x_1 + 3x_2 - 2x_1^2 - 2x_2^2$, Max $\psi_3(x) = 5x_1 + x_2 - x_1^2 - x_2^2$ .On solving, we get ψ_1^{max} (0.56, 0.86) =4.98, ψ_2^{max} (1.5, 0.5) = 7, and ψ_3^{max} (2, 0)= 6. It is observed that $\psi_1, \psi_2, \psi_3 \ge 0$ for each value of x of the feasible region S.

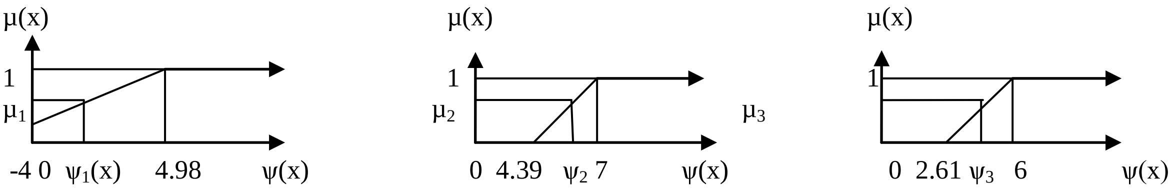

Figure 2: Membership functions $\mu_1(x)$, $\mu_2(x)$ and $\mu_3(x)$

Hence, the MOQOP convert into LP problem as

Max $\theta(\mu) = \mu_1 + \mu_2 + \mu_3$

Subject to $8.98\,\mu_1 - 0.64\,x_1 - 2.8\,x_2 = 6.3816$

$2.706\,\mu_2 - x_1 - x_2 = 0.706$

$3.394\,\mu_3 - x_1 - x_2 = 0.607$

$x_1 + 4\,x_2 \le 4,\ x_1 + x_2 \le 2$

$\mu_1 \le 1,\ \mu_2 \le 1,\ \mu_3 \le 1$

$x_1, x_2, \mu_1, \mu_2, \mu_3 \ge 0$ (6)

Table - 4.1: Computational Results of the problem (6)

Input Results								
	x_1	x_2	μ_1	μ_2	μ_3	Total	Sign	Limits
Objective Function						2.7681		
Constraint I	-0.64	-2.8	8.98	0	0	6.3816	=	6.3816
Constraint II	-1	-1	0	2.706	0	0.706	=	0.706
Constraint III	-1	-1	0	0	3.394	0.607	=	0.607
Constraint IV	1	4	0	0	0	3.8311	≤	4
Constraint V	1	1	0	0	0	2	≤	2
Constraint VI	0	0	1	0	0	1	≤	1
Constraint VII	0	0	0	1	0	1	≤	1
Constraint VIII	0	0	0	0	1	0.7681	≤	1
Non-Negativity	≥ 0	≥ 0	≥ 0	≥ 0	≥ 0			
Output Results								
	x_1	x_2	$\mu_1(x)$	$\mu_2(x)$	$\mu_3(x)$	$\theta(\mu)$		
	1.3896	0.6104	1	1	0.7681	2.7681		
Obj. Functions		$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$				
		2.6824	6.9513	5.2549				

Analysis of Results

Name	Original Value		
Objective Function (Max)	2.768120212		
Name	**Original Value**		
x_1	1.38962963		
x_2	0.61037037		
μ_1	1		
μ_2	1		
μ_3	0.768120212		
Name	**Total Value**	**Status**	**Slack**
Constraint III	0.607	Not Binding	0
Constraint II	0.706	Not Binding	0
Constraint I	6.3816	Not Binding	0
Constraint IV	3.831111111	Not Binding	0.168888889
Constraint V	2	Binding	0
Constraint VII	1	Binding	0
Constraint VI	1	Binding	0
Constraint VIII	0.768120212	Not Binding	0.231879788
x_1	1.38962963	Not Binding	1.38962963
x_2	0.61037037	Not Binding	0.61037037
μ_1	1	Not Binding	1
μ_2	1	Not Binding	1
μ_3	0.768120212	Not Binding	0.768120212

Analysis

Name	Final Value	Reduced Gradient
x_1	1.38962963	0
x_2	0.61037037	0
μ_1	1	0
μ_2	1	0
μ_3	0.768120212	0

Constraints Name	Total Value	Lagrange Multiplier
Constraint III	0.607	0.294637602
Constraint II	0.706	0.369549137
Constraint I	6.3816	0
Constraint IV	3.831111111	0
Constraint V	2	0.66418678
Constraint VII	1	0
Constraint VI	1	0.999999738
Constraint VIII	0.768120212	0

Limitations of the variables

Target Name	Value				
Obj. Function	2.7681				
Adjus. Name	**Value**	**Low. Limit**	**Tar. Result**	**Upp. Limit**	**Tar. Result**
x_1	1.3896	1.3896	2.7681	1.3896	2.7681
x_2	0.6104	0.6104	2.7681	0.61034	2.7681
μ_1	1	1	2.7681	1	2.7681
μ_2	1	1	2.7681	1	2.7681
μ_3	0.7681	0.7681	2.7681	0.7681	2.7681

The optimal solution of the problem (1) is x_1= 1.3896, x_2 = 0.6104 and $\theta(\mu)$ = 2.7681 and values of membership functions are μ_1 =1, μ_2 =1 and μ_3= 0.7681. The membership function values at (1.3896, 0.6104) indicate that the goals ψ_1= 2.6824, ψ_2= 6.9513 and ψ_3 = 5.2549 are satisfied 100%, 100% and 76.81% respectively, for the obtained solution.

5. CONCLUSION

The aim of this paper is to present a method to solve the MOQOP with the help of TPS method at the optimal point of each quadratic objective function in the feasible region. The proposed method to solve MOQOP always yields efficient solution. The complexity in solving it has reduced to easy computation. Hence this method gives more accurate solution. This methodology can be extended to solve multi-objective QFOP and NLP problem which may be the problems as future research studies.

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ABBREVIATIONS MEANING

QOPs - Quadratic Optimization Problems
LP - Linear Programming
LFP - Linear Fractional Programming
GP - Goal Programming
FGP - Fuzzy Goal Programming
MOQOP – Multi-Objective Quadratic Optimization Problem
NLP - Non Linear Programming
TPS – Taylor Polynomial Series

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