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BIANCHI TYPE-1 STRING MAGNETISED COSMOLOGICAL MODEL IN BIMETRIC THEORY

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ABSTRACT

Spatially homogeneous and anisotropic Bianchi type-1 space time is studied in context of bimetric relativity taking the source cosmic cloud strings coupled with electromagnetic field directed along X-axis. It is shown that there is no contribution from Maxwell's fields. However the solution obtained in case-2 represents geometric string model and vacuum models are obtained in both case-1 and case-3.It is interesting that models found in all the three cases are free from singularities.

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Keywords: *Cosmic strings, bimetric theory, magnetic field, cosmological model.*

1. INTRODUCTION

In order to get rid of the singularities that occur in general relativity and appear in the big-bang cosmological models, Rosen [1] proposed a new theory of gravitation known as bimetric relativity. In this theory there exists two metric tensors at each point of the space time. The first metric tensor g_{ij} determines the Riemannian geometry of curved space time which plays the same role as in general relativity and it interacts with matter. The back ground metric tensor γ_{ij} refers to the geometry of the empty universe and described the inertial forces. It has no direct physical significance but appears in the field equations. Also it interacts with g_{ij} but not directly with matter. One can regards γ_{ij} as giving the geometry that would exist if there were no matter. This theory also satisfies the covariant and equivalence principles.

In recent days there has been a lot of interest in the study of cosmic strings which are topologically stable objects. These might be found during a phase transition in the early universe i.e. after the big-bang explosion as the temperature goes down below some critical temperature as predicted by Grand Unified Theories (Zel'dovich *et al* [2]; Kibble [3, 4]; Everett [5], Vilenkin [6]). It is believed that cosmic strings give rise to density perturbations leading to the formation of galaxies (Zel'dovich[7]). These cosmic strings have stress energy and couple to the gravitational field for which it is interesting to study the effects of gravitation that arise from strings. So Letelier [8] and Satchel [9] are the authors who had started the general relativistic treatment of strings.

Moreover, electromagnetic field which contains highly ionized matter plays a key role for description of the energy distribution in the universe. String magnetic fields may be created due to adiabatic compression in cluster of galaxies and cosmic anisotropies may be attributed to the large scale magnetic fields. It is believed that in anisotropic models, the presence of electromagnetic field can alter the rate of creation of the particles and directly affects the rate of expansion of the universe.

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The present day universes we study are satisfactorily described by homogeneous and isotropic models. Since the universe in smaller scale is neither homogeneous nor isotropic nor do we expect the universe in its early stages to have these properties. Also homogeneous and anisotropic cosmological models are much studied in general relativity for the sake of realistic picture of the universe in its early stages. So in this paper we have chosen to study the Bianchi type -1 model because it is supposed to be a reasonable representation of the universe in early epochs and it is simple enough that many of its features can be studied exactly.

Deo[10], Katore[11], Rosen[1,12], Yilmaz [13], Israelit[14,15,16] Karade[17], Karade and Dhoble[18], and Sahoo [19,20] are some of the authors who have investigated several aspects of bimetric relativity. Mohanty *et al* [21] have studied Bianchi type-1 space time in bimetric relativity when source of the gravitation is governed either by scalar meson field or by mesonic perfect fluid. Reddy [22] has studied Bianchi type-1 space time in presence of cosmic cloud string while Reddy and Venketswarlu [23] have studied Bianchi type-1 space time in Rosen's bimetric theory when source of the gravitational field is a perfect fluid. Deo and Thengane [24] have shown that Bianchi type-1 space time in context of bimetric theory taking the source as perfect fluid coupled with magnetic field. Gaikwad *et al* [25] have investigated the Bianchi type-1 massive string magnetized barotropic perfect fluid cosmological model with and without a magnetic field.

But to our knowledge no author has studied Bianchi type-1 space time in bimetric relativity taking the source cosmic cloud strings coupled with electromagnetic field. So in order to extend the work of Reddy [23], we have considered this problem and obtained cosmological models of the universe for three different cases. It is established that the vacuum model of the universe exists in case-1, geometric string model of the universe exists in case-2 and anisotropic vacuum model of the universe exists in case-3.It is more interesting that all the models we obtained are free from singularities.

2. **FIELD EQUATION**

Consider the spatially homogeneous and anisotropic Bianchi type-1 metric

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{1}$$

where the metric potentials A, B, C are functions of cosmic time t only.

The background flat metric corresponding to the metric (1) is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad . \tag{2}$$

The energy momentum tensor for cosmic cloud string coupled with electromagnetic field as given by Letelier [8] and Stachel [9]

$$T_j^i = T_{j\,\text{strings}}^i + E_{j\,\text{mag}}^i \tag{3}$$

where $$T_{j\,\text{strings}}^i = \rho u^i u_j - \lambda x^i x_j \tag{4}$$

Together with $u^i u_i = -1 = x^i x_i$ and $u^i x_i = 0$. (5)

From equations (1) and (5), we write $u^i = u_i = (-1, 0, 0, 0)$ (6)

and x^i can be taken parallel to any of the directions $\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}$.

We choose x^i parallel to $\dfrac{\partial}{\partial x}$, so that

$$x^i = \left(A^{-1}, 0, 0, 0\right). \tag{7}$$

In above ρ is the rest energy density of the system of strings with massive particles attached to them and λ is the tension density of the system of strings. As pointed out by Letelier [8], λ may be +ve or -ve, u^i describes the four - velocity vector and x^i represents the anisotropic direction i.e. the direction of the string.

In the co-moving coordinate system taking the string in radial direction, (4) takes the form

$$T_{1\text{strings}}^1 = -\lambda \;,\; T_{4\,\text{strings}}^4 = -\rho$$

and $T_{j\ strings}^i = 0 \ for\ i, j = 2,3\ and\ i \neq j$.

The electromagnetic tensor described by

$$E^i_{j\,mag} = F_{jr}F^{ir} - \frac{1}{4}F_{ab}F^{ab}g^i_j \tag{8}$$

where F^i_j is the electromagnetic field tensor.

Again we consider the electromagnetic field along the x-axis, so the non-vanishing component of the electromagnetic field tensor F_{ij} is F_{23}. A cosmological model containing global magnetic field is necessarily anisotropic because the magnetic field specifies a preferred spatial direction. Bronnikov *et al* [26] have studied the evolution of Bianchi type-1 space-time with a global unidirectional electromagnetic field (F_{ij}). Hence due to assumption of infinite electrical conductivity, $F_{14} = 0 = F_{24} = F_{34.}$.

The Maxwell's equation for magnetic field

$$F_{[ij,k]} = F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \tag{9}$$

leads to the result $F_{23} = \text{Constant=H (Say)}$.

Now from (8), we obtain

$$-E^1_{1\text{mag}} = E^2_{2\text{mag}} = E^3_{3\text{mag}} = -E^4_{4\text{mag}} = \frac{H^2}{2B^2C^2} = \eta\,(\text{say}). \tag{10}$$

Thus equation (3) leads to

$$T^1_1 = -(\lambda+\eta), T^2_2 = \eta = T^3_3, T^4_4 = -(\rho+\eta) . \tag{11}$$

The field equations of bimetric relativity proposed by Rosen [1] are

$$N^i_j - \frac{1}{2}N\delta^i_j = -8\pi k T^i_j \tag{12}$$

where

$$N^i_j = \frac{1}{2}\gamma^{ab}\left(g^{hi}g_{hj|a}\right)_b \tag{13}$$

and $N = N^i_i = N^1_1 + N^2_2 + N^3_3 + N^4_4, g = \det(g_{ij}), \gamma = \det(\gamma_{ij}), k = \sqrt{\left(g/r\right)}.$

Here the vertical bar (|) represents covariant differentiation w. r .to γ_{ij} and T^i_j is the energy momentum tensor of matter fields.

The Rosen's equations (12) for the metrics (1) and (2) with help of equation (11) can be written as

$$-\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 16\pi k(\lambda+\eta), \tag{14}$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k\eta, \tag{15}$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = -16\pi k\eta \tag{16}$$

and

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k(\rho+\eta). \tag{17}$$

where $A_4 = \frac{dA}{dt}, B_4 = \frac{dB}{dt}, C_4 = \frac{dC}{dt},$

Now there are four field equations (14) to (17) with five unknowns A, B, C, λ and ρ. So in order to get explicit solutions one extra condition is required. Let the condition be

$$\rho+\omega\lambda=0 \tag{18}$$

where $-1\le\omega\le 1.$

Adding (14) to (17), and equating (15) and (16) ,we get

$$2\left[\left(\frac{A_4}{A}\right)_4+\left(\frac{B_4}{B}\right)_4+\left(\frac{C_4}{C}\right)_4\right]=16\pi k\left(\lambda+\rho\right), \tag{19}$$

and

$$\left(\frac{B_4}{B}\right)_4=\left(\frac{C_4}{C}\right)_4. \tag{20}$$

Applying (17) in (19), we obtain

$$\rho-\lambda+2\eta=0. \tag{21}$$

Case-1: (Vacuum Model)

Suppose $\omega\neq 0$ and $\rho=\lambda=0$.

In this case, equation (19) and (21) reduces to

$$\left(\frac{A_4}{A}\right)_4+\left(\frac{B_4}{B}\right)_4+\left(\frac{C_4}{C}\right)_4=0 \tag{22}$$

and $\eta=0$. (23)

Now use of (20) and (23) in (15), one gets

$$\left(\frac{A_4}{A}\right)_4=0. \tag{24}$$

Again, use of (20) and (24) in (22), one finds

$$\left(\frac{A_4}{A}\right)_4=\left(\frac{B_4}{B}\right)_4=\left(\frac{C_4}{C}\right)_4=0\ . \tag{25}$$

Integrating separately, we can obtain from equation (25) that

$$A=e^{a_1t+b_1},B=e^{a_2t+b_2}\ \ and\ \ C=e^{a_3t+b_3} \tag{26}$$

where $a_i,b_i;i=1,2,3$ are constants of integration.

As $\rho=\lambda=0=\eta$, it concludes that Bianchi type-1 cosmological model does not survive in bimetric theory when source is cosmic strings coupled with electromagnetic field. Thus in view of equation (26), the metric (1) takes the form

$$ds^2=-dt^2+e^{2(a_1t+b_1)}dx^2+e^{2(a_2t+b_2)}dy^2+e^{2(a_3t+b_3)}dz^2. \tag{27}$$

Choosing $a_1=a_2=a_3$ and $b_1=b_2=b_3$, we can write (27) as

$$ds^2=-dt^2+e^{2(a_1t+b_1)}\left[dx^2+dy^2+dz^2\right]. \tag{28}$$

After proper choice of coordinates, equation (28) reduces to

$$ds^2=\exp\left[2T\left(dX^2+dY^2+dZ^2-dT^2\right)\right], \tag{29}$$

which is a vacuum model with no singularity at T= 0.

Case-2: (Geometric String Model).

Suppose $\omega = -1$. So equation (18) turns to

$$\rho = \lambda \,. \tag{30}$$

Applying equation (30) in (21), we get

$$\eta = 0\,. \tag{31}$$

Now use of (31) and (20) in (15) and (16), we obtain

$$\left(\frac{A_4}{A}\right)_4 = 0. \tag{32}$$

Integrating (32), we find $A = e^{k_1 t + k_2}$ (33)

where k_1, k_2 are integration constants.

With help of (31) and (32), equations (17) reduces to

$$\rho = \frac{1}{8\pi k}\left(\frac{B_4}{B}\right)_4 = \lambda = \frac{1}{8\pi k}\left(\frac{C_4}{C}\right)_4 . \tag{34}$$

By assigning different functions to B" or "C", one can get different values of $\rho \, and \, \lambda$ and accordingly different geometric string models of the universe can be obtained.

In order to get physically meaningful solution as considered by Sahoo [19] on Kantowski Sachs model, we consider the metric potentials B and C as

$$C = B = \left(m_1 t + m_2\right)^n . \exp\left(m_1 t + m_2\right) \tag{35}$$

where n, m_1, m_2 are constants and $m_1 \neq 0$.

Putting the value of 'B' from (35) in (34), we get

$$\rho = \frac{1}{8\pi k} . \frac{-n m_1^2}{\left(m_1 t + m_2\right)^2} = \lambda. \tag{36}$$

In view of equations (33) and (35), the corresponding model of metric (1) can be written as

$$ds^2 = -dt^2 + \exp\left[2\left(k_1 t + k_2\right)\right] dx^2 + \left(m_1 t + m_2\right)^{2n} \exp\left[2\left(m_1 t + m_2\right)\right]\left(dy^2 + dz^2\right). \tag{37}$$

The model (37) reduces to plane symmetric vacuum model and hence there is no contribution from Maxwell's field to Bianchi type-1 model in bimetric relativity. It is interesting to point out that the field equations obtained here are similar to as studied by Sahoo [20] for the case of Bianchi type VI_0 space time where the author has shown that the field equations have no solutions (in case of geometric string).

Case-3: Suppose $\omega = 1$.So equation of state (18) can be written as

$$\rho = -\lambda \,. \tag{38}$$

Use of (38) and (20) in equation (19), we get

$$\left(\frac{A_4}{A}\right)_4 + 2\left(\frac{B_4}{B}\right)_4 = 0\,. \tag{39}$$

Integrating (39), we obtain

$$AB^2 = e^{k_3 t + k_4}. \tag{40}$$

Similarly, we can find

$$AC^2 = e^{k_5 t + k_6}. \tag{41}$$

From (40) and (41), we have

$$A^2 B^2 C^2 = e^{(k_3+k_5)t+(k_4+k_6)} \quad . \tag{42}$$

Taking square root both sides, equation (42) yields to

$$ABC = e^{k_7 t + k_8}, \tag{43}$$

where $k_7 = \dfrac{k_3 + k_5}{2}$ and $k_8 = \dfrac{k_4 + k_6}{2}$.

Since A, B, C are functions of 't', we can express equation (1) as

$$A = e^{p_1(k_7 t + k_8)}, B = e^{p_2(k_7 t + k_8)}, C = e^{p_3(k_7 t + k_8)}. \tag{44}$$

where $\sum_{i=1}^{3} p_i = 1.$

Use of $\rho = -\lambda$, equation (21) reduces to

$$\eta = 0. \tag{45}$$

Putting the values of A, B, C and η in (14) and (17), we get

$$\lambda = \rho = 0. \tag{46}$$

Thus in view of equation (44), the metric (1) takes the form

$$ds^2 = -dt^2 + \exp\left[2p_1\left(k_7 t + k_8\right)\right]dx^2 + \exp\left[2p_2\left(k_7 t + k_8\right)\right]dy^2 + \exp\left[2p_3\left(k_7 t + k_8\right)\right]dz^2. \tag{47}$$

The model described by (47) is an anisotropic vacuum model and no singularity at t = 0.

3. CONCLUSIONS

It is observed from the solution in Section-2 that the Bianchi type-1 cosmological model does not accommodate cosmic strings coupled with electromagnetic field directed along X-axis in Rosen's bimetric theory. However, we have shown that geometric string models exist in case-2 and vacuum models in case-1 and case-3.Moreover; it is shown that the models found in exercise are free from singularities. The work reported in this paper is an extension to the work already done by Reddy [22].Moreover, keeping in view of result interest in electromagnetic field in free space, we have tried to develop the idea of cosmic string taking Bianchi type-1 cosmological models in Rosen's bimetric theory, which will help for better understanding of the readers. Further, our investigation reveals that the bimetric theory doesn't admit singularities which are of physical nature. Hence the difficulty faced in theory of relativity as regards to physical singularities doesn't exist in bimetric theory. Thus the basic aim of Rosen to develop bimetric theory has been fulfilled in present work.

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