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GENERALIZED FUZZY SOFT SETS: A NEW APPROACH

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ABSTRACT

In this paper, we redefined null, absolute, union, and intersection of generalized fuzzy soft sets and study some of their properties. We also defined disjunctive sum, difference and product of two generalized fuzzy soft sets and their basic properties.

***Key Words**: Fuzzy soft sets, Generalized fuzzy soft set, Disjunctive sum, Difference and Product of fuzzy soft set.*

1. INTRODUCTION

Maji *et al.* [4] introduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. In 2011, Neog and Sut [6] put forward some more propositions regarding fuzzy soft set theory. In 2009 Majumder and Samanta [5] initiated another important part, known as Generalized Fuzzy Soft Set Theory. While generalizing the concept of Fuzzy Soft Sets, Majumder and Samanta [5] considered the same set of parameter. But in 2012 Borah, Neog and Sut [3] considering generalized fuzzy soft sets different sets of parameters. In our work, we redefined null, absolute, union, and intersection of generalized fuzzy soft sets and study some of their properties. We also defined disjunctive sum, difference and product of two generalized fuzzy soft sets and their basic properties.

2. PRELIMINARIES

In this section, we recall some basic concepts and definitions regarding fuzzy soft sets and generalized fuzzy soft sets.

Definition 2.1: [4] A pair (F, A) is called a fuzzy soft set over U where $F : A \to \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$.

Definition 2.2: [4] A soft set (F, A) over U is said to be null fuzzy soft set denoted by φ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of U where $\overline{0}(x) = 0 \forall x \in U$.

Definition 2.3: [4] A soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\overline{1}$ of U where $\overline{1}(x) = 1 \forall x \in U$

Definition 2.4: [4] For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E), we say that (F, A) is a fuzzy soft subset of (G, B), if
(*i*) $A \subseteq B$
(*ii*) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \tilde{\subseteq} (G, B)$.

Definition 2.5: [1] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions.
(*i*) $*$ is commutative and associative
(*ii*) $*$ is continuous
(*iii*) $a * 1 = a \quad \forall a \in [0,1]$
(*iv*) $a * b \le c * d$ Whenever $a \le c, b \le d$ and $a, b, c, d \in [0,1]$

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Definition 2.6: [1] A binary operation $\Diamond:[0,1]\times[0,1]\to[0,1]$ is continuous *t*-conorm if $\Diamond$ satisfies the following conditions:
(*i*) $\Diamond$ is commutative and associative
(*ii*) $\Diamond$ is continuous
(*iii*) $a\ \Diamond\ 0 = a \quad \forall a\in[0,1]$
(*iv*) $a\Diamond b\le c\Diamond d$ whenever $a\le c, b\le d$ and $a,b,c,d\in[0,1]$

Definition 2.7: [5] Let $U=\{x_1,x_2,x_3,............,x_n\}$ be the universal set of elements and $E=\{e_1,e_2,e_3,......e_m\}$ be the universal set of parameters. The pair (*U*, *E*) will be called a soft universe. Let $F:E\to I^U$ and μ be a fuzzy subset of *E, i.e.* $\mu:E\to I=[0,1]$, where I^U is the collection of all fuzzy subsets of *U*. Let F_μ be the mapping $F_\mu:E\to I^U\times I$ be a function defined as follows:

$F_\mu(e)=(F(e),\mu(e))$, where $F(e)\in I^U$. Then F_μ is called generalized fuzzy soft sets over the soft universe (*U*,*E*). Here for each parameter e_i, $F_\mu(e_i)=(F(e_i),\mu(e_i))$ indicates not only the degree of belongingness of the elements of *U* in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Definition 2.8: [5] Let F_μ and G_δ be two GFSS over (*U*, *E*).

Now, F_μ is said to be a generalized fuzzy soft subset of G_δ if
(i) μ is a fuzzy subset of δ
(ii) $F(e)$ is also a fuzzy subset of $G(e)\ \forall e\in E$.

In this case, we write $F_\mu \tilde{\subseteq} G_\delta$

Definition 2.9: [5] Let F_μ be a GFSS over (*U*, *E*). Then the complement of F_μ, denoted by $F_\mu{}^c$ and is defined by $F_\mu{}^c=G_\delta$, where $\delta(e)=\mu^c(e)$ and $G(e)=F^c(e),\ \forall e\in E.$

Definition 2.10: [5] The union of two GFSS F_λ and G_μ over (*U*, *E*) is denoted by $F_\lambda \tilde{\cup} G_\mu$ and defined by GFSS $H_\delta:E\to I^U\times I$ such that for each $e\in E$
$H_\delta(e)=(F(e)\Diamond G(e),\lambda(e)\Diamond\mu(e))$

Definition 2.11: [5] The intersection of two GFSS F_λ and G_μ over (*U*, *E*) is denoted by $F_\lambda \tilde{\cap} G_\mu$ and defined by GFSS $H_\delta:E\to I^U\times I$ such that for each $e\in E$
$H_\delta(e)=(F(e)*G(e),\lambda(e)*\mu(e))$

Definition 2.12: [5] A GFSS is said to be a generalized null fuzzy soft set, denoted by θ_ϕ, if $\theta_\phi:E\to I^U\times I$ such that $\theta_\phi(e)=(F(e),\phi(e))$ Where $F(e)=\bar{0},\phi(e)=0,\forall e\in E$

Definition 2.13: [5] A GFSS is said to be a generalized absolute fuzzy soft set, denoted by $\tilde{A}_\lambda$, if $\tilde{A}_\lambda:E\to I^U\times I$ such that
$A_\phi(e)=(F(e),\lambda(e))$ Where $F(e)=\bar{1},\lambda(e)=1,\forall e\in E$

Definition 2.14: [3] Let $X=\{x_1,x_2,x_3,..........,x_n\}$ be the universal set of elements and $E=\{e_1,e_2,e_3,........,e_m\}$ be the set of parameters. Let $A\subseteq E$ and $F:A\to I^U$ and λ be a fuzzy subset of *A* i.e. $\lambda:A\to I=[0,1]$, where I^U is the collection of all fuzzy subsets of *U*. Let $F:A\to I^U\times I$ be a function defined as follows:

$F_\lambda{}^A(e)=(F(e),\lambda(e))$, where $F(e)\in I^U$ .Then $F_\lambda{}^A$ is called a generalized fuzzy soft set (GFSS) over (*U*, *E*).
Here for each parameter $e_i, F_\lambda{}^A(e_i)$ indicates not only degree of belongingness of the elements of *U* in $F(e_i)$ but also degree of preference of such belongingness which is represented by $\lambda(e_i)$.

Example 2.1: Let $U=\{S_1,S_2,S_3\}$, E = $\{e_1, e_2, e_3, e_4\}$ and $A=\{e_1,e_3,e_4\}\subseteq E$.

We define $F_\lambda{}^A$ as follows:

$F_\lambda{}^A(e_1)=(\{S_1/0.3, S_2/0.5, S_3/0.7\}, 0.4)$,

$F_\lambda{}^A(e_3)=(\{S_1/0.6, S_2/0.1, S_3/0.3\}, 0.5)$, $F_\lambda{}^A(e_4)=(\{S_1/0.3, S_2/0.9, S_3/0.7\}, 0.8)$ is the generalized fuzzy soft set.

Definition 2.15: [3] Let $F_\lambda{}^A$ and $G_\mu{}^B$ be two generalized fuzzy soft set over (*U*, *E*). Now $F_\lambda{}^A$ is called a generalized fuzzy soft subset of $G_\mu{}^B$ if

(i) $A\subseteq B$,

(ii) λ is a fuzzy subset of μ,

(iii) $\forall\lambda\in A, F(e)$ is a fuzzy subset of $G(e)$ *i.e.* $F(e)\subseteq G(e), \forall e\in E$

We write $F_\lambda{}^A \,\tilde{\subseteq}\, G_\mu{}^B$

Example 2.2: We consider the GFSS $F_\lambda{}^A$ given in **Example 2.1** and let $B=\{e_1,e_2,e_3,e_4\}\subseteq E$

We define $G_\mu{}^B$ as follows:

$G_\mu{}^B(e_1)=(\{S_1/0.6, S_2/0.9, S_3/0.8\}, 0.4)$,

$G_\mu{}^B(e_2)=(\{S_1/0.6, S_2/0.1, S_3/0.3\}, 0.3)$,

$G_\mu{}^B(e_3)=(\{S_1/0.8, S_2/0.7, S_3/0.5\}, 0.7)$, $G_\mu{}^B(e_4)=(\{S_1/0.6, S_2/0.9, S_3/0.8\}, 0.9)$

Then $F_\lambda{}^A \,\tilde{\subseteq}\, G_\mu{}^B$.

Definition 2.16: [3] Let $F_\mu{}^A$ be a GFSS over (*U*, *E*). Then the complement of $F_\mu{}^A$, denoted by $F_\mu{}^{A^c}$ and is defined by $F_\mu{}^{A^c}=G_\delta{}^A$, where $\delta(e)=\mu^c(e)$ and $G^A(e)=F^{A^c}(e), \ \forall e\in A.$

3. GENARALIZED FUZZY SOFT SET REDEFINED

In this section, we put forward the notion of generalized fuzzy soft sets considering different sets of parameters and accordingly redefine the notions of null, absolute, union, intersection, complement etc. of generalized fuzzy soft sets in the following manner:

Definition 3.1: A generalized fuzzy soft sets (GFSS) is said to be a generalized null fuzzy soft set denoted by $\theta_\phi{}^A$, if $\theta_\phi{}^A : A\to I^U\times I$ such that $\theta_\phi{}^A(e)=(F(e),\phi(e))$ where $F(e)=\bar{0}, \phi(e)=1, \forall e\in A\subseteq E$

It is clear from our definition that the generalized fuzzy soft null set is not unique in our way, it would depend upon the set of parameters under consideration.

Definition 3.2: A GFSS is said to be a generalized absolute fuzzy soft set, denoted by $\tilde{A}_\lambda$, if $\tilde{A}_\lambda : A\to I^U\times I$ such that $\tilde{A}_\phi(e)=(F(e),\lambda(e))$ Where $F(e)=\bar{1}, \lambda(e)=0, \forall e\in A\subseteq E$

It is clear from our definition that the generalized fuzzy soft absolute set is also not unique in our way, it would depend upon the set of parameters under consideration.

Definition 3.3: The union of two GFSS $F_\lambda{}^A$ and $G_\mu{}^B$ over (*U*, *E*) is denoted by $F_\lambda{}^A \,\tilde{\cup}\, G_\mu{}^B$ and defined by GFSS $H_\delta{}^{A\cup B} : A\cup B\to I^U\times I$ such that for each $e\in A\cup B$ and $A, B\subseteq E$

where $$H_\delta{}^{A\cup B}(e)=\begin{cases}(F(e),\lambda(e)), & \text{if } e\in A-B\\ (G(e),\mu(e)), & \text{if } e\in B-A\\ (F(e)\Diamond G(e), \lambda(e)*\mu(e)), & \text{if } e\in A\cap B\end{cases}$$

Example 3.1: From **Example 2.1**and **2.2**

$H_\delta^{A\cup B}(e_1) = (\{S_1/0.72, S_2/0.95, S_3/0.94\}, 0.16)$, $H_\delta^{A\cup B}(e_2) = (\{S_1/0.60, S_2/0.10, S_3/0.30\}, 0.30)$,
$H_\delta^{A\cup B}(e_3) = (\{S_1/0.92, S_2/0.73, S_3/0.65\}, 0.35)$, $H_\delta^{A\cup B}(e_4) = (\{S_1/0.72, S_2/0.99, S_3/0.94\}, 0.72)$

Remark 3.1: If we consider $F(e)\Diamond G(e) = \max\{F(e), G(e)\}$, $\lambda(e) * \mu(e) = \min\{\lambda(e), \mu(e)\}$.
Then

$H_\delta^{A\cup B}(e_1) = (\{S_1/0.6, S_2/0.9, S_3/0.8\}, 0.4)$, $H_\delta^{A\cup B}(e_2) = (\{S_1/0.6, S_2/0.1, S_3/0.3\}, 0.3)$,
$H_\delta^{A\cup B}(e_3) = (\{S_1/0.8, S_2/0.7, S_3/0.5\}, 0.5)$, $H_\delta^{A\cup B}(e_4) = (\{S_1/0.6, S_2/0.9, S_3/0.8\}, 0.8)$

Definition 3.4: The intersection of two GFSS F_λ^A and G_μ^B over (*U*, *E*) is denoted by $F_\lambda^A \tilde{\cap} G_\mu^B$ and defined by GFSS $K_\delta^{A\cap B} : A\cap B \to I^U \times I$ such that for each $e \in A\cap B$ and $A, B \subseteq E$ $K_\delta^{A\cap B}(e) = (K(e), \delta(e))$,
Where $K(e) = F(e) * G(e), \delta(e) = \lambda(e)\Diamond\mu(e)$. In order to avoid degenerate case, we assume here that $A\cap B \neq \varphi$.

Example 3.2: From **Example 2.1** and **2.2**

$K_\delta^{A\cap B}(e_1) = (\{S_1/0.18, S_2/0.45, S_3/0.56\}, 0.64)$, $K_\delta^{A\cap B}(e_3) = (\{S_1/0.48, S_2/0.07, S_3/0.15\}, 0.85)$,
$K_\delta^{A\cap B}(e_4) = (\{S_1/0.18, S_2/0.81, S_3/0.56\}, 0.98)$

Remark 3.2: Let us define $K_\delta^{A\cap B}(e) = (K(e), \delta(e))$,
where $K(e) = \min\{F(e), G(e)\}$,
$\delta(e) = \max\{\lambda(e), \mu(e)\}$.
Then

$K_\delta^{A\cap B}(e_1) = (\{S_1/0.3, S_2/0.5, S_3/0.7\}, 0.4)$,
$K_\delta^{A\cap B}(e_3) = (\{S_1/0.6, S_2/0.1, S_3/0.3\}, 0.5)$, $K_\delta^{A\cap B}(e_4) = (\{S_1/0.3, S_2/0.9, S_3/0.7\}, 0.8)$

Proposition 3.1: If F_λ^A be a GFSS over (*U*, *E*), then

(i) $F_\lambda^A \tilde{\bigcup} \theta_\phi^A = F_\lambda^A$

(ii) $F_\lambda^A \tilde{\bigcap} \theta_\phi^A = \theta_\phi^A$

(iii) $F_\lambda^A \tilde{\bigcup} \tilde{\mathrm{A}}_\alpha = \tilde{\mathrm{A}}_\alpha$

(iv) $F_\lambda^A \tilde{\bigcap} \tilde{\mathrm{A}}_\alpha = F_\lambda^A$

Proposition 3.2: If $F_\lambda^A, G_\mu^B, H_\delta^C$ be a GFSS over (*U*, *E*), then

(i) $F_\lambda^A \tilde{\bigcap} G_\mu^B = G_\mu^B \tilde{\bigcap} F_\lambda^A$

(ii) $F_\lambda^A \tilde{\bigcup} G_\mu^B = G_\mu^B \tilde{\bigcup} F_\lambda^A$

(iii) $F_\lambda^A \tilde{\bigcap} (G_\mu^B \tilde{\bigcap} H_\delta^C) = (F_\lambda^A \tilde{\bigcap} G_\mu^B) \tilde{\bigcap} H_\delta^C$

$F_\lambda^A \tilde{\bigcup} (G_\mu^B \tilde{\bigcup} H_\delta^C) = (F_\lambda^A \tilde{\bigcup} G_\mu^B) \tilde{\bigcup} H_\delta^C$

Proof: Since the *t* - norm function and *t* - co norm functions are commutative and associative, therefore the theorem follows.

Proposition 3.3: If F_λ^A, G_μ^A be a GFSS over (*U*, *E*), then

(i) $(F_\lambda^A \tilde{\bigcap} G_\mu^A)^C = (G_\mu^A)^C \tilde{\bigcup} (F_\lambda^A)^C$

(ii) $(F_\lambda^A \tilde{\bigcup} G_\mu^A)^C = (G_\mu^A)^C \tilde{\bigcap} (F_\lambda^A)^C$

Proof: The proof is straight forward and follows from definition.

Proposition 3.4: The following results are valid if we take *max* and *min* operations. If $F_\lambda{}^A, G_\mu{}^B, H_\delta{}^C$ be a GFSS over (*U*, *E*), then

(i) $F_\lambda{}^A \tilde{\bigcap} (G_\mu{}^B \tilde{\bigcup} H_\delta{}^C) = (F_\lambda{}^A \tilde{\bigcap} G_\mu{}^B) \tilde{\bigcup} (F_\lambda{}^A \tilde{\bigcap} H_\delta{}^C)$

(ii) $F_\lambda{}^A \tilde{\bigcup} (G_\mu{}^B \tilde{\bigcap} H_\delta{}^C) = (F_\lambda{}^A \tilde{\bigcup} G_\mu{}^B) \tilde{\bigcap} (F_\lambda{}^A \tilde{\bigcup} H_\delta{}^C)$

Definition 3.5: The equal of two GFSS $F_\lambda{}^A$ and $G_\mu{}^B$ over (*U*, *E*) is denoted by $F_\lambda{}^A = G_\mu{}^B$,

Where $F(e) = G(e), \lambda(e) = \mu(e)$

Proposition 3.5: If $F_\lambda{}^A, G_\mu{}^B, H_\delta{}^C$ be a GFSS over (*U*, *E*), then $F_\lambda{}^A = G_\mu{}^B, G_\mu{}^B = H_\delta{}^C \Leftrightarrow F_\lambda{}^A = H_\delta{}^C$

Proof: Since

$F_\lambda{}^A = G_\mu{}^B, G_\mu{}^B = H_\delta{}^C$

Therefore

$\{F(e) = G(E), \lambda(e) = \mu(e), \ \{G(e) = H(E), \mu(e) = \delta(e)$

Therefore

$\{F(e) = H(E), \lambda(e) = \delta(e)$

Hence

$F_\lambda{}^A = H_\delta{}^C$.

4. SOME NEW OPERATIONS OF GENERALIZED FUZZY SOFT SETS

Definition 4.1: Let $F_\lambda{}^A$ and $G_\mu{}^B$ be two GFSS over (*U*, *E*).We define the disjunctive sum of $F_\lambda{}^A$ and $G_\mu{}^B$ be as the generalized fuzzy soft set $K_\delta{}^C$ over (U, E). Written as '$F_\lambda{}^A \tilde{\oplus} G_\mu{}^B = K_\delta{}^C$'

where $C = A \cap B \neq \phi, \forall e \in C$ and $A, B \subseteq E$

$K_\delta{}^C(e) = (K(e), \delta(e)),$

Where $K(e) = \max(F(e) * \mu(e), G(e) * \lambda(e))$

$\delta(e) = \min(\min(G(e) \Diamond \lambda(e), F(e) \Diamond \mu(e))$

Example 4.1: Let $U = \{S_1, S_2, S_3\}$ and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters and $A = \{e_1, e_3, e_4\} \subseteq E$, $B = \{e_1, e_2, e_3\} \subseteq E$

We define $F_\lambda{}^A$ and $G_\mu{}^B$ as follows:

$F_\lambda{}^A(e_1) = (\{S_1/0.3, S_2/0.5, S_3/0.7\}, 0.4)$,

$F_\lambda{}^A(e_3) = (\{S_1/0.6, S_2/0.1, S_3/0.3\}, 0.5)$, $F_\lambda{}^A(e_4) = (\{S_1/0.3, S_2/0.9, S_3/0.7\}, 0.8)$

$G_\mu{}^B(e_1) = (\{S_1/0.6, S_2/0.9, S_3/0.8\}, 0.4)$, $G_\mu{}^B(e_2) = (\{S_1/0.6, S_2/0.1, S_3/0.3\}, 0.3)$,

$G_\mu{}^B(e_3) = (\{S_1/0.8, S_2/0.7, S_3/0.5\}, 0.7)$,

Then '$F_\lambda{}^A \tilde{\oplus} G_\mu{}^B = K_\delta{}^C$' Where $C = A \cap B = \{e_1, e_3\}$

$K_\delta{}^C = \{$

$$K_\delta{}^C(e_1) = (\{S_1/\max\{0.12, 0.24\}, S_2/\max\{0.20, 0.36\}, S_3/\max\{0.28, 0.32\};$$
$$\min\{\min(0.76, 0.58), \min(0.94, 0.70), \min(0.88, 0.82)\})$$

$K_\delta{}^C(e_3) = (\{S_1 / \max\{0.42, 0.40\}, S_2 / \max\{0.07, 0.35\}, S_3 / \max\{0.21, 0.25\};$
$\min\{\min(0.90, 0.88), \min(0.85, 0.73), \min(0.75, 0.79)\}\})$

i.e

$K_\delta{}^C(e_1) = (\{S_1 / 0.24, S_2 / 0.36, S_3 / 0.32\}, 0.58)$

$K_\delta{}^C(e_3) = (\{S_1 / 0.42, S_2 / 0.35, S_3 / 0.25\}, 0.73)$

Proposition 4.1:

Let $F_\lambda{}^A$ and $G_\mu{}^B$ be two GFSS over (*U*, *E*).Then the following results hold.

(i) $F_\lambda{}^A \tilde{\oplus} G_\mu{}^B = G_\mu{}^B \tilde{\oplus} F_\lambda{}^A$

(ii) $F_\lambda{}^A \tilde{\oplus} (G_\mu{}^B \tilde{\oplus} H_\delta{}^C) = (F_\lambda{}^A \tilde{\oplus} G_\mu{}^B) \tilde{\oplus} H_\delta{}^C$

Proof: Since

$F_\lambda{}^A(e) = (F(e), \lambda(e))$, where $e \in E, A \subseteq E$, $G_\mu{}^B(e) = (G(e), \mu(e))$, where $e \in E, B \subseteq E$

Now $(F_\lambda{}^A \tilde{\oplus} G_\mu{}^B)(e) = K_\delta{}^C(e) where C = A \cap B$, $K_\delta{}^C(e) = (K(e), \delta(e))$

Where $K(e) = \max(F(e) * \mu(e), G(e) * \lambda(e))$, $\delta(e) = \min(\min(G(e) \Diamond \lambda(e), F(e) \Diamond \mu(e))$

Let $(G_\mu{}^B \tilde{\oplus} F_\lambda{}^A)(e) = H_\zeta{}^C(e) where C = A \cap B$, $H_\zeta{}^C(e) = (H(e), \zeta(e))$

Where $H(e) = \max(G(e) * \lambda(e), F(e) * \mu(e))$
$= \max(F(e) * \mu(e), G(e) * \lambda(e)) = K(e)$

$\zeta(e) = \min(\min(F(e) \Diamond \mu(e), G(e) \Diamond \lambda(e))$
$= \min(\min(G(e) \Diamond \lambda(e), F(e) \Diamond \mu(e)) = \delta(e)$

Therefore

$K_\delta{}^C(e) = H_\zeta{}^C(e)$

Hence

$F_\lambda{}^A \tilde{\oplus} G_\mu{}^B = G_\mu{}^B \tilde{\oplus} F_\lambda{}^A$.

Proof of (ii) can be done in a similar way.

Proposition 4.2:

$F_\lambda{}^A \tilde{\oplus} \theta_\phi{}^A = F_\lambda{}^A$

Proof:

$F_\lambda{}^A(e) = (F(e), \lambda(e))$, where $e \in E, A \subseteq E$, $\theta_\phi{}^A(e) = (\overline{0}, 1)$, where $e \in E, B \subseteq E$

Now $(F_\lambda{}^A \tilde{\oplus} \theta_\phi{}^A)(e) = K_\delta{}^A(e)$

$K_\delta{}^A(e) = (K(e), \delta(e))$

Where $K(e) = \max(F(e) * 1, 0 * \lambda(e)) = \max(F(e), 0) = F(e)$
$\delta(e) = \min(\min(0 \Diamond \lambda(e), F(e) \Diamond 1) = \min\{\min(\lambda(e), 1)\} = \lambda(e)$
$K_\delta{}^A(e) = (K(e), \delta(e)) = (F(e), \lambda(e)) = F_\lambda{}^A(e)$

Hence

$$F_\lambda{}^A \tilde{\oplus}\, \theta_\phi{}^A = F_\lambda{}^A.$$

Definition 4.2: Let $F_\lambda{}^A$ and $G_\mu{}^B$ be two GFSS over (*U*, *E*).We define the difference of $F_\lambda{}^A$ and $G_\mu{}^B$ be as the generalized fuzzy soft set $H_\delta{}^C$ over (U, E). Written as '$F_\lambda{}^A \tilde{\Theta} G_\mu{}^B = H_\delta{}^C$'

where $C = A \cap B \neq \phi, \forall e \in C$ and $A, B \subseteq E$

$$H_\delta{}^C(e) = (H(e), \delta(e)),$$

Where $H(e) = F(e) * \mu(e)$, $\delta(e) = \max(G(e) \Diamond \lambda(e))$

Example 4.2: From **Example 4.1**

$$K_\delta{}^C(e_1) = (\{S_1/0.12, S_2/0.20, S_3/0.28\}, \max\{0.76, 0.94, 0.88\})$$

$$K_\delta{}^C(e_3) = (\{S_1/0.42, S_2/0.07, S_3/0.21\}, \max\{0.90, 0.85, 0.75\})$$

i.e.

$$K_\delta{}^C(e_1) = (\{S_1/0.12, S_2/0.20, S_3/0.28\}, 0.94)$$

$$K_\delta{}^C(e_3) = (\{S_1/0.42, S_2/0.07, S_3/0.21\}, 0.90)$$

Proposition 4.3

$$F_\lambda{}^A \tilde{\Theta} \theta_\phi{}^A = F_\lambda{}^A$$

Proof:

$F_\lambda{}^A(e) = (F(e), \lambda(e))$, where $e \in E, A \subseteq E$, $\theta_\phi{}^A(e) = (\overline{0}, 1)$, where $e \in E, B \subseteq E$

Now $(F_\lambda{}^A \tilde{\Theta} \theta_\phi{}^A)(e) = K_\delta{}^A(e)$

$$K_\delta{}^A(e) = (K(e), \delta(e))$$

Where $K(e) = F(e) * 1 = F(e)$, $\delta(e) = \max(0 \Diamond \lambda(e)) = \lambda(e)$

Therefore

$$K_\delta{}^A(e) = (K(e), \delta(e)) = (F(e), \lambda(e)) = F_\lambda{}^A(e)$$

Hence

$$F_\lambda{}^A \tilde{\Theta} \theta_\phi{}^A = F_\lambda{}^A$$

Proposition 4.4:

$$F_\lambda{}^A \tilde{\Theta} \tilde{A}_\mu = \theta_\phi{}^A$$

Proof:

$F_\lambda{}^A(e) = (F(e), \lambda(e))$, where $e \in E, A \subseteq E$, $\tilde{A}_\mu(e) = (1, 0)$

Now $(F_\lambda{}^A \tilde{\Theta} \tilde{A}_\mu)(e) = K_\delta{}^A(e)$

$$K_\delta{}^A(e) = (K(e), \delta(e))$$

Where $K(e) = F(e) * \mu(e) = F(e) * 0 = 0$, $\delta(e) = \max(1 \Diamond \lambda(e)) = 1$

Therefore

$$K_\delta{}^A(e) = (K(e), \delta(e)) = (\tilde{0}, 1) = \theta_\phi{}^A(e)$$

Hence

$$F_\lambda{}^A \tilde{\Theta} \tilde{A}_\mu = \theta_\phi{}^A.$$

5. PROUCT OF GENERALIZED FUZZY SOFT SETS

Definition 5.1: The $\wedge$-Product of two GFSS $F_\lambda{}^A$ and $G_\mu{}^B$ over (*U*, *E*) is denoted by $F_\lambda{}^A \wedge G_\mu{}^B$ and defined by GFSS $H_\delta{}^{A\wedge B} : A\times B \to I^U \times I$ such that for each $(\alpha,\beta)\in A\times B$ and $A,B\subseteq E$

$$H_\delta{}^{A\wedge B}(\alpha,\beta) = (H(\alpha,\beta),\delta(\alpha,\beta)),$$

Where $H(\alpha,\beta) = F(\alpha) * G(\beta), \delta(\alpha,\beta) = \lambda(\alpha) * \mu(\beta)$

Example 5.1:
We define $F_\lambda{}^A$ and $G_\mu{}^B$ as follows:

$F_\lambda{}^A(e_1) = (\{S_1/0.4, S_2/0.6, S_3/0.5\}, 0.3)$, $F_\lambda{}^A(e_2) = (\{S_1/0.2, S_2/0.9, S_3/1.0\}, 0.7)$
and
$G_\mu{}^B(e_2) = (\{S_1/0.8, S_2/0.2, S_3/1.0\}, 0.4)$, $G_\mu{}^B(e_3) = (\{S_1/0.2, S_2/0.3, S_3/0.6\}, 0.2)$

Then

$$H_\delta{}^{A\wedge B}(e_1,e_2) = (\{S_1/0.32, S_2/0.12, S_3/0.50\}, 0.12)$$
$$H_\delta{}^{A\wedge B}(e_1,e_3) = (\{S_1/0.08, S_2/0.18, S_3/0.30\}, 0.06)$$
$$H_\delta{}^{A\wedge B}(e_2,e_2) = (\{S_1/0.16, S_2/0.18, S_3/1.00\}, 0.28)$$
$$H_\delta{}^{A\wedge B}(e_2,e_3) = (\{S_1/0.04, S_2/0.27, S_3/0.06\}, 0.14)$$

Definition 5.2: The V-product of two GFSS $F_\lambda{}^A$ and $G_\mu{}^B$ over (*U*, *E*) is denoted by $F_\lambda{}^A \vee G_\mu{}^B$ and defined by GFSS $K_\delta{}^{A\vee B} : A\times B \to I^U \times I$ such that for each $(\alpha,\beta)\in A\times B$ and $A,B\subseteq E$

$$K_\delta{}^{A\vee B}(\alpha,\beta) = (K(\alpha,\beta),\delta(\alpha,\beta)),$$

Where $K(\alpha,\beta) = F(\alpha)\lozenge G(\beta), \delta(\alpha,\beta) = \lambda(\alpha)\lozenge\mu(\beta)$

Example 5.2: From **Example 5.1**

$$K_\delta{}^{A\vee B}(e_1,e_2) = (\{S_1/0.88, S_2/0.68, S_3/1.00\}, 0.58)$$
$$K_\delta{}^{A\vee B}(e_1,e_3) = (\{S_1/0.52, S_2/0.72, S_3/0.80\}, 0.44)$$
$$K_\delta{}^{A\vee B}(e_2,e_2) = (\{S_1/0.84, S_2/0.92, S_3/1.00\}, 0.82)$$

Definition 5.3: The $\overline{\wedge}$-product of two GFSS $F_\lambda{}^A$ and $G_\mu{}^B$ over (*U*, *E*) is denoted by $F_\lambda{}^A \overline{\wedge} G_\mu{}^B$ and defined by GFSS $H_\delta{}^{A\overline{\wedge} B} : A\times B \to I^U \times I$ such that for each $(\alpha,\beta)\in A\times B$ and $A,B\subseteq E$

$$H_\delta{}^{A\overline{\wedge} B}(\alpha,\beta) = (H(\alpha,\beta),\delta(\alpha,\beta)), \text{ Where } H(\alpha,\beta) = F(\alpha) * G^C(\beta), \delta(\alpha,\beta) = \lambda(\alpha) * \mu^C(\beta)$$

Example 5.3: From **Example 5.1**

$$H_\delta{}^{A\overline{\wedge} B}(e_1,e_2) = (\{S_1/0.08, S_2/0.48, S_3/0.00\}, 0.18)$$
$$H_\delta{}^{A\overline{\wedge} B}(e_1,e_3) = (\{S_1/0.32, S_2/0.42, S_3/0.20\}, 0.24)$$
$$H_\delta{}^{A\overline{\wedge} B}(e_2,e_2) = (\{S_1/0.04, S_2/0.72, S_3/0.00\}, 0.42)$$
$$H_\delta{}^{A\overline{\wedge} B}(e_2,e_3) = (\{S_1/0.16, S_2/0.63, S_3/0.40\}, 0.56)$$

Definition 5.4: The $\underline{\vee}$-product of two GFSS $F_\lambda{}^A$ and $G_\mu{}^B$ over (*U*, *E*) is denoted by $F_\lambda{}^A \underline{\vee} G_\mu{}^B$ and defined by GFSS $K_\delta{}^{A\underline{\vee} B} : A\times B \to I^U \times I$ such that for each $(\alpha,\beta)\in A\times B$ and $A,B\subseteq E$

$$K_\delta{}^{A\underline{\vee} B}(\alpha,\beta) = (K(\alpha,\beta),\delta(\alpha,\beta)), \text{ Where } K(\alpha,\beta) = F(\alpha)\lozenge G^C(\beta), \delta(\alpha,\beta) = \lambda(\alpha)\lozenge\mu^C(\beta)$$

Example 5.4: From **Example 5.1**

$$K_{\delta}^{A \underline{\vee} B}(e_1, e_2) = (\{S_1/0.52, S_2/0.92, S_3/0.50\}, 0.72)$$
$$K_{\delta}^{A \underline{\vee} B}(e_1, e_3) = (\{S_1/0.88, S_2/0.88, S_3/0.70\}, 0.86)$$
$$K_{\delta}^{A \underline{\vee} B}(e_2, e_2) = (\{S_1/0.36, S_2/0.98, S_3/1.00\}, 0.88)$$
$$K_{\delta}^{A \underline{\vee} B}(e_2, e_3) = (\{S_1/0.84, S_2/0.97, S_3/1.00\}, 0.94)$$

Proposition 5.1:

If $F_{\lambda}^{A}, G_{\mu}^{A}$ be a GFSS over (*U*, *E*), then

(i) $F_{\lambda}^{A} \wedge F_{\lambda}^{A} \subseteq F_{\lambda}^{A}$

(ii) $F_{\lambda}^{A} \vee F_{\lambda}^{A} \supseteq F_{\lambda}^{A}$

(iii) $F_{\lambda}^{A} \overline{\wedge} F_{\lambda}^{A} \subseteq F_{\lambda}^{A}$

(iv) $F_{\lambda}^{A} \underline{\vee} F_{\lambda}^{A} \supseteq F_{\lambda}^{A}$

(v) $(F_{\lambda}^{A} \vee G_{\mu}^{A})^{C} = (G_{\mu}^{A})^{C} \wedge (F_{\lambda}^{A})^{C}$

(vi) $(F_{\lambda}^{A} \wedge G_{\mu}^{A})^{C} = (G_{\mu}^{A})^{C} \vee (F_{\lambda}^{A})^{C}$

(vii) $(F_{\lambda}^{A} \underline{\vee} G_{\mu}^{A})^{C} = (G_{\mu}^{A})^{C} \overline{\wedge} (F_{\lambda}^{A})^{C}$

(viii) $(F_{\lambda}^{A} \overline{\wedge} G_{\mu}^{A})^{C} = (G_{\mu}^{A})^{C} \underline{\vee} (F_{\lambda}^{A})^{C}$

Proof: The proof is straight forward and follows from definition.

6. CONCLUSION

In this paper, we redefined null, absolute, union, and intersection of generalized fuzzy soft sets and study some of their properties. Also we have put forward some new idea such as disjunctive sum, difference and product of two generalized fuzzy soft sets and their basic properties. It is hoped that our findings will help enhancing this study on generalized fuzzy soft sets.

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7. REFERENCES

[1] B. Schweirer, A. Sklar, "Statistical metric space", Pacific Journal of Mathematics 10(1960), 314-334.

[2] H. L.Yang, "Notes On Generalized Fuzzy Soft Sets", Journal of Mathematical Research and Exposition, Vol. 31, No. 3, May - 2011, pp.567-570.

[3] M.J.Borah, T.J.Neog and D.K.Sut, "Some Results on Generalized Fuzzy Soft Sets", International Journal of Computer Technology and Application, Vol. 3(2), 583-591.

[4] P. K. Maji, R. Biswas and A.R. Roy, "Fuzzy Soft Sets", Journal of Fuzzy Mathematics, Vol. 9, no.3, pp.-589-602,2001.

[5] P. Majumdar, S. K. Samanta, "Generalized Fuzzy Soft Sets", Computers and Mathematics with Applications, 59(2010), pp.1425-1432.

[6] T. J. Neog, D. K. Sut, "On Union and Intersection of Fuzzy Soft Sets", Int.J. Comp. Tech. Appl., Vol. 2 (5), 1160-11.

[7] T. J. Neog, D. K. Sut, "On Fuzzy Soft Complement and Related Properties", Accepted for publication in International Journal of Energy, Information and communications (IJEIC), Japan.

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