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ON SEMI SYMMETRIC METRIC S -CONNECTION ON AN UNIFIED PARA-NORDEN CONTACT METRIC STRUCTURE MANIFOLD

Shalini Singh*
Department of Applied Mathematics,
JSS Academy of Technical Education, Noida–201301, India.

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ABSTRACT

The present paper deals with the different geometrical properties of an unified para-norden contact metric structure manifold [12] equipped with semi-symmetric metric S -connection. We find the expression for the curvature tensor of an unified para-norden contact metric structure manifold that admits a type of semi-symmetric metric S -connection .Finally we obtain certain conditions under which two different curvature tensors with respect to the Riemannian connection are identical. Also we study the properties of curvature tensors, conformal curvature tensor, projective curvature tensor, con-harmonic curvature tensor and con-circular curvature tensor with respect to the semi-symmetric metric S -connection.

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Keywords and Phrases: *C^∞ -manifold, unified para- norden contact metric structure manifold, semi-symmetric metric S -connection, curvature tensors.*

1. INTRODUCTION

In 1924, A..Friedmann and J. A.Schouten [6] introduced the idea of a semi-symmetric linear connection in a differentiable manifold. In 1970, K. Yano [14] considered a semi-symmetric metric connection and studied some of its properties. He proved that a Riemannian manifold is conformally flat if and only if it admits a semi-symmetric metric connection whose curvature tensor vanishes identically. Also in 1985, R. H. Ojha and S. Prasad [8] introduce semi-symmetric metric S -connection in a Sasakian manifold and study its properties. The Riemannian manifold equipped with a semi-symmetric metric connection has been studied by O.C. Andonie [2], M.C. Chaki and A.. Konar [3], U. C. De [4] etc., while a special type of semi-symmetric metric connection on a Riemannian manifold has been studied by U.C. De and S.C. Biswas [5]. P. N. Pandey and S. K. Dubey [11] discussed an almost Grayan manifold admitting a semi-symmetric metric connection while a Kahler manifold equipped with semi-symmetric metric connection and an almost Hermitian manifold with semi-symmetric recurrent connection have been studied by P. N. Pandey and B.B. Chaturvedi [9][10].

In this paper we define a semi-symmetric metric S -connection $\tilde{\nabla}$ on an unified para-norden contact metric structure manifold M_n and define the curvature tensor of M_n with respect to the semi-symmetric metric S -connection $\tilde{\nabla}$ and Riemannian connection $\tilde{R}$.

*Corresponding author: Shalini Singh**
Department of Applied Mathematics, JSS Academy of Technical Education, Noida–201301, India.

2. PRELIMINARIES

Let M_n be a differentiable manifold of differentiability class C^∞. Let there exist in M_n a vector valued C^∞ - linear function Φ, a C^∞ - vector field η and a C^∞ -one form ξ such that

$$\Phi^2(X) = a^2 X - \xi(X)\eta \tag{2.1}$$

$$\overline{\eta} = 0, \tag{2.2}$$

$$G(\overline{X}, \overline{Y}) = a^2 G(X,Y) - \xi(X)\xi(Y) \tag{2.3}$$

where $\Phi(X) = \overline{X}$, a is a nonzero complex number.

Let us agree to say that Φ gives to M_n a differentiable structure define by algebraic equation (2.1). We shall call (Φ, η, a, ξ) as an almost unified para-norden contact structure.

Remark 2.1: The manifold M_n equipped with an almost unified para-norden contact structure (Φ, η, a, ξ) will be called an almost unified para-norden contact structure manifold.

Remark 2.2: The C^∞ -manifold M_n satisfying (2.1), (2.2) and (2.3) is called an almost unified para-norden contact metric structure manifold. (Φ, η, a, G, ξ)

It is easy to calculate in M_n that

$$\xi(\eta) = a^2 \tag{2.4}$$

$$G(X, \eta) \underline{\underline{def}} \ \xi(X) \tag{2.5}$$

and

$$\Phi(\overline{X}) = 0 \tag{2.6}$$

Remark 2.3: The almost unified para-norden contact metric structure manifold (Φ, η, a, G, ξ) gives an almost para contact metric manifold [1] or an almost Norden contact metric manifold [13] according as $(a^2 = \pm 1)$ or $(a^2 = \pm i)$

Definition 2.1: A C^∞ -manifold M_n, satisfying

$$D_X \eta = \Phi(X) \ \underline{\underline{def}} \ \overline{X} \tag{2.7}$$

will be denoted by M_n^*

In M_n^*, we can easily shown that

$$(D_X \xi)(Y) = {}^{`}\Phi(X,Y) = (D_Y \xi)(X) \tag{2.8}$$

where

$${}^{`}\Phi(X,Y) \ \underline{\underline{def}} \ G(\overline{X}, Y) = G(X, \overline{Y}) = {}^{`}\Phi(Y, X) \tag{2.9}$$

$${}^{`}\Phi(\overline{X}, \overline{Y}) = a^2 \, {}^{`}\Phi(X,Y) \tag{2.10}$$

and

$${}^{`}\Phi(\overline{X}, Y) = {}^{`}\Phi(X, \overline{Y})$$

In view of (2.9) it may be noted that $`\Phi$ is symmetric and $`\Phi$ is pure or hybrid in view of (2.10) according as $a^2 = \pm i$ or $a^2 = \pm 1$ respectively

Definition 2.2: Let $\tilde{\nabla}$ be an affine connection is said to be metric if

$$\tilde{\nabla}_X G = 0 \tag{2.11}$$

The metric connection $\tilde{\nabla}$ satisfying

$$\left(\tilde{\nabla}_X \Phi\right)(Y) = \xi(Y) X - G(X,Y)\eta \tag{2.12}$$

is called S -connection.

A metric S -connection $\tilde{\nabla}$ is called semi-symmetric metric S -connection if

$$\tilde{\nabla}_X Y = D_X Y - \xi(X)\overline{Y} \tag{2.13}$$

Where D is the Riemannian connection. Also equation (2.13) implies

$$S(X,Y) = \xi(Y) X - \xi(X) Y \tag{2.14}$$

where S is the torsion tensor of connection $\tilde{\nabla}$.

Replacing Y by η in (2.12), we have

$$\left(\tilde{\nabla}_X \Phi\right)(\eta) = \xi(\eta) X - G(X,\eta)\eta$$

Using (2.1), (2.4) and (2.6) in the above equation, we get

$$\left(\tilde{\nabla}_X \Phi\right)(\eta) = \overline{\overline{X}} \tag{2.15}$$

From (2.2), we have

$$\Phi\eta = 0$$

Differentiating covariantly above equation with respect to X , we get

$$\left(\tilde{\nabla}_X \Phi\right)(\eta) + \Phi\left(\tilde{\nabla}_X \eta\right) = 0$$

Using (2.15) in the above equation, we get

$$\tilde{\nabla}_X \eta = -\overline{X} \tag{2.16}$$

Now, from (2.6), we have

$$G(Y,\eta) = \xi(X)$$

Differentiating covariantly above equation with respect to X and using (2.9), (2.11) and (2.16), we get

$$-\Phi(X,Y) = \left(\tilde{\nabla}_X \xi\right)(Y) \tag{2.17}$$

We know that

$$\Phi Z = \overline{Z}$$

Differentiating covariantly above equation with respect to X and using (2.13), we get

$$(D_X \Phi)(Z) = \left(\tilde{\nabla}_X \Phi\right)(Z) \tag{2.18}$$

Let $\tilde{R}$ and K be the curvature tensors with respect to the connection $\tilde{\nabla}$ and D respectively then

$$\tilde{R}(X,Y,Z) \underline{\underline{def}} \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z \tag{2.19}$$

and

$$K(X,Y,Z) \underline{\underline{def}} D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z \tag{2.20}$$

Using (2.13) and (2.20) in (2.19), we get

$$\tilde{R}(X,Y,Z)=K(X,Y,Z)-\xi(Y)(D_X\Phi)(Z)+\left\{(\tilde{\nabla}_Y\xi)(X)-(\tilde{\nabla}_X\xi)(Y)\right\}\overline{Z}$$
$$+\left\{\xi(\tilde{\nabla}_Y X)-\xi(\tilde{\nabla}_X Y)\right\}\overline{Z}+\xi(D_X Y-D_Y X)\overline{Z}+\xi(X)(D_Y\Phi)(Z)$$

Using (2.8), (2.12), (2.13), (2.17) and (2.18) in the above equation, we get

$$\tilde{R}(X,Y,Z)=K(X,Y,Z)-\xi(Y)\xi(Z)X+\xi(Y)G(X,Z)\eta$$
$$+\left\{-\xi(D_X Y)+\xi(X)\xi(\overline{Y})+\xi(D_Y X)-\xi(Y)\xi(\overline{X})\right\}\overline{Z}$$
$$+\xi(D_X Y-D_Y X)\overline{Z}+\xi(X)\xi(Z)Y-G(Y,Z)\xi(X)\eta$$

Using (2.5) in the above equation, we get

$$\tilde{R}(X,Y,Z)=K(X,Y,Z)-\xi(Y)\xi(Z)X+\xi(Y)G(X,Z)\eta+\xi(X)\xi(Z)Y-\xi(X)G(Y,Z)\eta \tag{2.21}$$

Let us consider that $\tilde{R}(X,Y,Z)=0$ then above equation implies

$$K(X,Y,Z)=\xi(Y)\xi(Z)X-\xi(Y)G(X,Z)\eta-\xi(X)\xi(Z)Y+\xi(X)G(Y,Z)\eta \tag{2.22}$$

Contracting X in the above equation, we get

$$Ric(Y,Z)=(n-2)\xi(Y)\xi(Z)+a^2G(Y,Z) \tag{2.23}$$

Contracting with respect to Z in the above equation, we get

$$rY=(n-2)\xi(Y)\eta+a^2Y \tag{2.24}$$

Contracting Y in the above equation, we get

$$\tilde{R}=2a^2(n-1) \tag{2.25}$$

Where Ric and $\tilde{R}$ are Ricci tensor and scalar curvature respectively.

The Projective curvature tensor W, Conformal curvature tensor V, Conharmonic curvature tensor L and Concircular curvature tensor C in a Riemannian manifold are given by [7]

$$W(X,Y,Z)=K(X,Y,Z)-\frac{1}{(n-1)}\left[Ric(Y,Z)X-Ric(X,Z)Y\right] \tag{2.26}$$

$$V(X,Y,Z)=K(X,Y,Z)-\frac{1}{(n-2)}\left[Ric(Y,Z)X-Ric(X,Z)Y\right. \tag{2.27}$$
$$\left.+G(Y,Z)r(X)-G(X,Z)r(Y)\right]+\frac{\tilde{R}}{(n-1)(n-2)}\left[G(Y,Z)X-G(X,Z)Y\right]$$

$$L(X,Y,Z)=K(X,Y,Z)-\frac{1}{(n-2)}\left[Ric(Y,Z)X-Ric(X,Z)Y\right.$$
$$\left.+G(Y,Z)r(X)-G(X,Z)r(Y)\right] \tag{2.28}$$

$$C(X,Y,Z)=K(X,Y,Z)-\frac{\tilde{R}}{n(n-2)}\left[G(Y,Z)X-G(X,Z)Y\right] \tag{2.29}$$

where

$$'W(X,Y,Z,T)\overset{def}{=}G(W(X,Y,Z),T) \tag{2.30}$$

$$'V(X,Y,Z,T)\overset{def}{=}G(V(X,Y,Z),T) \tag{2.31}$$

$$'L(X,Y,Z,T)\overset{def}{=}G(L(X,Y,Z),T) \tag{2.32}$$

$$'C(X,Y,Z,T)\overset{def}{=}G(C(X,Y,Z),T) \tag{2.33}$$

3. CURVATURE TENSORS

Theorem 3.1: If an almost unified para-norden contact metric structure manifold admits a semi-symmetric metric S-connection whose curvature tensor is locally isometric to the unit sphere $S^{(n)}(1)$, then the Conformal and Con-harmonic curvature tensors with respect to the Riemannian connection are identical iff $n+2a^2=0$

Proof: If the curvature tensor with respect to the semi-symmetric non metric S - connection is locally isometric to the unit sphere $S^{(n)}(1)$, then

$$\tilde{R}(X,Y,Z)=G(Y,Z)X-G(X,Z)Y \tag{3.1}$$

Inconsequence of (3.1) and (2.21) becomes

$$G(Y,Z)X-G(X,Z)Y=K(X,Y,Z)-\xi(Y)\xi(Z)X+\xi(Y)G(X,Z)\eta$$
$$+\xi(X)\xi(Z)Y-\xi(X)G(Y,Z)\eta$$

Contracting above with respect to X , we get

$$Ric(Y,Z)=(a^2+n-1)G(Y,Z)+(n-2)\xi(Y)\xi(Z) \tag{3.2}$$

Contracting above equation with respect to Z , we get

$$rY=(a^2+n-1)Y+(n-2)\xi(Y)\eta$$

Contracting above equation with respect to Y , we get

$$\tilde{R}=(n-1)(2a^2+n) \tag{3.3}$$

Where Ric and $\tilde{R}$ are Ricci tensor and scalar curvature of the manifold respectively.

From (3.3), (2.27) and (2.28), we obtain the necessary part of the theorem. Converse part is obvious from (2.27) and (2.28).

Theorem 3.2: If an almost unified para-norden contact metric structure manifold M_n admits a semi-symmetric metric S -connection whose curvature tensor is locally isometric to the unit sphere $S^{(n)}(1)$, then the Con-circular curvature tensor coincides with the Riemannian connection iff $n+2a^2=0$

Proof: Using (3.3) in (2.29), we get

$$C(X,Y,Z)=K(X,Y,Z)-\left(\frac{2a^2+n}{n}\right)\left[G(Y,Z)X-G(X,Z)Y\right] \tag{3.4}$$

which is the required proves of the theorem.

Now, let us consider that the curvature tensor of the semi-symmetric metric S -connection has the form

$$\tilde{R}(X,Y,Z)='\Phi(X,Z)\overline{Y}-'\Phi(Y,Z)\overline{X} \tag{3.5}$$

Using above equation in (2.21), we get

$$K(X,Y,Z)=\xi(Y)\xi(Z)X-\xi(X)\xi(Z)Y+\xi(X)G(Y,Z)\eta$$
$$-\xi(Y)G(X,Z)\eta+'\Phi(X,Z)\overline{Y}-'\Phi(Y,Z)\overline{X} \tag{3.6}$$

Contracting X in the above equation and using (2.1), (2.4) and (2.6), we get

$$Ric(Y,Z)=(n-3)\xi(Y)\xi(Z)+2a^2G(Y,Z) \tag{3.7}$$

Contracting above equation with respect to Z , we get

$$rY = (n-3)\xi(Y)\eta + 2a^2 Y \tag{3.8}$$

Contracting above equation with respect to Y , we get

$$\tilde{R} = 3a^2(n-1) \tag{3.9}$$

Using (3.6) and (3.7) in (2.26), we get

$$W(X,Y,Z) = `\Phi(X,Z)\overline{Y} - `\Phi(Y,Z)\overline{X} + \xi(X)G(Y,Z)\eta - \xi(Y)G(X,Z)\eta + \frac{1}{n-1}\left[2\xi(Y)\xi(Z)X - 2\xi(X)\xi(Z)Y\right] + \frac{2a^2}{(n-1)}\left[G(X,Z)Y - G(Y,Z)X\right] \tag{3.10}$$

Now operating G on both sides of above equation and using (2.6), (2.9) and (2.30), we get

$$`W(X,Y,Z,T) = `\Phi(X,Z)`\Phi(Y,T) - `\Phi(Y,Z)`\Phi(X,T) + \left[\xi(X)G(Y,Z)\xi(T) - G(X,Z)\xi(Y)\xi(T)\right] + \frac{1}{(n-1)}\left[2\xi(Y)\xi(Z)G(X,T) - 2\xi(X)\xi(Z)G(Y,T)\right] + \left(\frac{2a^2}{n-1}\right)\left[G(X,Z)G(Y,T) - G(Y,Z)G(X,T)\right] \tag{3.11}$$

Now, using (3.6), (3.7) and (3.8) in (2.28), we get

$$L(X,Y,Z) = `\Phi(X,Z)\overline{Y} - `\Phi(Y,Z)\overline{X} + \frac{1}{n-2}\left[\xi(X)G(Y,Z)\eta - \xi(Y)G(X,Z)\eta + \xi(Y)\xi(Z)X - \xi(X)\xi(Z)Y\right] + \left(\frac{4a^2}{n-2}\right)\left[G(X,Z)Y - G(Y,Z)X\right] \tag{3.12}$$

Operating G on both sides of above equation and using (2.6), (2.9) and (2.32), we get

$$`L(X,Y,Z,T) = `\Phi(X,Z)`\Phi(Y,T) - `\Phi(Y,Z)`\Phi(X,T) + \frac{1}{(n-2)}\left[\xi(X)G(Y,Z)\xi(T) - \xi(Y)G(X,Z)\xi(T) + \xi(Y)\xi(Z)G(X,T) - \xi(X)\xi(Z)G(Y,T)\right] + \left(\frac{4a^2}{n-2}\right)\left[G(X,Z)G(Y,T) - G(Y,Z)G(X,T)\right] \tag{3.13}$$

Using (3.6) in (2.27), we get

$$V(X,Y,Z) = `\Phi(X,Z)\overline{Y} - `\Phi(Y,Z)\overline{X} + G(Y,Z)\xi(X)\eta - G(X,Z)\xi(Y)\eta + \xi(Y)\xi(Z)X - \xi(X)\xi(Z)Y - \frac{1}{(n-2)}\left[Ric(Y,Z)X - Ric(X,Z)Y - G(X,Z)r(Y) + G(Y,Z)r(X)\right] + \frac{\tilde{R}}{(n-1)(n-2)}\left[G(Y,Z)X - G(X,Z)Y\right] \tag{3.14}$$

Using (3.7), (3.8) and (3.9) in the above equation, we get

$$V(X,Y,Z) = `\Phi(X,Z)\overline{Y} - `\Phi(Y,Z)\overline{X} + \frac{a^2}{(n-2)}\left[G(X,Z)Y - G(Y,Z)X\right] + \frac{1}{(n-2)}\left[G(Y,Z)\xi(X)\eta - G(X,Z)\xi(Y)\eta + \xi(Y)\xi(Z)X - \xi(X)\xi(Z)Y\right] \tag{3.15}$$

Operating G both sides of above equation and using (2.6), (2.9) and (2.31), we get

$$\begin{aligned} {}'V(X,Y,Z,T) &= {}'\Phi(X,Z)\,{}'\Phi(Y,T) - {}'\Phi(Y,Z)\,{}'\Phi(X,T) + \frac{a^2}{(n-2)}\big[G(X,Z)G(Y,T) \\ &\quad - G(Y,Z)G(X,T)\big] + \frac{1}{(n-2)}\big[G(Y,Z)\xi(X)\xi(T) - G(X,Z)\xi(Y)\xi(T) \\ &\quad + \xi(Y)\xi(Z)G(X,T) - \xi(X)\xi(Z)G(Y,T)\big] \end{aligned} \tag{3.16}$$

Using (3.6) and (3.9) in (2.29), we get

$$\begin{aligned} C(X,Y,Z) &= {}'\Phi(X,Z)\bar{Y} - {}'\Phi(Y,Z)\bar{X} + \big[G(Y,Z)\xi(X)\eta - G(X,Z)\xi(Y)\eta \\ &\quad + \xi(Y)\xi(Z)X - \xi(X)\xi(Z)Y\big] - \frac{3a^2}{n}\big[G(Y,Z)X - G(X,Z)Y\big] \end{aligned} \tag{3.17}$$

Operating G both sides of above equation and using (2.6), (2.9) and (2.33), we get

$$\begin{aligned} {}'C(X,Y,Z,T) &= {}'\Phi(X,Z)\,{}'\Phi(Y,T) - {}'\Phi(Y,Z)\,{}'\Phi(X,T) + \big[G(Y,Z)\xi(X)\xi(T) \\ &\quad - G(X,Z)\xi(Y)\xi(T) + G(X,T)\xi(Y)\xi(Z) + G(Y,T)\xi(X)\xi(Z)\big] \\ &\quad - \frac{3a^2}{n}\big[G(Y,Z)G(X,T) - G(X,Z)G(Y,T)\big] \end{aligned} \tag{3.18}$$

Theorem 3.3: On a manifold M_n, we have

$$ {}'W(\eta,Y,Z,T) = a^2\left(\frac{n-3}{n-1}\right)G(Y,Z)\xi(T) - \frac{2a^2}{(n-1)}\xi(Z)G(Y,T) + \left(\frac{n-3}{n-1}\right)\xi(Y)\xi(Z)\xi(T) \tag{3.19a}$$

$$ {}'W(\eta,Y,\bar{Z},\bar{T}) = 0 \tag{3.19b}$$

$$ {}'W(X,Y,Z,\eta) = a^2\left(\frac{n-3}{n-1}\right)\big[G(Y,Z)\xi(X) - G(X,Z)\xi(Y)\big] \tag{3.19c}$$

$$ {}'W(\eta,Y,Z,\eta) = a^2\left(\frac{n-3}{n-1}\right)\big[a^2G(Y,Z) - \xi(Y)\xi(Z)\big] \tag{3.19d}$$

$$ {}'W(\bar{X},\bar{Y},Z,\eta) = 0 \tag{3.19e}$$

$$ {}'W(X,Y,\eta,\eta) = 0 \tag{3.19f}$$

Proof: Replacing X by η in (3.11) and using (2.2), (2.4), (2.6) and (2.9), we get (3.19a).

Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (3.19a) and using (2.4), we get (3.19b).

Replacing T by η in (3.11) and using (2.2), (2.4), (2.6) and (2.9), we get (3.19c).

Replacing X by η in (3.19c) and using (2.4) and (2.6) we get (3.19d).

Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (3.19c) and using (2.4), we get (3.19e).

Replacing Z by η in (3.19c) and using (2.6), we get (3.19f).

Theorem 3.4: On a manifold M_n, we have

$$ {}'V(\eta,Y,Z,T) = 0 \tag{3.20a}$$

$$ {}'V(X,Y,Z,\eta) = 0 \tag{3.20b}$$

$$V(\eta, Y, Z, \eta) = 0 \tag{3.20c}$$

$$V(X, Y, \eta, \eta) = 0 \tag{3.20d}$$

$$V(\bar{X}, \bar{Y}, Z, \eta) = 0 \tag{3.20e}$$

$$V(\eta, Y, \bar{Z}, \bar{T}) = 0 \tag{3.20f}$$

$$V(X, Y, \eta) = 0 \tag{3.20g}$$

$$V(\eta, Y, \eta) = 0 \tag{3.20h}$$

Proof: Replacing X by η in (3.16) and using (2.2), (2.4), (2.6) and (2.9), we get (3.20a).

Replacing T by η in (3.16) and using (2.2), (2.4), (2.6) and (2.9), we get (3.20b).

Replacing T by η in (3.20a) we get (3.20c).

Replacing Z by η in (3.20b), we get (3.20d).

Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (3.20b), we get (3.20e).

Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (3.20a), we get (3.20f).

Replacing Z by η in (3.15) and using (2.4), (2.6) and (2.9), we get (3.20g).

Replacing X by η in (3.20g), we get (3.20h).

Theorem 3.5: On a manifold M_n, we have

$$'L(X, Y, Z, \eta) = \frac{a^2}{(n-2)}\left[3G(X, Z)\xi(Y) - 3\xi(X)G(Y, Z)\right] \tag{3.21a}$$

$$'L(\eta, Y, Z, T) = \frac{a^2}{(n-2)}\left[3G(Y, T)\xi(Z) - 3\xi(T)G(Y, Z)\right] \tag{3.21b}$$

$$'L(\eta, Y, Z, \eta) = \frac{a^2}{(n-2)}\left[3\xi(Y)\xi(Z) - 3a^2G(Y, Z)\right] \tag{3.21c}$$

$$'L(X, Y, \eta, \eta) = 0 \tag{3.21d}$$

$$'L(\bar{X}, \bar{Y}, Z, \eta) = 0 \tag{3.21e}$$

$$'L(\eta, Y, \bar{Z}, \bar{T}) = 0 \tag{3.21f}$$

Proof: Replacing T by η in (3.13) and using (2.2), (2.4), (2.6) and (2.9), we get (3.21a).

Replacing X by η in (3.13) and using (2.2), (2.4), (2.6) and (2.9), we get (3.21b).

Replacing T by η in (3.21b) and using (2.4) and (2.6), we get (3.21c).

Replacing Z by η in (3.21a) and using (2.6), we get (3.21d).

Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (3.21a) and using (2.4), we get (3.21e).

Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (3.21b) and using (2.4), we get (3.21f).

Theorem 3.6: On a manifold M_n, we have

$$\breve{C}(\eta,Y,Z,\eta)=a^2\left[\left(1-\frac{3}{n}\right)G(Y,Z)\xi(T)+\left(1+\frac{3}{n}\right)G(Y,T)\xi(Z)\right] \tag{3.22a}$$

$$\breve{C}(X,Y,Z,\eta)=1-\frac{3}{n}\left[a^2G(Y,Z)\xi(X)-a^2G(X,Z)\xi(Y)\right]+2\xi(X)\xi(Y)\xi(Z) \tag{3.22b}$$

$$\breve{C}(X,Y,Z,\eta)=a^2\left[a^2\left(1-\frac{3}{n}\right)G(Y,Z)+\left(1+\frac{3}{n}\right)\xi(Y)\xi(Z)\right] \tag{3.22c}$$

$$\breve{C}(X,Y,\eta,\eta)=2a^2\xi(X)\xi(Y) \tag{3.22d}$$

$$\breve{C}(\bar{X},\bar{Y},Z,\eta)=0 \tag{3.22e}$$

$$\breve{C}(\eta,Y,\bar{Z},\bar{T})=0 \tag{3.22f}$$

Proof: Replacing X by η in (3.18) and using (2.2), (2.4), (2.6) and (2.9), we get (3.22a).

Replacing T by η in (3.18) and using (2.2), (2.4), (2.6) and (2.9), we get (3.22b).

Replacing T by η in (3.22a) and using (2.4) and (2.6), we get (3.22c).

Replacing Z by η in (3.22b) and using (2.6), we get (3.22d).

Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (3.22b) and using (2.4), we get (3.22e).

Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (3.22a) and using (2.4), we get (3.22f).

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