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A COMPARISON STUDY OF FUNCTION APPROXIMATION USING FOURIER AND WAVELET TRANSFORMS

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ABSTRACT

The present paper focuses on function approximation using Fourier Transform and Wavelet Transform. We have used three functions i.e., Continuous Exponential Function (Case-1), Continuous Periodic Function (Case-2) and Piecewise continuous function (Case-3). The equations generating the functions and the respective plots are given below.

***Key words**: Function Approximation, Fourier transform, Wavelet Transform*

INTRODUCTION:

In *Fourier transform* (FT) we represent a function in terms of signals. FT provides a function which is localized only in the frequency domain. It does not give any information of the function in the time domain. Basis functions of the *wavelet transform* (WT) are *small waves* located in different times. They are obtained using scaling and translation of a scaling function and wavelet function. Therefore, the WT is localized in both time and frequency. In addition, the WT provides a multiresolution system.Multiresolution is useful in several applications. For instance, image communications and image data base are such applications. If a function has a discontinuity, FT produces many coefficients with large magnitude (*significant* coefficients)

But WT generates a few significant coefficients around the discontinuity. Nonlinear approximation is a method to benchmark the approximation power of a transform. In nonlinear approximation we keep only a few significant coefficients of a function and set the rest to zero. Then we reconstruct the function using the significant coefficients.WT produces a few significant coefficients for the functions with discontinuities. Thus, we obtain better results for WT nonlinear approximation when compared with the FT.Most natural functions are smooth with a few discontinuities (are *piecewise continuous function*).Speech and natural images are such functions. Hence, WT has better capability for representing this function when compared with the FT.

DISCRETE WAVELET TRANSFORM:

Wavelet transform decomposes a signal into a set of basis functions. These basis functions are called *wavelets.* Wavelets are obtained from a single prototype wavelet *y(t)* called mother *wavelet* by *dilations* and *shifting:*

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

where a is the scaling parameter and b is the shifting parameter

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THEORY OF WT:

The wavelet transform is computed separately for different segments of the time-domain signal at different frequencies. Multi-resolution analysis: analyzes the signal at different frequencies giving different resolutions.MRA is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies. Good for signal having high frequency components for short durations and low frequency components for long duration.e.g. Images and video frames. Wavelet transform decomposes a signal into a set of basis functions. These basis functions are called *wavelets.* Wavelets are obtained from a single prototype wavelet *y(t)* called mother *wavelet* by *dilations* and *shifting:*

The 1-D wavelet transform is given by:

$$W_f(a,b) = \int_{-\infty}^{\infty} x(t)\psi_{a,b}(t)dt$$

The inverse 1-D wavelet transform is given by:

$$x(t) = \frac{1}{C}\int_{0}^{\infty}\int_{-\infty}^{\infty} W_f(a,b)\psi_{a,b}(t)db\frac{da}{a^2}$$

$$\text{where } C = \int_{-\infty}^{\infty}\frac{|\psi\omega)|^2}{\omega}d\omega < \infty$$

Discrete wavelets transform (DWT), which transforms a discrete time signal to a discrete wavelet representation.

(a) 1-D Continuous Exponential Function (Case-1):

$$y = (x+1)exp(-3x+3) \tag{1}$$

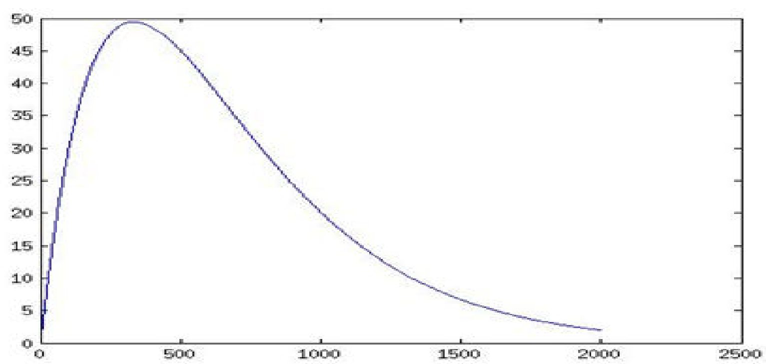

Figure 9.1 Original data of Continuous Exponential Function (Case-1).

(b) Continuous Periodic Function (Case-2):

The continuous periodic function used in in this study is given by

$$y = sin(4\pi x)exp(-|5x|) \tag{2}$$

The plot of function is shown in Fig. 9.2

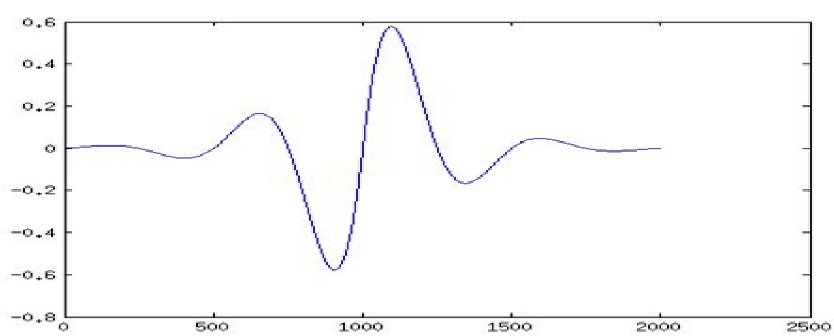

Figure 9.2 Continuous periodic function Case 2 2000 points

(c) Piecewise Continuous Function (Case 3):

A piecewise continuous function is generated

$$f(x)=\begin{cases} -2.186x-12.864 & -10 \le x \le -2 \\ 4.264x & -2 \le x \le 0 \\ 10e^{-0.05x-0.5}\sin[(0.03x+0.7)x] & 0 \le x \le 10 \end{cases} \quad (9.3)$$

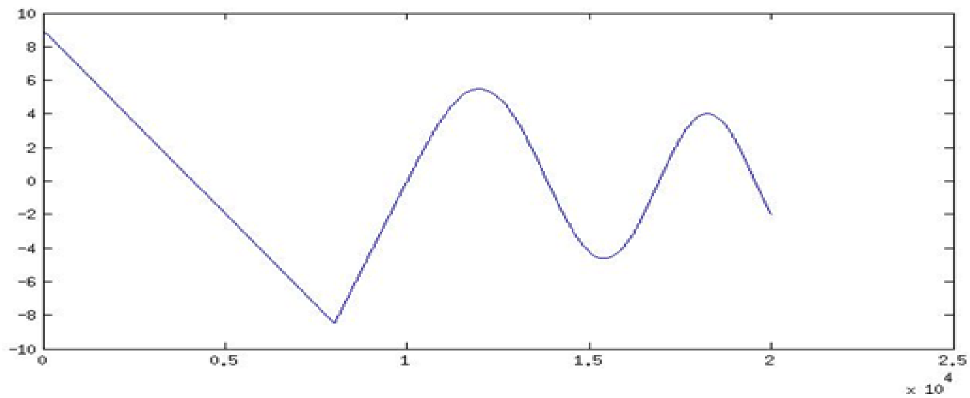

Figure 9.3 Piecewise Continuous function Simulation result for Case 3

These functions have been evaluated for 2000 data points and sampled uniformly for 100 points. These 100 points have been used for Fourier approximations. The using result of Fourier approximation formula we have reevaluated for 200 points. The analysis of 95% confidence interval. The accuracy of the function approximation can be seen from the relative RMSE (Root Mean Square Error) of 100 and 200 point approximation and evaluation.

PERFORMANCE COMPARISON OF WAVELET APPROXIMATIONS:

Names of wavelets	Exponential function	Periodic function	Piecewise continuous function
Haar	4.5092e-016	3.3559e-018	2.0985e-017
db	4.6624e-014	2.7361e-015	1.7179e-016
sym	4.8542e-015	6.0941e-017	6.0827e-017
coif	1.6384e-010	2.8961e-012	2.0553e-013
bior	4.8283e-016	3.4384e-018	2.1714e-017

COMPARISON BETWEEN FOURIER TRANSFORM AND WAVELET TRANSFORM:

The numerical experiments have revealed that the RMSE of Fourier approximations are higher then the wavelet approximations. Only the continuous periodic function has the RMSE of 0.1584 for the Fourier approximation .The exponential function is the highest in its approximation error and similar high RMSE 2.769 is seen for piecewise continuous function .In contrast Fourier all the wavelet functions have shown approximation error in the power of 10^{-10} to 10^{-18} . It establishes that the wavelet approximation is superior to the Fourier method.

RESULTS AND CONCLUSION:

The Fourier Transform (FT) and the discrete wavelet transform (DWT) are both linear operations that generate a data structure which may contain data segments of various lengths. These approximating functions usually perform filling and transforming functions into a different data vectors. The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain. For the FT, this new domain contains basis functions that are sines and cosines($f(x) = a0 + a1 * cos(x * w) + b1 * sin(x * w)$). For the wavelet transform, this new domain contains more complicated basis functions called wavelets, mother wavelets, or analyzing wavelets

Both transforms have another similarity. The basis functions are localized in frequency, making mathematical tools such as power spectra useful at picking out frequencies and calculating power distributions. The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes many functions and operators using wavelets "sparse" when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series. One way to see the time-frequency resolution differences between the Fourier transform and the wavelet transform is to look at the basis function coverage of the time-frequency plane. Fourier is confined to a fixed window whereas the wavelet has advantage of the windows that vary. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. A way to achieve this is to have short high-frequency basis functions and long low-frequency ones. Wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

The thesis focuses on approximation of continuous exponential, periodic, piecewise continuous function using Fourier transform and wavelet transform. The main aim has been to compare and contrast the FT and Wavelet transform in function approximation. Five wavelet forms (Haar, db4, symlet, coif5, bio3.3) have been used in this study. The approximation has been compare with the original and RMSE‘s have been computed for each approximations. The results unequivocally establish the superiority of wavelets over Fourier transforms.

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