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ON TOPOLOGICAL $\ddot{g}_{\alpha}$-QUOTIENT MAPPINGS

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ABSTRACT

The aim of this paper is to introduce $\ddot{g}_{\alpha}$-quotient maps using $\ddot{g}_{\alpha}$-closed sets and study their basic properties. We also study the relation between weak and strong forms of $\ddot{g}_{\alpha}$-quotient maps. We also derive the relation between strongly $\ddot{g}_{\alpha}$-continuous maps and $\ddot{g}_{\alpha}$-quotient maps. Examples are given to illustrate the results.

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1. INTRODUCTION:

Levine [2] offered a new and useful notion in General Topology that is the notion of a generalized closed set. A subset A of a topological space (X, τ) is called generalized closed (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ). This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets had led to several new and interesting concepts. After the introduction of generalized closed sets there are many research papers which deal with different types of generalized closed sets. Recently Ravi et al [9] have introduced $\ddot{g}_{\alpha}$-closed sets and studied their properties using sg-open set [1]. In this paper we introduce $\ddot{g}_{\alpha}$-quotient maps. Using these new types of maps, several characterizations and its properties have been obtained. Also the relationship between strong and weak forms of $\ddot{g}_{\alpha}$-quotient maps have been established.

2. PRELIMINARIES:

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition: 2.1

A subset A of a space (X, τ) is called:
(i) semi-open set [3] if A ⊆ cl(int(A));

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(ii) α-open set [5] if $A \subseteq int(cl(int(A)))$.

The complement of semi-open set (resp. α-open set) is said to be semi-closed (resp. α-closed).
The α-closure of a subset A of X, denoted by αcl(A), is defined as the intersection of α-closed sets of X containing A.

The semi-closure of a subset A of X, denoted by scl(A), is defined as the intersection of semi-closed sets of X containing A.

Definition: 2.2

A subset A of a space (X, τ) is called:

(i) a semi-generalized closed (briefly sg-closed) set [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The complement of sg-closed set is called sg-open set;

(ii) a $\ddot{g}$-closed set [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ). The complement of $\ddot{g}$-closed set is called $\ddot{g}$-open set;

(iii) a $\ddot{g}_\alpha$-closed set [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ). The complement of $\ddot{g}_\alpha$-closed set is called $\ddot{g}_\alpha$-open set.

Remark: 2.3

The collection of all $\ddot{g}_\alpha$-closed (resp. $\ddot{g}_\alpha$-open) sets are denoted by $\ddot{G}_\alpha C(X)$ (resp. $\ddot{G}_\alpha O(X)$) respectively.

Definition: 2.4

A map $f : (X, \tau) \to (Y, \sigma)$ is called

(i) $\ddot{g}_\alpha$-continuous [10] if $f^{-1}(V)$ is a $\ddot{g}_\alpha$-closed set of (X, τ) for each closed set V of (Y, σ).

(ii) Strongly $\ddot{g}_\alpha$-continuous [10] if $f^{-1}(V)$ is a closed set of (X, τ) for each $\ddot{g}_\alpha$-closed set V of (Y, σ).

(iii) α-continuous [11] if $f^{-1}(V)$ is a α-closed set of (X, τ) for each closed set V of (Y, σ).

(iv) $\ddot{g}$-continuous [8] if $f^{-1}(V)$ is a $\ddot{g}$-closed set of (X, τ) for each closed set V of (Y, σ).

Definition: 2.5

A map $f : (X, \tau) \to (Y, \sigma)$ is called

(i) $\ddot{g}_\alpha$-irresolute [10] if $f^{-1}(V)$ is a $\ddot{g}_\alpha$-open in (X, τ) for each $\ddot{g}_\alpha$-open set V of (Y, σ).

(ii) α-irresolute [11] if $f^{-1}(V)$ is an α-open in (X, τ) for each α-open set V of (Y, σ).

(iii) $\ddot{g}$-irresolute [8] if $f^{-1}(V)$ is a $\ddot{g}$-open in (X, τ) for each $\ddot{g}$-open set V of (Y, σ).

Definition: 2.6

A surjective map $f : (X, \tau) \to (Y, \sigma)$ is said to be

(i) a quotient map [4], provided a subset U of (Y, σ) is open in (Y, σ) if and only if $f^{-1}(U)$ is open in (X, τ).

(ii) a α-quotient map [11] if f is α-continuous and $f^{-1}(V)$ is open in (X, τ) implies V is an α-open set in (Y, σ).

(iii) a α*-quotient map [11] if f is α-irresolute and $f^{-1}(V)$ is α-open set in (X, τ) implies V is an open set in (Y, σ).

(iv) a $\ddot{g}$-quotient map [6] if f is $\ddot{g}$-continuous and $f^{-1}(V)$ is open in (X, τ) implies V is a $\ddot{g}$-open set in (Y, σ).

Remark: 2.7 [9]

(i) Every closed set is $\ddot{g}_\alpha$-closed but not conversely.

(ii) Every α-closed set is $\ddot{g}_\alpha$-closed but not conversely.

(iii) Every $\ddot{g}$-closed set is $\ddot{g}_\alpha$-closed but not conversely.

Remark: 2.8 [10]

(i) Every continuous map is $\ddot{g}_\alpha$-continuous but not conversely.

(ii) Every α-continuous map is $\ddot{g}_\alpha$-continuous but not conversely.

(iii) Every $\ddot{g}$-continuous map is $\ddot{g}_\alpha$-continuous but not conversely.

Remark: 2.9 [11]

Every quotient map is α-quotient but not conversely.

Definition: 2.10 [5]

A map f : X → Y is called α-open if f(V) is α-open in Y for each open set V of X.

Remark: 2.11

(i) Every α-irresolute map is $\ddot{g}_\alpha$-irresolute but not conversely [10].

(ii) Every α-irresolute map is α-continuous but not conversely [11].

(iii) Every $\ddot{g}_\alpha$-irresolute map is $\ddot{g}_\alpha$-continuous but not conversely [10].

3. $\ddot{g}_\alpha$-QUOTIENT MAPS:

Definition: 3.1

A surjective map f : (X, τ) → (Y, σ) is said to be a $\ddot{g}_\alpha$-quotient map if f is $\ddot{g}_\alpha$-continuous and $f^{-1}(V)$ is open in (X, τ) implies V is a $\ddot{g}_\alpha$-open set in (Y, σ).

Example: 3.2

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}, {a, c, d}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f(a) = 1, f(b) = 3, f(c) = f(d) = 2. The map f is $\ddot{g}_\alpha$-quotient.

Definition: 3.3

A map f : (X, τ) → (Y, σ) is said to be $\ddot{g}_\alpha$-open if f(U) is $\ddot{g}_\alpha$-open in (Y, σ) for each open set U in (X, τ).

Definition: 3.4

A map f : (X, τ) → (Y, σ) is said to be strongly $\ddot{g}_\alpha$-open if f(U) is $\ddot{g}_\alpha$-open in (Y, σ) for each $\ddot{g}_\alpha$-open set U in (X, τ).

Proposition: 3.5

If a map f : (X, τ) → (Y, σ) is surjective, $\ddot{g}_\alpha$-continuous and $\ddot{g}_\alpha$-open, then f is a $\ddot{g}_\alpha$-quotient map.

Proof: We only need to prove that $f^{-1}(V)$ is open in (X, τ) implies V is a $\ddot{g}_\alpha$-open set in (Y, σ). Let $f^{-1}(V)$ be open in (X, τ). Then f ($f^{-1}(V)$) is a $\ddot{g}_\alpha$-open set, since f is $\ddot{g}_\alpha$-open. Hence V is a $\ddot{g}_\alpha$-open set, as f is surjective, f ($f^{-1}(V)$) = V. Thus, f is a $\ddot{g}_\alpha$-quotient map.

Proposition: 3.6

Let f : (X, $\tau^{\ddot{g}_\alpha}$) → (Y, $\sigma^{\ddot{g}_\alpha}$) be a quotient map. Then f : (X, τ) → (Y, σ) is a $\ddot{g}_\alpha$-quotient map.

Proof: Let V be any open set in (Y, σ). Then V is $\ddot{g}_\alpha$-open set in (Y,σ) and V∈ $\sigma^{\ddot{g}_\alpha}$. Then $f^{-1}(V)$ is open in (X,τ), because f is a quotient map that is, $f^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (X,τ). Hence f is $\ddot{g}_\alpha$-continuous. Suppose $f^{-1}(V)$ is open in (X,τ), that is, $f^{-1}(V) \in \tau^{\ddot{g}_\alpha}$. Since f is a quotient map, V is open in (Y,σ) and V is a $\ddot{g}_\alpha$-open set in (Y,σ). This shows that f : (X, τ) → (Y, σ) is a $\ddot{g}_\alpha$-quotient map.

4. STRONG FORM OF $\ddot{g}_\alpha$-QUOTIENT MAPS:

Definition: 4.1

Let f: (X,τ) → (Y,σ) be a surjective map. Then f is called strongly $\ddot{g}_\alpha$-quotient map provided a set U of (Y,σ) is open in Y if and only if $f^{-1}(U)$ is a $\ddot{g}_\alpha$-open in (X,τ).

Example: 4.2

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {b}, {a, b}, {b ,c}, {a, b, c}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}, {2}, {1, 2}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {2}, {1, 2}}. The map f is defined as f(a) = 1, f(b) = 2 = f(c), f(d) = 3. The map f is strongly $\ddot{g}_\alpha$-quotient.

Theorem: 4.3

Every strongly $\ddot{g}_\alpha$-quotient map is $\ddot{g}_\alpha$-open.

Proof: Let f: (X,τ) → (Y, σ) be a strongly $\ddot{g}_\alpha$-quotient map. Let V be an open set in (X,τ). Since every open set is $\ddot{g}_\alpha$-open and hence V is $\ddot{g}_\alpha$-open in (X, τ). That is $f^{-1}(f(V))$ is $\ddot{g}_\alpha$-open in (X, τ). Since f is strongly $\ddot{g}_\alpha$-quotient, f (V) is open and hence $\ddot{g}_\alpha$-open in (Y, σ). This shows that f is a $\ddot{g}_\alpha$-open.

Remark: 4.4

The converse of Theorem 4.3 need not be true.

Example: 4.5

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {b}, {a, b}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}, {2}, {1, 2}, {1,3}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {2}, {1, 2}, {1, 3}}. The map f is defined as f(a) = 1 = f(b), f(c) = 3, f(d) = 2. The map f is $\ddot{g}_\alpha$-open but not strongly $\ddot{g}_\alpha$-quotient, since $f^{-1}(\{2\})$ = {d} is not $\ddot{g}_\alpha$-open in (X, τ).

Theorem: 4.6
Every strongly $\ddot{g}_\alpha$-quotient map is strongly $\ddot{g}_\alpha$-open.

Proof: Let f: (X,τ) → (Y,σ) be a strongly $\ddot{g}_\alpha$-quotient map. Let V be a $\ddot{g}_\alpha$-open set in (X,τ). That is f^{-1}(f (V)) is $\ddot{g}_\alpha$-open in (X,τ). Since f is strongly $\ddot{g}_\alpha$-quotient, f (V) is open and hence $\ddot{g}_\alpha$-open in (Y,σ). This shows that f is strongly $\ddot{g}_\alpha$-open.

Remark: 4.7

The converse of Theorem 4.6 need not be true.

Example: 4.8

Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1, 2}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {b}, {a, b}} and $\ddot{G}_\alpha$ O(Y) = {ϕ, Y, {1}, {2}, {1, 2}}. The map f is defined as f (a) = 1, f(b) = 2, f(c) = 3. The map f is strongly $\ddot{g}_\alpha$-open but not strongly $\ddot{g}_\alpha$-quotient because f^{-1}({1}) = {a} is $\ddot{g}_\alpha$-open in (X,τ) but {1} is not open in (Y,σ).

Definition: 4.9

Let f: (X,τ) → (Y, σ) be a surjective map. Then f is called $\ddot{g}_\alpha$*-quotient map if f is $\ddot{g}_\alpha$-irresolute and f^{-1}(U) is a $\ddot{g}_\alpha$-open in (X,τ) implies U is open in (Y,σ).

Example: 4.10

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}, {a, c, d}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}, {1, 2}, {1, 3}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f (a) = 1, f(b) = 3, f(c) = f(d) = 2. The map f is $\ddot{g}_\alpha$*-quotient.

Proposition: 4.11

Every $\ddot{g}_\alpha$*-quotient map is $\ddot{g}_\alpha$-irresolute.

Proof: It follows from the Definition 4.9.

Remark: 4.12

The converse of Proposition 4.11 need not be true.

Example: 4.13

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}, {a, c, d}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}, {1, 2}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f (a) = 1, f (b) = 3, f(c) = f (d) = 2. The map f is $\ddot{g}_\alpha$-irresolute but not $\ddot{g}_\alpha$*-quotient because f^{-1}({1, 3}) = {a, b} is $\ddot{g}_\alpha$-open in (X, τ) but {1, 3} is not open in (Y, σ).

Theorem: 4.14

Every $\ddot{g}_\alpha$*-quotient map is strongly $\ddot{g}_\alpha$-open.

Proof: Let f: (X,τ) → (Y, σ) be a $\ddot{g}_\alpha$*-quotient map. Let V be an $\ddot{g}_\alpha$-open set in (X, τ). Then f^{-1}(f (V)) is $\ddot{g}_\alpha$-open in (X,τ). Since f is $\ddot{g}_\alpha$*-quotient, this implies that f (V) is open in (Y,σ) and thus $\ddot{g}_\alpha$-open in (Y,σ). Hence f is strongly $\ddot{g}_\alpha$-open.

Remark: 4.15

The converse of Theorem 4.14 need not be true.

Example: 4.16

Let X = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1, 2}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {b}, {a, b}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {2}, {1, 2}}. The map f is defined as f (a) = 1, f (b) = 2, f(c) = 3. The map f is strongly $\ddot{g}_\alpha$-open but not $\ddot{g}_\alpha$*-quotient because f^{-1}({1}) = {a} is $\ddot{g}_\alpha$-open in (X,τ) but {1} is not open in (Y,σ).

Proposition: 4.17

Every $\ddot{g}$-irresolute (α-irresolute map) is $\ddot{g}_\alpha$-irresolute.

Proof: Let U be a $\ddot{g}$-closed (α-closed) set in (Y, σ). Since every $\ddot{g}$-closed (α-closed) set is $\ddot{g}_\alpha$-closed [9], U is $\ddot{g}_\alpha$-closed. Since f is $\ddot{g}$-irresolute (α-irresolute), f^{-1}(U) is $\ddot{g}$-closed (α-closed) which is $\ddot{g}_\alpha$-closed.

5. COMPARISON:

Proposition: 5.1

(i) Every quotient map is a $\ddot{g}_\alpha$-quotient map.

(ii) Every α-quotient map is a $\ddot{g}_\alpha$-quotient map.

Proof: Since every continuous (α-continuous) map is $\ddot{g}_\alpha$-continuous and every open (α-open) set is $\ddot{g}_\alpha$-open [9], the proof follows from the definitions 2.6 and 3.1.

Remark: 5.2

The separate converses of Proposition 5.1 need not be true.

Example: 5.3

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}, {a, c, d}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f (a) = 1, f (b) = 3, f(c) = f (d) = 2. The map f is $\ddot{g}_\alpha$-quotient but not quotient. Since for the $\ddot{g}_\alpha$-open set {1, 2} in (Y,σ) f^{-1}({1, 2}) = {a, c, d} is open in (X,τ) but {1, 2} is not open in (Y,σ).

Example: 5.4

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {b}, {a, b}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1, 2}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {2}, {1, 2}}. The map f is defined as f(a) = 1, f(b) = 2, f(c) = f(d) = 3. The map f is $\ddot{g}_\alpha$-quotient but not α-quotient because f^{-1}({1}) = {a} is open in (X,τ) but {1} is not α-open in (Y, σ).

Theorem: 5.5

Every strongly $\ddot{g}_\alpha$-quotient map is $\ddot{g}_\alpha$-quotient but not conversely.

Proof: Let V be an open set in (Y, σ). Since f is strongly $\ddot{g}_\alpha$-quotient, f^{-1}(V) is $\ddot{g}_\alpha$-open set in (X,τ). Thus f is $\ddot{g}_\alpha$-continuous. Let f^{-1}(V) be open in (X,τ). Then f^{-1}(V) is $\ddot{g}_\alpha$-open in (X,τ). Since f is strongly $\ddot{g}_\alpha$-quotient V is open in (Y,σ). It implies that V is $\ddot{g}_\alpha$-open in (Y,σ). This shows that f is a $\ddot{g}_\alpha$-quotient map.

Example: 5.6

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}, {a, c, d}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f (a) = 1, f (b) = 3, f(c) = f (d) = 2. The map f is $\ddot{g}_\alpha$-quotient but not strongly $\ddot{g}_\alpha$-quotient because f^{-1}({1, 2}) = {a, c, d} is $\ddot{g}_\alpha$-open in (X,τ) but {1, 2} is not open in (Y,σ).

Proposition: 5.7

Every α*-quotient map is $\ddot{g}_\alpha$*-quotient map.

Proof: Let f be a α*-quotient map. Then f is surjective, α-irresolute and f^{-1}(U) is an α-open set in (X, τ) implies U is an open set in (Y, σ). Then U is $\ddot{g}_\alpha$-open set in (Y,σ). Since every α-irresolute map is $\ddot{g}_\alpha$-irresolute and every α-irresolute map is α-continuous, f^{-1}(U) is an α-open set which is a $\ddot{g}_\alpha$-open set in (X,τ). Since f is α*-quotient map, U is open in (Y,σ). Hence f is a $\ddot{g}_\alpha$*-quotient map.

Remark: 5.8

The converse of Proposition 5.7 need not be true.

Example: 5.9

Let X = {a, b, c, d}, τ = {ϕ, X, {b, c}, {b, c, d}, {a, b, c}}, Y = {1, 2, 3}and σ = {ϕ, Y, {2}, {3}, {2, 3}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {b}, {c}, {b, c}, {a, b, c}, {b, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {2}, {3}, {2, 3}}. The map f is defined as f(a) = 1 = f(d), f(b) = 2, f(c) = 3. The map f is $\ddot{g}_\alpha$*-quotient but not α*-quotient because f^{-1}({2}) = {b}, f is not α-irresolute.

Proposition: 5.10

Every $\ddot{g}_\alpha$*-quotient map is strongly $\ddot{g}_\alpha$-quotient.

Proof: Let V be an open set in (Y,σ). Then it is $\ddot{g}_\alpha$-open set in (Y,σ). Since, by Proposition 4.11 f is $\ddot{g}_\alpha$-irresolute f^{-1}(V) is a $\ddot{g}_\alpha$-open set in (X, τ). Thus V is open set in (Y,σ) implies f^{-1}(V) is a $\ddot{g}_\alpha$-open set in (X,τ). Conversely, if f^{-1}(V) is a $\ddot{g}_\alpha$-open set in (X, τ), since f is a $\ddot{g}_\alpha$*-quotient map, V is an open set in (Y, σ). Hence f is a strongly $\ddot{g}_\alpha$-quotient map.

Remark: 5.11

The converse of Proposition 5.10 need not be true.

Example: 5.12

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f (a) = 1, f (b) = 3 = f(c), f (d) = 2. The map f is strongly $\ddot{g}_\alpha$-quotient but not $\ddot{g}_\alpha$*-quotient because f^{-1}({1, 3}) = {a, b, c} is $\ddot{g}_\alpha$-open in (X,τ) but {1, 3} is not open in (Y,σ).

Remark: 5.13

quotient maps and strongly $\ddot{g}_\alpha$-quotient maps are independent of each other.

Example: 5.14

Let X = {1, 2, 3}, τ = {ϕ, X, {1}, {2}, {1, 2}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1, 2}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {1}, {2}, {1, 2}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {2}, {1, 2}}. The map f is defined as identity map. The map f is strongly $\ddot{g}_\alpha$-quotient but not quotient because f^{-1}({1}) = {1} is open in (X,τ) but {1} is not open in (Y,σ).

Example: 5.15

Let X = {1, 2, 3}, τ = {ϕ, X, {1}, {1, 2}, {1, 3}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {1}, {1, 2}, {1, 3}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as identity map. The map f is quotient but not strongly $\ddot{g}_\alpha$-quotient because f^{-1}({1, 3}) = {1, 3} is $\ddot{g}_\alpha$-open in (X,τ) but {1, 3} is not open in (Y,σ).

Theorem: 5.16

Every $\ddot{g}_\alpha$*-quotient map is $\ddot{g}_\alpha$-quotient.

Proof: Let f be a $\ddot{g}_\alpha$*-quotient map. Then f is $\ddot{g}_\alpha$-irresolute. By Remark 2.11, f is $\ddot{g}_\alpha$-continuous. Let f^{-1}(V) be an open set in (X,τ). Then f^{-1}(V) is a $\ddot{g}_\alpha$-open set in (X,τ). Since f is $\ddot{g}_\alpha$*-quotient, V is open in (Y,σ). It means V is $\ddot{g}_\alpha$-open in (Y,σ). Therefore f is $\ddot{g}_\alpha$-quotient map.

Remark: 5.17

The converse of Theorem 5.16 need not be true.

Example: 5.18

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}, {a, c, d}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f (a) = 1, f (b) = 3, f(c) = f (d) = 2. The map f is $\ddot{g}_\alpha$-quotient but not $\ddot{g}_\alpha$*-quotient because f^{-1}({1, 2}) = {a, c, d} is $\ddot{g}_\alpha$-open in (X, τ) but {1, 2} is not open in (Y, σ).

Theorem: 5.19 [6]

α-quotient maps and $\ddot{g}$-quotient maps are independent of each other.

Theorem: 5.20

Every $\ddot{g}$-quotient map is $\ddot{g}_\alpha$-quotient.

Proof: Let f be $\ddot{g}$-quotient map. Then by definition, f is $\ddot{g}$-continuous and hence, by Remark 2.8, f is $\ddot{g}_\alpha$-continuous. Let f^{-1}(V) be an open set in (X,τ). By definition of $\ddot{g}$-quotient map, V is $\ddot{g}$-open in (Y,σ). By Remark 2.7, V is $\ddot{g}_\alpha$-open in (Y,σ). Therefore f is $\ddot{g}_\alpha$-quotient map.

Remark: 5.21

The converse of Theorem 5.20 need not be true.

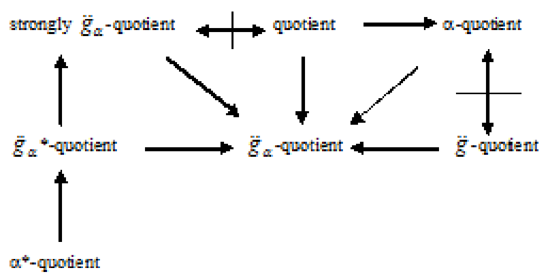
Example: 5.22

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {a, b}, {a, c, d}}, Y = {1, 2, 3} and σ = {ϕ, Y, {1}}. We have $\ddot{G}_\alpha$O(X) = {ϕ, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $\ddot{G}_\alpha$O(Y) = {ϕ, Y, {1}, {1, 2}, {1, 3}}. The map f is defined as f (a) = 1, f (b) = 3, f(c) = f (d) = 2. The map f is $\ddot{g}_\alpha$-quotient but not $\ddot{g}$-quotient because $f^{-1}(\{1, 2\}) = \{a, c, d\}$ is $\ddot{g}$-open in (X,τ) but {1, 2} is not open in (Y,σ).

Remark: 5.23

From the previous Theorems, Propositions, Examples and Remarks, we obtain the following diagram, where A → B (resp. A $\nleftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent of each other).

DIAGRAM



6. APPLICATIONS :

Proposition : 6.1

Let f: (X,τ) → (Y, σ) be an open surjective $\ddot{g}_\alpha$-irresolute map and g : (Y, σ) → (Z, η) be a $\ddot{g}_\alpha$-quotient map. Then their composition g o f: (X,τ) → (Z,η) is a $\ddot{g}_\alpha$-quotient map.

Proof: Let V be any open set in (Z,η). Then $g^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y, σ) since g is a $\ddot{g}_\alpha$-continuous map. Since f is $\ddot{g}_\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (X, τ). This implies $(g \circ f)^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (X, τ). This shows that g o f is a $\ddot{g}_\alpha$-continuous map. Also, assume that $(g \circ f)^{-1}(V)$ is open in (X,τ) for V ⊆ Z, that is, $(f^{-1}(g^{-1}(V)))$ is open in (X, τ). Since f is open $f(f^{-1}(g^{-1}(V)))$ is open in (Y, σ). It follows that $g^{-1}(V)$ is open in (Y,σ), because f is surjective. Since g is a $\ddot{g}_\alpha$-quotient map, V is a $\ddot{g}_\alpha$-open set in (Z,η). Thus g o f: (X,τ) → (Z,η) is a $\ddot{g}_\alpha$-quotient map.

Proposition: 6.2

If h : (X, τ) → (Y, σ) is a $\ddot{g}_\alpha$-quotient map and g : (X, τ) → (Z, η) is a continuous map that is constant on each set $h^{-1}(y)$, for y∈ Y, then g induces a $\ddot{g}_\alpha$-continuous map f : (Y, σ) → (Z, η) such that f o h = g.

Proof: Since g is constant on $h^{-1}(y)$, for each y ∈ Y, the set $g(h^{-1}(y))$ is a one point set in (Z,η). If f(y) denote this point, then it is clear that f is well defined and for each x ∈ X, f (h(x)) = g(x). We claim that f is $\ddot{g}_\alpha$-continuous. For if we let V be any open set in (Z,η), then $g^{-1}(V)$ is an open set in (X,τ) as g is continuous. But $g^{-1}(V) = h^{-1}(f^{-1}(V))$ is open in (X,τ). Since h is $\ddot{g}_\alpha$-quotient map, $f^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y,σ). Hence f is $\ddot{g}_\alpha$-continuous.

Proposition: 6.3

Let f : (X, τ) → (Y, σ) be an strongly $\ddot{g}_\alpha$-open surjective and $\ddot{g}_\alpha$-irresolute map and g : (Y, σ) → (Z, η) be a strongly $\ddot{g}_\alpha$-quotient map then g o f : (X, τ) → (Z, η) is a strongly $\ddot{g}_\alpha$-quotient map.

Proof: Let V be any open set in (Z, η). Then $g^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y,σ) (since g is strongly $\ddot{g}_\alpha$-quotient). Since f is $\ddot{g}_\alpha$-irresolute, $f^{-1}(g^{-1}(V))$ is a $\ddot{g}_\alpha$-open set in (X, τ). Conversely, assume that $(g \circ f)^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (X,τ) for V ⊆ Z. Then $f^{-1}(g^{-1}(V))$ is a $\ddot{g}_\alpha$-open in (X, τ). Since f is strongly $\ddot{g}_\alpha$-open, $f(f^{-1}(g^{-1}(V)))$ is a $\ddot{g}_\alpha$-open set in

(Y, σ). It follows that $g^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y,σ). This gives that V is an open set in (Z,η) (since g is strongly $\ddot{g}_\alpha$-quotient). Thus g o f is a strongly $\ddot{g}_\alpha$-quotient map.

Definition: 6.4

A space (X, τ) is called a $^{\#}T\ddot{g}_\alpha$-space if every $\ddot{g}_\alpha$-closed set in it is closed.

Theorem: 6.5

Let p: (X, τ) → (Y, σ) be a $\ddot{g}_\alpha$-quotient map where (X, τ) and (Y, σ) are $^{\#}T\ddot{g}_\alpha$-spaces. Then f : (Y, σ) → (Z, η) is a strongly $\ddot{g}_\alpha$-continuous if and only if the composite map f o p : (X, τ) → (Z, η) is strongly $\ddot{g}_\alpha$-continuous.

Proof: Let f be strongly $\ddot{g}_\alpha$-continuous and U be any $\ddot{g}_\alpha$-open set in (Z, η). Then $f^{-1}(U)$ is open in (Y, σ). Then $(f \circ p)^{-1}(U) = p^{-1}(f^{-1}(U))$ is $\ddot{g}_\alpha$-open (X,τ). Since (X,τ) is a $^{\#}T\ddot{g}_\alpha$-space, $p^{-1}(f^{-1}(U))$ is open in (X,τ). Thus the composite map is strongly $\ddot{g}_\alpha$-continuous. Conversely let the composite map f o p be strongly $\ddot{g}_\alpha$-continuous. Then for any $\ddot{g}_\alpha$-open set U in (Z,η), $p^{-1}(f^{-1}(U))$ is open in (X,τ). Since p is a $\ddot{g}_\alpha$-quotient map, it implies that $f^{-1}(U)$ is $\ddot{g}_\alpha$-open in (Y,σ). Since (Y,σ) is a $^{\#}T\ddot{g}_\alpha$-space, $f^{-1}(U)$ is open in (Y,σ). Hence f is strongly $\ddot{g}_\alpha$-continuous.

Theorem: 6.6

Let f : (X, τ) → (Y, σ) be a surjective strongly $\ddot{g}_\alpha$-open and $\ddot{g}_\alpha$-irresolute map and g : (Y, σ) → (Z, η) be a $\ddot{g}_\alpha$*-quotient map then g o f is $\ddot{g}_\alpha$*-quotient map.

Proof: Let V be $\ddot{g}_\alpha$-open set in (Z, η). Then $g^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y, σ) because g is a $\ddot{g}_\alpha$*-quotient map. Since f is $\ddot{g}_\alpha$-irresolute, $f^{-1}(g^{-1}(V))$ is a $\ddot{g}_\alpha$-open set in (X, τ). Then g o f is a $\ddot{g}_\alpha$-irresolute. Suppose $(g \circ f)^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (X, τ) for a subset $V \subseteq Z$. That is $(f^{-1}(g^{-1}(V)))$ is $\ddot{g}_\alpha$-open in (X,τ). Since f is strongly $\ddot{g}_\alpha$-open, $f(f^{-1}(g^{-1}(V)))$ is $\ddot{g}_\alpha$ set in (Y, σ). Thus $g^{-1}(V)$ is $\ddot{g}_\alpha$-open set in (Y,σ). Since g is a $\ddot{g}_\alpha$*-quotient map, V is an open set in (Z,η). Hence g o f is a $\ddot{g}_\alpha$*-quotient map.

Proposition: 6.7

Let f : (X, τ) → (Y, σ) be an strongly $\ddot{g}_\alpha$-quotient $\ddot{g}_\alpha$-irresolute map and g : (Y, σ) → (Z, η) be a $\ddot{g}_\alpha$*-quotient map then g o f is a $\ddot{g}_\alpha$*-quotient map.

Proof: Let V be any $\ddot{g}_\alpha$-open set in (Z, η). Then $g^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y, σ) (Since g is $\ddot{g}_\alpha$*-quotient map). We have $f^{-1}(g^{-1}(V))$ is also $\ddot{g}_\alpha$-open set in (X, τ) (Since f is $\ddot{g}_\alpha$-irresolute). Thus, $(g \circ f)^{-1}(V)$ is $\ddot{g}_\alpha$-open set in (X,τ). Hence g o f is αgs-irresolute. Let $(g \circ f)^{-1}(V)$ be a $\ddot{g}_\alpha$-open set in (X, τ) for $V \subseteq Z$. i.e., $f^{-1}(g^{-1}(V))$ is a $\ddot{g}_\alpha$-open in (X, τ). Then $g^{-1}(V)$ is an open set in (Y,σ) because f is a strongly $\ddot{g}_\alpha$-quotient map. This means that $g^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y,σ). Since g is αgs*-quotient map, V is an open set in (Z, η). Thus g o f is a $\ddot{g}_\alpha$*-quotient map.

Theorem: 6.8

The composition of two $\ddot{g}_\alpha$*-quotient maps is $\ddot{g}_\alpha$*-quotient.

Proof: Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be two $\ddot{g}_\alpha$*-quotient map. Let V be a $\ddot{g}_\alpha$-open set in (Z, η). Since g is $\ddot{g}_\alpha$*-quotient, $g^{-1}(V)$ is $\ddot{g}_\alpha$-open set in (Y, σ) . Since f is $\ddot{g}_\alpha$*-quotient, $f^{-1}(g^{-1}(V))$ is $\ddot{g}_\alpha$-open set in (X,τ). That is $(g \circ f)^{-1}(V)$ is $\ddot{g}_\alpha$-open set in (X, τ). Hence g o f is $\ddot{g}_\alpha$-irresolute. Let $(g \circ f)^{-1}(V)$ be $\ddot{g}_\alpha$-open in

(X, τ). Then $f^{-1}(g^{-1}(V))$ is $\ddot{g}_\alpha$-open in (X, τ). Since f is $\ddot{g}_\alpha$*-quotient, $g^{-1}(V)$ is an open in (Y, σ). Then $g^{-1}(V)$ is a $\ddot{g}_\alpha$-open set in (Y, σ). Since g is $\ddot{g}_\alpha$*-quotient, V is open set in (Z, η). Thus g o f is $\ddot{g}_\alpha$*-quotient.

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