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Note on δ –semiopen sets

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ABSTRACT

We characterize preopen sets via δ – semiopen sets. Also, we characterize spaces which are s– closed, semiconnected, semi – T_2, s – Urysohn, s – regular, semi – regular, s –normal and semi – normal by δ – semiopen sets.

Key words and Phrases: *δ – semiopen, δ – open, semiopen, preopen, b – open and β – open sets, δ – semiclosure and δ – semiinterior, s – closed, semiconnected, semi – T_2, s – Urysohn, s – regular, semi – regular, s – normal and semi – normal spaces.*

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1. INTRODUCTION AND PRELIMINARIES:

In 1997, Park, Lee and Son [9] have introduced and studied δ – semiopen sets in topological spaces. Also, in 1997, *Á.Császár* [4] have introduced and studied generalized open subsets of a set *X* defined in terms of monotonic functions $\gamma: \wp(X) \rightarrow \wp(X)$. The family $\mu = \{A \subset X \mid A \subset \gamma(A)\}$ is called the family of γ – open sets which is closed under arbitrary union and $\emptyset \in \mu$. μ is called a *generalized topology*. The family of δ – semiopen sets in a space is a particular kind of generalized topology but is different from the well known family of generalized open sets, namely semiopen sets, preopen sets, b – open sets and β – open sets. In this paper, we characterize spaces which are s – closed, semiconnected, semi – T_2, s – Urysohn, s – regular, semi – regular, s – normal and semi – normal by δ – semiopen sets.

By a space *X*, we will *always* mean the topological space (X, τ). A subset *A* of a space *X* is said to be *regular open* if *A=int(cl(A))* where *int* and *cl* are the *interior* and *closure* operators in the space *X*. The family of all regular open sets is a base for a topology τ_s coarser than τ, which is called the *semiregularization* of the topology τ. Elements of τ_s are called δ – open sets. δ*int* and δ*cl* are the *interior* and *closure* operators in (X, τ_s). A space (X, τ) is said to be *semiregular* if $\tau = \tau_s$. A subset *A* of a space *X* is said to be *α – open* [15](resp. *semiopen* [10], *preopen* [14], *b – open* [1], *β – open* [2]) if $A \subset \text{int}(\text{cl}(\text{int}(A)))$ (resp. $A \subset \text{cl}(\text{int}(A))$, $A \subset \text{int}(\text{cl}(A))$, $A \subset \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$, $A \subset \text{cl}(\text{int}(\text{cl}(A)))$). A subset *A* of a space *X* is said to be *α – closed* (resp. *semiclosed, preclosed, b – closed, β – closed*) if *X –A* is α – open (resp. semiopen, preopen, b – open, β – open). The family of all α – open (resp. semiopen, preopen, b – open, β – open) of a space will be denoted by $\tau^\alpha(X)$, $\sigma(X)$, $\pi(X)$, $b(X)$ and $\beta(X)$. The closure and interior operators of these families are denoted by c_α, i_α, c_σ, i_σ, c_π, i_π. c_b, i_b and c_β, i_β respectively. A subset *A* of a space *X* is said to be *δ – semiopen* [18] if $A \subset cl(\delta int(A))$. *A* is said to be *δ – semiclosed* if *X – A* is *δ – semiopen*. We will denote the family of all *δ – semiopen* sets by $\xi(X)$. For any subset *A* of X, the *δ – semiinterior* of *A*, denoted by $i_\xi(A)$, is given by $i_\xi(A) = \cup \{U \in \xi \mid U \subset A\}$ and the *δ – semiclosure* of *A*, denoted by $c_\xi(A)$, is given by $c_\xi(A) = \cup\{X - U \mid U \in \xi, A \subset X - U\}$. Every δ – open set is a δ – semiopen set and δ – semiopen set is a semiopen set but the converses are not true. The following lemma will be useful in the sequel.

LEMMA: 1.1 *Let X be a space and $A \subset X$. Then the following hold.*

(a) *If A is semiopen, then $cl(A) = c_\alpha(A)$* [19, Lemma 1].

(b) *If A is semiopen, then $cl(A) = \delta cl(A) = c_\alpha(A)$* [8, Proposition 2.2].

(c) *A is preopen if and only if $c_\sigma(A) = int(cl(A))$* [8, Proposition .7(a)].

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2. δ – SEMIOPEN SETS:

We state the following Theorem 2.1 without proof, which gives the relation between the operators c_ξ and i_ξ with other operators which are essential to characterize the spaces already stated.

Theorem: 2.1 *Let X be a space and $A \subset X$. Then the following hold.*

(a) $\delta int(c_\sigma(A)) = \delta int(cl(A))$.

(b) $c_b(i_\xi(A)) = c_\sigma(i_\xi(A))$.

(c) $c_\xi(i_\sigma(A)) = i_\sigma(A) \cup int(\delta cl(int(A)))$.

(d) $c_\beta(i_\xi(A)) = c_\xi(i_\xi(A))$.

(e) $cl(i_\xi(A)) = cl(\delta int(A))$.

(f) $c_\xi(i_\pi(A)) = int(cl(A))$.

(g) $c_\pi(i_\xi(A)) = cl(\delta int(A))$.

(h) $c_\xi(i_\alpha(A)) = int(\delta cl(int(A)))$.

The following Theorem 2.2 gives characterizations of preopen and δ – open sets. Also, it gives properties of semiopen, δ – semiopen and δ – open sets.

Theorem: 2.2 *Let X be a space and $A \subset X$. Then the following hold.*

(a) *If A is δ – semiopen, then* $c_b(A) = c_\sigma(A)$.

(b) *If A is semiopen, then* $c_\xi(A) = c_\sigma(A)$ [16, Theorem 3.3].

(c) *If A is δ – semiopen, then* $c_\xi(A) = c_\sigma(A) = c_b(A)$.

(d) *If A is δ – semiopen, then* $c_\beta(A) = c_\xi(A)$ *and so* $c_\beta(A) = c_b(A) = c_\sigma(A) = c_\xi(A)$.

(e) *A is δ – semiopen if and only if* $cl(A) = cl(\delta int(A))$ *if and only if* $cl(\delta int(A)) = \delta cl(A)$ *if and only if* $cl(\delta int(A)) = c_\pi(A)$ *if and only if* $cl(\delta int(A)) = c_\alpha(A)$.

(f) *A is preopen if and only if* $c_\xi(A) = int(cl(A))$ *if and only if* $int(cl(A)) = c_\sigma(A)$ Lemma 1.1(c).

(g) *If A is α –open, then* $c_\xi(A) = c_\beta(A)$ *and so* $c_\xi(A) = c_\beta(A) = c_b(A) = c_\sigma(A)$.

Proof: (a) The proof follows from Theorem 2.1(b).

(b) By Theorem 2.1(c), $c_\xi(A) = A \cup int(\delta cl(int(A))) = A \cup int(cl(int(A))) = A \cup int(cl(A)) = c_\sigma(A)$.

(c) The proof follows from (a) and (b), since every δ –semiopen set is a semiopen set.

(d) The proof follows from Theorem 2.1(d) and the fact that $\xi(X) \subset \sigma(X) \subset b(X) \subset \beta(X)$.

(e) By Theorem 2.1(e), it follows that $cl(A) = cl(\delta int(A))$. Since *A* is δ – semiopen, $\delta cl(A) = cl(\delta int(A))$. Again, by Theorem 2.1(g), $c_\pi(A) = cl(\delta int(A))$ and by Lemma 1.1(a), $c_\alpha(A) = cl(A)$. Hence the proof follows. The converses are clear.

(f) If *A* is preopen, then by Theorem 2.1(f), $c_\xi(A) = int(cl(A))$ *and so* $c_\xi(A) = A \cup int(cl(A)) = c_\sigma(A)$. The converses are clear.

(g) If *A* is α –open, then by Theorem 2.1(h), $c_\xi(A) = int(\delta cl(int(A))) = A \cup int(cl(int(A))) = c_\beta(A)$ *and so* $c_\xi(A) = c_\beta(A) = c_b(A) = c_\sigma(A)$.

Remark: 2.1 *Theorem 2.2(e) is a generalization of Lemma 1.1(b) and characterizes δ –semiopens. Theorem 2.2(f) is a generalization of Lemma 1.1(c) which characterizes preopen sets in terms of δ – semiopen sets and also shows that*

Theorem 3.3 of [18] is partially true for preopen sets. Theorem 2.2(g) shows that Theorem 2.2 (d) is also true for α – open sets.

A space X is said to be *s – closed* [5] if for every cover $\{V_\alpha \mid \alpha \in \Delta\}$ of X by semiopen sets of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{c_\sigma(V_\alpha) \mid \alpha \in \Delta_0\}$ or equivalently, for every cover $\{V_\alpha \mid \alpha \in \Delta\}$ of X by δ – semiopen sets of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{c_\sigma(V_\alpha) \mid \alpha \in \Delta_0\}$ [16, Theorem 6.1(2)]. The following Theorem 2.3 gives more characterizations of s – closed spaces, the proof of which follows from Theorem 2.2(d).

Theorem: 2.3 *Let X be a space. Then the following are equivalent.*

(a) *X is s – closed.*

(b) *For every cover $\{V_\alpha \mid \alpha \in \Delta\}$ of X by δ – semiopen sets of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{c_b(V_\alpha) \mid \alpha \in \Delta_0\}$.*

(c) *For every cover $\{V_\alpha \mid \alpha \in \Delta\}$ of X by δ – semiopen sets of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{c_\beta(V_\alpha) \mid \alpha \in \Delta_0\}$.*

A space X is said to be *semiconnected* [17] if X cannot be expressed as the disjoint union of two nonempty semiopen sets. The following Theorem 2.4 gives more characterizations of semi -connected spaces.

Theorem: 2.4 *Let X be a space. Then the following are equivalent.*

(a) *X is semiconnected.*

(b) $c_\xi(A) = X$ *for every nonempty δ – semiopen set A.*

(c) $c_\sigma(A) = X$ *for every nonempty δ – semiopen set A.*

(d) $c_b(A) = X$ *for every nonempty δ – semiopen set A.*

(e) $c_\beta(A) = X$ *for every nonempty δ – semiopen set A.*

Proof: (a) and (b) are equivalent by Theorem 6.3 of [16]. (b), (c), (d) and (e) are equivalent by Theorem 2.2(d). A space X is said to be *semi–T_2* [12] if for each pair of distinct points x and y, there exist semiopen sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. Theorem 6.5 of [16] gives some characterizations of semi –T_2 spaces in terms of δ – semiopen sets. The following Theorem 2.5 gives some more characterizations of semi–T_2 spaces in terms of δ – semiopen sets.

Theorem: 2.5 Let *X be a space. Then the following are equivalent.*

(a) *X is semi–T_2.*

(b) *For each pair of distinct points x and y, there exist δ – semiopen sets U and V such that $x \in U$, $y \in V$ and $c_\xi(U) \cap c_\xi(V) = \emptyset$.*

(c) *For each pair of distinct points x and y, there exist δ – semiopen sets U and V such that $x \in U$, $y \in V$ and* $c_\beta(U) \cap c_\beta(V) = \emptyset$.

(d) *For each pair of distinct points x and y, there exist δ – semiopen sets U and V such that $x \in U$, $y \in V$ and $c_b(U) \cap c_b(V) = \emptyset$.*

Proof: (a) and (b) are equivalent by Theorem 6.5(3) of [16]. (b), (c) and (d) are equivalent by Theorem 2.2(d).
A space X is said to be *s –Urysohn* [3] if for each pair of distinct points x and y, there exist semiopen sets U and V such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \emptyset$. In Theorem 6.6 of [16], it is established that a space X is *s –Urysohn* if and only if for each pair of distinct points x and y, there exist δ – semiopen sets U and V such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \emptyset$. The following Theorem 2.6 gives more characterizations.

Theorem: 2.6 *Let X be a space. Then the following are equivalent.*

(a) *X* is *s – Urysohn.*

(b) *For each pair of distinct points x and y, there exist δ – semiopen sets U and V such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \emptyset$.*

(c) *For each pair of distinct points x and y, there exist δ – semiopen sets U and V such that* $x \in U$, $y \in V$ *and* $\delta cl(U) \cap \delta cl(V) = \emptyset$.

(d) *For each pair of distinct points x and y, there exist δ – semiopen sets U and V such that* $x \in U$, $y \in V$ *and* $c_\alpha(U) \cap c_\alpha(V) = \emptyset$.

(e) *For each pair of distinct points x and y, there exist δ – semiopen sets U and V such that* $x \in U$, $y \in V$ *and* $c_\pi(U) \cap c_\pi(V) = \emptyset$.

Proof: (a) and (b) are equivalent by Theorem 6.6 of [16]. (b), (c), (d) and (e) are equivalent by Theorem 2.2(e). A space *X* is said to be *s –regular* [11] (resp. *semi – regular* [6]) if for each closed (resp. semiclosed) set *F* of X and a point $x \notin F$, there exist semiopen sets *U* and *V* such that $x \in U$, $F \subset V$ and $U \cap V = \emptyset$. The following Theorem 2.7 gives more characterizations of s –regular and semi – regular spaces in terms of δ – semiopen sets.

Theorem: 2.7 *Let X be a space. Then the following are equivalent.*

(a) *X is s – regular (resp. semi – regular).*

(b) *For each point* $x \in X$ *and each open (resp. semiopen) set V containing x, there exists a δ – semiopen set U such that* $x \in U \subset c_\sigma(U) \subset V$.

(c) *For each point* $x \in X$ *and each open (resp. semiopen) set V containing x, there exists a δ – semiopen set U such that* $x \in U \subset c_b(U) \subset V$.

(d) *For each point* $x \in X$ *and each open (resp. semiopen) set V containing x, there exists a δ – semiopen set U such that* $x \in U \subset c_\beta(U) \subset V$.

Proof: The proof follows from Theorem 6.7(3) of [16] and Theorem 2.2(d). A space *X* is said to be *s –normal* [15] (resp. *sem –normal* [7]) if for each disjoint closed (resp. semiclosed) sets *F* and *K* of X, there exist semiopen sets *U* and *V* such that $F \subset U$, $K \subset V$ and $U \cap V = \emptyset$. The following Theorem 2.8 gives more characterizations of s–normal and semi– normal spaces in terms of δ – semiopen sets.

Theorem: 2.8 *Let X be a space. Then the following are equivalent.*

(a) *X is s – normal (resp. semi –normal).*

(b) *For each closed (resp. semiclosed) set F and each open (resp. semiopen) set V containing F, there exists a δ – semiopen set U such that* $F \subset U \subset c_\sigma(U) \subset V$.

(c) *For each closed (resp. semiclosed) set F and each open (resp. semiopen) set V containing F, there exists a δ – semiopen set U such that* $F \subset U \subset c_\beta(U) \subset V$.

(d) *For each closed (resp. semiclosed) set F and each open (resp. semiopen) set V containing F, there exists a δ – semiopen set U such that* $F \subset U \subset c_\beta(U) \subset V$.

Proof: The proof follows from Theorem 6.8(3) of [16] and Theorem 2.2(d).

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