International Journal of Mathematical Archive-5(4), 2014, 345-351
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ON δ – H CONTINUOUS FUNCTIONS IN GTS WITH HEREDITARY CLASSES

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(Received on: 28-02-14; Revised & Accepted on: 28-03-14)

ABSTRACT

In this paper, We introduce a new class of functions called δ-H continuous function. We Obtain several characterization and some of their properties. Also, we investigate its relationship with other types of functions.

Keywords: *δ-H cluster points, R-H-open set, θ−H -continuous, strongly δH- continuous, almost-H-continuous, SH−R space, AH −R space with hereditary classes.*

Mathematics Subject Classification 2010: *54A05, 54A10.*

1. INTRODUCTION

In 2007, Császár [3] defined a nonempty class of subsets of a nonempty set, called hereditary class and studied modification of generalized topology via hereditary classes. Also it is studied in [8]. The aim of the paper is to extend the study of the properties of the generalized topologies via hereditary classes. A subfamily μ of $\wp(X)$ is called a generalized topology (GT) [2] if $\emptyset \in \mu$ and μ is closed under arbitrary union. The pair (X, μ) is called a generalized topological space (GTS). Members of μ are called μ-open sets and its complement is called a μ-closed set. The largest μ - open set contained in a subset A of X is denoted by $i_\mu(A)$ [1] and is called the μ - interior of A. The smallest μ- closed set containing A is called the μ - closure of A and is denoted by $c_\mu(A)$ [1].

A generalized topology μ is said to be a quasi-topology if μ is closed under finite intersection. Let X be a nonempty set. A hereditary class H of X is a nonempty collection of subset of X such that $A \subset B$, $B \in H$ implies $A \in H$ [3].

A hereditary class of X is an ideal [8] if $A \cup B \in H$ whenever $A \in H$ and $B \in H$.

An ideal I in a topological space (X,τ) is said to be codense if $\tau \cap I = \{\emptyset\}$. With respect to the generalized topology μ of all μ - open sets and a hereditary class H, for each subset A of X, a subset $A^*(H)$ or simply A^* of X is defined by $A^* = \{x \in X \mid M \cap A \notin H$ for every $M \in \mu$ such that $x \in M\}$ [3].

In this paper, we introduce the notions of δ − H-open sets and δ − H-continuous functions in GTS with hereditary classes. We obtain several characterizations and some properties of δ − H-continuous functions. Also, we investigate the relationships with other related functions.

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2. δ −H−sets

In this section, we introduce δ−H-open sets and the δ −H-closure of a set in a GTS with hereditary class and investigate their basic properties. It turns out that they have similar properties with δ – open and the δ – closure [11].

A subset A of a GTS (X,μ) with hereditary class H is said to be an R−H-open set (resp. regular open set) if $i_\mu(c_\mu{}^*(A)) = A$ (resp. $i_\mu(c_{\mu^*}(A)) = A$. We call a subset A of X is R-H-closed if its complement is R –H - open. Let A be a subset of a GTS (X,μ) with a hereditary class H. A point x of X is called a δ−H-cluster point of A if $A \cap i_\mu(c^*(U)) \neq \emptyset$ for each μ− open neighborhood U of x. The family of all δ − H-cluster points of A is called the δ-H-closure of A and is denoted by $[A]_{\delta-H}$ and a subset A of X is said to be δ-H-closed if $[A]_{\delta-H} = A$. The complement of a δ-H-closed set of X is said to be δ−H-open.

Lemma: 2.1 Let A and B be subsets of a quasi topological space (X,μ) with a hereditary class H. Then, the following properties hold,
(a) $i_\mu(c^*_\mu(A))$ is R-H-open,
(b) If A and B are R –H -open, then $A \cap B$ is R − H-open,
(c) If A is regular open, then A is R − H- open,
(d) If A is R − H-open, then A is delta - H-open,
(e) Every delta − H-open set is the union of a family of R − H- open sets.

Proof:
(a) Let A be a subset of X and $V = i_\mu(c^*{}_\mu(A))$. Then, we have
$i_\mu(c^*{}_\mu(V)) = i_\mu(c^*{}_\mu(i_\mu(c^*{}_\mu(A)))) \subset i_\mu(c^*{}_\mu(c^*{}_\mu(A))) = i_\mu(c^*{}_\mu(A)) = V$ and also $V = i_\mu(V) \subset i_\mu(c^*{}_\mu(V))$.

Therefore, $i_\mu(c^*{}_\mu(V)) = V$.

(b) Let A and B be R-H open. Then,
$A \cap B = i_\mu(c^*{}_\mu(A)) \cap i_\mu(c^*{}_\mu(B)) = i_\mu(c^*{}_\mu(A) \cap c^*{}_\mu(B)) \supset i_\mu(c^*{}_\mu(A \cap B)) = A \cap B$.

Therefore $A \cap B$ is R − H- open.

(c) Let A be regular open. Since $\mu^*_\mu \supset \mu$, we have $A = i_\mu(A) \subset i_\mu(c^*(A)) \subset i_\mu(c_\mu(A)) = A$ and hence $i_\mu(c^*_\mu(A)) = A$. Therefore, A is R − H- open.

(d) Let A be any R − H- open set. For each $x \in A$, $(X - A) \cap A = \emptyset$ and A is R – H- open. Hence $x \notin [X - A]_{\delta-H}$ for each $x \in A$. Therefore $x \notin (X - A)$ implies $x \notin [X - A]_{\delta-H}$. Therefore, $[X - A]_{\delta-H} \subset (X - A)$. Since, $S \subset [S]_{\delta-H}$ for any subset S of X, $[X - A]_{\delta-H} = (X - A)$ and hence A is delta − H-open.

(e) Let A be a δ−H-open set. Then X-A is δ−H-closed and hence $[X-A]_{\delta-H} = (X-A)$. For each $x \in A$, $x \notin [X-A]_{\delta-H}$ and there exists an μ-open neighborhood V_x such that $i_\mu(c^*{}_\mu(V_x)) \cap (X - A) = \emptyset$.

Therefore, $x \in V_x \subset i_\mu(c^*{}_\mu(V_x)) \subset A$, hence $A = \cup\{i_\mu(c^*{}_\mu(V_x)) \mid x \in A\}$. By (a), $i_\mu(c^*_\mu(V_x))$ is R − H- open for each $x \in A$.

Lemma: 2.2 Let A and B be subsets of a quasi topological space (X,μ) with a hereditary class H. Then, the following properties hold:
(a) $A \subset [A]_{\delta-H}$;
(b) If $A \subset B$, then $[A]_{\delta-H} \subset [B]_{\delta-H}$;
(c) $[A]_{\delta-H} = \cap\{F \subset X \mid A \subset F$ and F is δ− H −closed$\}$;
(d) If A is a δ −H-closed set of X for each $\alpha \in \Delta$, then $\cap\{A_\alpha \mid \alpha \in \Delta\}$ is δ − H-closed;
(e) $[A]_{\delta-H}$ is δ− H-closed.

Proof:

(a) For any $x\in A$ and any μ-open neighborhood V of x, we have $\emptyset \neq A\cap V \subset A\cap i_\mu(c_\mu^*(V))$ and hence $x \in [A]_{\delta-H}$

Therefore, $A \subset [A]_{\delta-H}$.

(b) Suppose that $x \notin [B]_{\delta-H}$. There exists a μ- open neighborhood V of x such $\emptyset = i_\mu(c_\mu^*(V))\cap B$,
Hence $i_\mu(c_\mu^*(V))\cap A = \emptyset$. Therefore, $x \notin [A]_{\delta-H}$.

(c) Suppose that $x \in [A]_{\delta-H}$. For any μ- open neighborhood V of x and any $\delta-H$-closed set F containing A, $\emptyset \neq A\cap i_\mu(c_\mu^*(V))\subset F\cap i_\mu(c_\mu^*(V))$ and hence $x\in[F]_{\delta-H} = F$. Therefore $x\in\cap \{F \subset X \mid A\subset F$ and F is $\delta-H-$ closed$\}$. Conversely, suppose that $x \notin [A]_{\delta-H}$. There exists a μ- open neighborhood V of x such that $i_\mu(c_\mu^*(V)) \cap A= \emptyset$. By Lemma 2.1, $X - i_\mu(c_\mu^*(V))$ is a $\delta-H$-closed set which contains A and does not contain x. Therefore, $x \notin \cap\{F \subset X \mid A \subset F$ and F is $\delta-H-$closed$\}$.

(d) For each $\alpha \in\Delta$, $[\cap_{\alpha\in\Delta}A_\alpha]_{\delta-H} \subset [A_\alpha]_{\delta-H} = A_\alpha$ and hence $[\cap_{\alpha\in\Delta}A_\alpha]_{\delta-H}\subset[\cap_{\alpha\in\Delta}A_\alpha]$. By (a) $[\cap_{\alpha\in\Delta}A_\alpha]_{\delta-H} = [\cap_{\alpha\in\Delta}A_\alpha]$. Therefore $\cap_{\alpha\in\Delta}A_\alpha$ is $\delta-H$- closed.

(e) This follows immediately from (c) and (d).

A point x of a quasi topological space (X,μ) with a hereditary class H is called a δ-cluster point of a subset A of X if $i_\mu(c_\mu(V)) \cap A \neq \emptyset$ for every μ− open set V containing x. The set of all δ-cluster points of A is called the δ - closure of A and is denoted by $c_\delta(A)$. If $c_\delta(A) = A$, then A is said to be δ− closed [6]. The complement of a δ− closed set is said to be δ− open. It is well-known that the family of all regular open sets of (X,μ) with a hereditary class H is a basis for a quasi topological space which is weaker than μ. This is called the semi-regularization of μ and is denoted by μ_S is the same as the family of δ - open sets of (X,μ) with a hereditary class H.

Theorem: 2.3 Let (X,μ) be a quasi topological space with a hereditary class H and $\mu_{\delta-H}=\{A\subset X \mid A$ is a $\delta-H$-open set of $(X,\mu)\}$. Then $\mu_{\delta-H}$ is a topology such that $\mu_S\subset\mu_{\delta-H}\subset\mu$.

Proof: By Lemma 2.1, $\mu_S \subset \mu_{\delta-H} \subset \mu$. Next we show that $\mu_{\delta-H}$ is a topology.

(1) It is obvious that $\emptyset, X\in \mu_{\delta-H}$.

(2) Let $V_\alpha \in\mu_{\delta-H}$ for each $\alpha\in\Delta$. Then $X-V_\alpha$ is δ-H-closed for each $\alpha\in\Delta$. By Lemma 2.2, $\cap_{\alpha\in\Delta} (X - V_\alpha)$ is a δ−H-closed and $\cap_{\alpha\in\Delta} (X - V_\alpha) = X - U_{\alpha\in\Delta}V_\alpha$. Hence $U_{\alpha\in\Delta}V_\alpha$ is $\delta-H$-open.

(3) Let $A, B\in\mu_{\delta-H}$. By Lemma 2.1, $A=U_{\alpha\in\Delta_1} A_\alpha$ and $B=U_{\beta\in\Delta_2} B_\beta$, where A_α and B_β, are R-H-open sets for each $\alpha\in\Delta_1$ and $\beta\in\Delta_2$. Thus $A \cap B = U\{A_\alpha \cap B_\beta \mid \alpha\in\Delta_1, \beta\in\Delta_2\}$. Since $A_\alpha \cap B_\beta$ is R-H-open, $A \cap B$ is $\delta-H$-open set by Lemma 2.1.

The following Example 2.4 shows that the δ-H-open set need not be a R-H-open set.

Example: 2.4 Let $X = \{a,b,c,d\}$, $\mu = \{\emptyset,\{d\},\{b,c,d\}\}$ and $H = \{\emptyset,\{c\}\}$. If A= {b, d}. Then $i_\mu c_\mu^*(A) = \{b, c, d\}$ and so $c_\mu^*(i_\mu(A))=\{b, c, d\}$ which implies that A is δ–H-open. But A is not $R-H-$ open, since $i_\mu c_\mu^*(A) = \{b,c,d\}$.

Proposition: 2.5 Let (X,μ) be a quasi topological space with a hereditary class H.
(a) If $H = \{\emptyset\}$ or the hereditary class N of nowhere dense set of (X,μ), then $\mu_{\delta-H} = \mu_S$.
(b) If $H = P(X)$, then $\mu_{\delta-H} =\mu$.

Proof: Let $H = \{\emptyset\}$, then $S^* = c_\mu(S)$ for every subset S of X. Let A be R-H -open. Then $A = i_\mu(c_\mu^*(A)) = i_\mu(A \cup A^*) = i_\mu(c_\mu(A))$ and hence A is regular open. Therefore, every $\delta-H$-open set is δ -open and we obtain $\mu_{\delta-H} \subset\mu_S$. By Theorem 2.1, $\mu_{\delta-H} = \mu_S$. Next, let H = N.It is well know that $S^*= c_\mu(i_\mu(c_\mu(S)))$ for every subset S of X. Let A be any

R−H-open set. Then A is μ-open $A = i_\mu(c_\mu{}^*(A)) = i_\mu(A \cup A^*) = i_\mu(A \cup c_\mu(i_\mu(c_\mu(A))) = i_\mu(c_\mu(i_\mu(c_\mu(A)))) = i_\mu(c_\mu(A))$. Hence A is regular open. Similarly to the case of $H = \{\emptyset\}$, hence $\mu_{\delta-H} = \mu_S$.

(b) Let $H = P(X)$. Then $S^* = \emptyset$ for every subset S of X. Now, let A be any μ-open set of X. Then $A = i_\mu(A) = i_\mu(A \cup A^*) = i_\mu(c_\mu{}^*(A))$ and hence A is R−H-open. By Theorem 2.1, thus $\mu_{\delta-H} = \mu$.

3. δ−H− continuous functions

A function $f: (X, \mu_1, H) \to (Y, \mu_2, I)$ is said to be δ−H-continuous if for each $x \in X$ and each μ−open neighborhood V of f(x), there exists a μ-open neighborhood U of x such that $f(i_\mu(c^*{}_\mu(U))) \subset i_\mu(c^*{}_\mu(V))$.

Theorem: 3.1 For a function $f: (X, \mu_1, H) \to (Y, \mu_2, I)$, the following properties are equivalent:

(a) f is δ−H− continuous,

(b) For each x∈X and each R−H-open set V containing f(x), there exists an R−H-open set containing x such that $f(U) \subset V$,

(c) $f([A]_{\delta-H}) \subset [f(A)]_{\delta-H}$ for every $A \subset X$,

(d) $[f^{-1}(B)]_{\delta\text{-}H} \subset f^{-1}([B]_{\delta\text{-}H})$ for every $B \subset Y$,

(e) For every δ-H- closed set F of Y, $f^{-1}(F)$ is δ−H-closed in X,

(f) For every δ−H-open set V of Y, $f^{-1}(V)$ is δ−H-open in X;

(g) For every R−H-open set V of Y, $f^{-1}(V)$ is R−H-open in X;

(h) For every R−H-closed set F of Y, $f^{-1}(F)$ is R−H-closed in X.

Proof:

(a) ⇒ (b): The proof is obvious.

(b) ⇒ (c): Let $x \in X$ and $A \subset X$ such that $f(x) \in f([A]_{\delta-H})$. Suppose that $f(x) \notin [f(A)]_{\delta-H}$. Then, there exists an R−H-open neighborhood V of f(x) such that $f(A) \cap V = \emptyset$. By (b), there exists an R−H-open neighborhood U of x such that $f(U) \subset V$. Since $f(A) \cap f(U) \subset f(A) \cap V = \emptyset$, $f(A) \cap f(U) = \emptyset$.

Hence $U \cap A \subset f^{-1}(f(U)) \cap f^{-1}(f(A)) = f^{-1}(f(U) \cap f(A)) = \emptyset$. Hence $U \cap A = \emptyset$ and $x \notin [A]_{\delta-H}$.

Therefore $f(x) \notin f([A]_{\delta-H})$. This is a contradiction.

Therefore $f(x) \in [f(A)]_{\delta-H}$.

(c) ⇒ (d): Let $B \subset Y$ such that $A = f^{-1}(B)$. By (c), $f([f^{-1}(B)]_{\delta-H}) \subset [f(f^{-1}(B)]_{\delta-H}) \subset [B]_{\delta-H}$. Therefore $[f^{-1}(B)]_{\delta-H} \subset f^{-1}[f(f^{-1}(B))]_{\delta-H} \subset f^{-1}([B]_{\delta-H})$. Thus $[f^{-1}(B)]_{\delta-H} \subset f^{-1}([B]_{\delta-H})$.

(d) ⇒ (e): Let $F \subset Y$ be δ-H-closed. BY (d), $[f^{-1}(F)]_{\delta-H} \subset f^{-1}([F]_{\delta-H}) = f^{-1}(F)$.
Therefore $f^{-1}(F)$ is δ−H-closed.

(e) ⇒ (f): Let $V \subset Y$ be δ−H−open. Then $Y - V$ is δ−H−closed. By (e) $f^{-1}(Y - V) = X - f^{-1}(V)$ is δ−H -closed. Therefore, $f^{-1}(V)$ is δ−H− open.

(f) ⇒ (g): Let $V \subset Y$ be R−H-open. Since every R−H− open set is δ−H−open, V is δ−H− open, by (f), $f^{-1}(V)$ is δ−H− open.

(g) ⇒ (h): Let $F \subset Y$ be R − H− closed. Then $Y - F$ is R − H− open. By (g) $f^{-1}(Y - F) = X - f^{-1}(F)$ is R − H -open. Therefore $X - f^{-1}(F)$ is δ − H -open. Therefore, $f^{-1}(F)$ is δ − H− closed.

(h) ⇒ (a): Let $x \in X$ and V be a μ-open set containing f(x). Now, $V_0 = i_\mu(c^*(V))$, then by Lemma 2.1 $Y - V_0$ is an R-H-closed set. By (8), $f^{-1}(Y - V_0) = X - f^{-1}(V_0)$ is δ − H -closed set. Therefore, $f^{-1}(V_0)$ is δ − H− open. Since $x \in f^{-1}(V_0)$, by Lemma 2.1 there exists a μ-open neighborhood U of x such that $x \in U \subset i_\mu(c^*(U)) \subset f^{-1}(V_0)$. Hence $f(i_\mu(c^*_\mu(U))) \subset i_\mu(c^*_\mu(V))$. Hence f is a δ − H− continuous function.

Corollary: 3.2 A function $f : (X,\mu_1,H) \to (Y,\mu_2,I)$ is δ-H-continuous if and only if $f : (X,\mu_1,H) \to (Y,\mu_2,I)$ is continuous.

Proof: This is an immediate consequence of Theorem 2.3.

Theorem: 3.3 If $f: (X,\mu_1,H) \to (Y,\mu_2,I)$ and $g : (Y,\mu_2,I) \to (Z,\mu_3,J)$ are δ − H− continuous, then so is $g \circ f : (X,\mu_1,H) \to (Z,\mu_3,J)$.

Proof: It follows immediately from Corollary 3.1.

A function $f: (X, \mu_1, H) \to (Y, \mu_2, I)$ from one GTS (X, μ_1) with a hereditary class H to another (Y, μ_2) with a hereditary class I is said to be strongly θ-H-continuous (resp. θ-H-continuous, almost-H-continuous) if for each $x \in X$ and each μ-open neighborhood V of f(x), there exists a μ− open neighborhood U of x such that $f(c_\mu^*(U)) \subset V$ (resp. $f(c^*_\mu(U)) \subset c^*_\mu(V)$, $f(U) \subset i_\mu(c_\mu^*(V))$). A function $f : (X,\mu_1,H) \to (Y,\mu_2,I)$ is said to be almost-H-open if for each R-H-open set U of X, f(U) is μ− open in Y.

Theorem: 3.4

(a) If $f : (X,\mu_1,H) \to (Y,\mu_2,I)$ is strongly θ-H-continuous and $g : (Y,\mu_2,I) \to (Z,\mu_3,J)$ almost-H-continuous, then $g \circ f : (X,\mu_1,H) \to (Z,\mu_3,J)$ is δ-H-continuous.

(b) The following implications hold:

strongly θ-H-continuous ⇒ δ-H-continuous ⇒ almost- H-continuous.

Proof:

(a) Let $x \in X$ and W be any μ−open set of Z containing $(g \circ f)(x)$.

Since g is almost-H-continuous, there exists a μ− open neighborhood $V \subset Y$ of f(x) such that $g(V) \subset i_\mu(c_\mu^*(W))$. Since f is strongly θ-H-continuous, there exists a μ-open neighborhood $U \subset X$ of x such that $f(c^*_\mu(U)) \subset V$. Hence $g(f(c^*_\mu(U))) \subset g(V)$ and $g(f(i_\mu(c^*_\mu(U)))) \subset g(f(c_\mu^*(U))) \subset g(V) \subset i_\mu(c^*_\mu(W))$. Hence, $g \circ f : (X,\mu_1,H) \to (Z,\mu_3,J)$ is δ-H-continuous.

(b) Let f be strongly θ-H-continuous. Let $x \in X$ and V be any μ-open neighborhood of f(x). Then, there exists a μ-open neighborhood $U \subset X$ of x such that $f(c^*_\mu(U)) \subset V$. Also $f(i_\mu(c^*_\mu(U))) \subset f(c^*_\mu(U)) \subset V$. Since V is μ-open, $f(i_\mu(c^*_\mu(U))) \subset i_\mu(c^*_\mu(V))$. Thus f is δ-H-continuous. Let f be δ-H-continuous.

Now we prove that f is almost H-continuous. Then, for each $x \in X$ and each μ-open neighborhood $V \subset Y$ of f(x), there exists a μ-open neighborhood $U \subset X$ of x such that $f(i_\mu(c^*_\mu(U))) \subset i_\mu(c^*_\mu(V))$. Since $U \subset i_\mu(c^*_\mu(U))$, $f(U) \subset i_\mu(c^*_\mu(V))$.

Hence f is almost H-continuous. A GTS (X,μ) with a hereditary class H is said to be SI-R space if for each $x \in X$ and each μ-open neighborhood V of x, there exists a μ-open neighborhood U of x such that $x \in U \subset i_\mu(c^*_\mu(U)) \subset V$.

Theorem: 3.5 For a function f: $(X,\mu_1,H) \rightarrow (Y,\mu_2,I)$, the following are true:

(a) If Y is an SH-R space and f is δ-H-continuous, then f is continuous.
(b) If X is an SH-R space and f is almost H-continuous, then f is δ-H-continuous.

Proof:
(a) Let Y be an SH-R space. Then, for each μ-open neighborhood V of f(x), there exists a μ-open neighborhood $V_\circ$ of f(x) such that $f(x) \in V \subset i_\mu(c^*_\mu(V)) \subset V$. Since f is δ-H-continuous, there exists a μ-open neighborhood $U_\circ$ of x such that $f(i_\mu(c^*_\mu(U_\circ))) \subset i_\mu(c^*_\mu(V_\circ))$. Thus $f(U_\circ) \subset V$, hence f is continuous.

(b) Let x ∈ X and V be a μ-open neighborhood of f(x). Since f is almost- H continuous, there exists a μ-open neighborhood U of x such that $f(U) \subset i_\mu(c^*(V))$. Since X is an SH-R space, there exists a μ-open neighborhood U_1 of x such that $i_\mu(c_\mu^*(U_1)) \subset U$. Thus $f(i_\mu(c_\mu^*(U_1))) \subset f(U) \subset i_\mu(c_\mu^*(V))$. Therefore f is δ− H− continuous.

Corollary: 3.6 If (X,μ_1) with hereditary class H and (Y,μ_2) with hereditary class I are SH-R spaces, then the following concepts on a function f : $(X,\mu_1,H) \rightarrow (Y,\mu_2,I)$: δ-H-continuity, continuity, almost-H-continuity are equivalent.

Proof: The proof follows from Theorem 3.7. A quasi topological space (X,μ) with a hereditary class H is said to be an AH-R space if for each R-H-closed set F ⊂ X and each x ∉ F, there exist disjoint μ− open sets U and V in X such that x ∈ U and F ⊂ V.

Theorem: 3.7 A quasi topological space (X,μ) with a hereditary class H is an AH-R space if and only if each x ∈ X and each R-H- open neighborhood V of x, there exists an R-H- open neighborhood U of x such that $x \in U \subset c^*_\mu(U) \subset c_\mu(U) \subset V$.

Proof: Suppose (X,μ) with a hereditary class H is an AI-R space. Let x ∈ V and V be R-H- open. Then $\{x\} \cap (X-V) = \emptyset$. Since X is an AI-R space, there exist μ-open sets U_1 and U_2 containing x and X-V respectively, such that $U_1 \cap U_2 = \emptyset$. Then $c_\mu(U_1) \cap U_2 = \emptyset$. and hence $c_\mu^*(U_1) \subset c_\mu(U_1) \subset (X-U_2) \subset V$. Thus $x \in U_1 \subset c^*(U_1) \subset c_\mu(U_1) \subset V$ and we have $U_1 \subset i_\mu(c_\mu^*(U_1)) \subset c_\mu^*(U_1)$.

Let $i_\mu(c_\mu^*(U_1)) = U$. Thus $c\mu(U_1) = c\mu(i\mu(c\mu^*(U_1))) \subset c\mu(c\mu^*(U_1)) \subset c\mu(c\mu(U_1)) = c\mu(U_1) \subsetneq c\mu(U)$ and $U_1 \subset U \subset c\mu^*(U) \subset c\mu^*(U_1) \subset c\mu(U_1) \subset V$. Therefore, there exists an R-H-open set U such that $x \in U \subset c^*(U) \subset c_\mu(U) \subset V$. Conversely, let x ∈ X and an R-H- closed set F such that x ∉ F. Then, X-F is an R-H-open neighborhood of x.

By hypothesis, there exists an R -H-open neighborhood V of x such that $x \in V \subset c^*(V) \subset c_\mu(V) \subset X-F$. Thus $F \subset X - c_\mu(V) \subset (X - c^*(V)) \subset$, where $X - c_\mu(V)$ is a μ−open set.

Also, we have $V \cap (X - c_\mu(V)) = \emptyset$ and V is μ−open. Therefore, X is an AH-R space.

Theorem: 3.8 For a function f: $(X,\mu_1,H) \rightarrow (Y,\mu_2,I)$, the following are hold:
(a) If Y is an AI-R space and f is θ-H-continuous, then f is δ-H-continuous.
(b) If X is an AI-R space, Y is an SH-R space and f is δ–H-continuous, then f is strongly θ–H-continuous.

Proof:
(a) Let Y be an AH-R space. Then for each x ∈ X and each R-H- open neighborhood V of f(x), there exists an R-H-open neighborhood $V_\circ$ of f(x) such that $f(x) \in V \subset c^*(V) \subset V$. Since f is θ-H-continuous, there exists a μ−open

neighborhood U of x such that $f(c\mu\, U)) \subset c_\mu(V_\circ)$. Hence $f(i\,(c\mu^*(U))) \subset f(c\mu^*(U)) \subset c\mu^*(V_\circ) \subset V$ and thus $f(i_\mu(c\mu^*(U))) \subset V$. By Theorem 3.1, f is δ− H− continuous.

(b) Let X be an AH-R space, Y an SH-R space. For each $x \in X$ and each μ-open neighborhood V of f(x), there exists a μ-open set $V_\circ$ such that $f(x) \in V_\circ \subset i_\mu(c_\mu^*(V_\circ)) \subset V'$, since Y is an SH−R space. Since f is δ−H− continuous, there exists a μ−open set U of x such that $f(i_\mu(c_\mu^*(U))) \subset i_\mu(c_\mu^*(V_\circ))$. By Lemma 2.1, $i_\mu(c_\mu^*(U))$ is R-H-open and since X is an AI-R space, by Theorem 3.7. there exists an R-H-open set $U_\circ$ such that $x \in V_\circ \subset c_\mu^*(U_\circ) \subset i_\mu(c_\mu^*(U))$. But every R-H-open set is μ− open, hence U is μ− open. Also, $f(c^*(U)) \subset V$ Hence f is strongly θ-H− continuous.

Theorem: 3.9 If a function $f: (X, \mu_1, H) \to (Y, \mu_2, I)$ is θ − H−continuous and almost- H−open, then f is δ − H−continuous.

Proof: Let $x \in X$ and V be a μ− open neighborhood of f(x). Since f is θ − H−continuous, there exists a μ−open neighborhood U of x such that $f(c_\mu^*(U)) \subset c_\mu^*(V)$. Hence $f(i_\mu(c^*(U))) \subset c_\mu^*(V)$. Since f is almost- H−open, $f(i_\mu(c_\mu^*(U))) \subset i_\mu(c_\mu^*(V))$. This shows f is strongly θ − H−continuous.

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Source of support: Nil, Conflict of interest: None Declared