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ON a-OPEN SETS IN A TOPOLOGICAL SPACE

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ABSTRACT

The purpose of this research article is to explain the meaning of α-open sets, which would be more understandable to the readers.

I. TOPOLOGICAL SPACE:

Let X be a non-empty set. A class $\mathcal{T}$ of subsets of X is a topology on X, iff $\mathcal{T}$ satisfies the following axioms,

(i) X and ϕ belong to $\mathcal{T}$.

(ii) The union of any numbers of sets in $\mathcal{T}$ belongs to $\mathcal{T}$

(iii) The interaction of any two sets in $\mathcal{T}$ belongs to $\mathcal{T}$.

The members of $\mathcal{T}$ are then called $\mathcal{T}$-open sets (or open sets) and pair $(X, \mathcal{T})$ is called a topological space.

II. INTERIOR OF SET:

Let A be a subset of a topological space X. Any point $p\in A$ is said to be interior of A, if p belongs to an open set G contained in A, i.e. $p\in G\subset A$. The set of interior points of A is denoted by $int\,(A)$ or $A°$, which is called the interior of A.

III. CLOUSER OF SET:

Let A be a subset of a topological space X. The closure of A is defined as the interaction of all closed super sets of A. The Closure of A is denoted by $Cl(A)$ or $\overline{A}$.

IV. α-OPEN SET (AND α-CLOSED SET):

Let A be a subset of a topological space $(X, \mathcal{T})$, then A is said to be α-open set if $A \subseteq A^{\circ-\circ}$. Complement of α-open set is called α-closed set, such that, $A^{-\circ-} \subseteq A$.

Now, Let $X = \{a, b, c, d, e\}$ be a non-empty set and

$$\mathcal{T} = \{\phi, X, \{a, b, c\}, \{d, e, c\}, \{d, e\}, \{c\}\}$$

is a collection of subset of X.

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A. SHOW THAT $\mathcal{T}$ IS A TOPOLOGY DEFINED ON *X*.

(i) ϕ, $X \in \mathcal{T}$

(ii) $\phi \cup X = X \in \mathcal{T}$
$X \cup \{a, b, c\} = X \in \mathcal{T}$
$\{a, b, c\} \cup \{d, e\} = \{a, b, c, d, e\} = X \in \mathcal{T}$
$\{d, e\} \cup \{c\} = \{d, e, c\} \in \mathcal{T}$
$\{c\} \cup \{d, e, c\} = \{d, e, c\} \in \mathcal{T}$

(iii) $\phi \cap X = \phi \in \mathcal{T}$
$X \cap \{a, b, c\} = \{a, b, c\} \in \mathcal{T}$
$\{a, b, c\} \cap \{d, e\} = \phi \in \mathcal{T}$
$\{d, e\} \cap \{c\} = \phi \in \mathcal{T}$
$\{c\} \cap \{d, e, c\} = \{c\} \in \mathcal{T}$

Here, we see that all three conditions for topological space are satisfied, it means that $\mathcal{T}$ is a topology on *X*.

Now, we have all possible subsets of $X = 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, which are given below.
X, ϕ, {a}, {b}, {c}, {d}, {e}, {a, b}, {b, c}, {c, d}, {d, e}, {a, e}, {a, c}, {a, d}, {b, d}, {b, e}, {c, e}, {a, b, c}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {b, c, d}, {b, c, e}, {c, d, e}, {d, e, a}, {d, e, b}, {a, b, c, d}, {a, b, c, e}, {b, c, d, e}, {c, e, d, a}, {d, e, a, b}

B. VERIFICATIONS OF α-OPEN SETS:

As given, $X = \{a, b, c, d, e\}$
And $\mathcal{T} = \{\phi, X, \{a, b, c\}, \{d, e\}, \{c\}, \{d, e, c\}\}$

So that we have

Open sets: ϕ, X, {a, b, c}, {d, e}, {c}, {d, e, c}

Closes sets: X, ϕ, {d, e}, {a, b, c}, (a, b, d, e}, {a, b}

Now as per definition of α-open set, here we are verifying for all (32) subsets of *X*. Let *A* be a subset of a topological space $(X, \mathcal{T})$, then

(i) Let A = ϕ, so that $A^{\circ-\circ} = \phi^{\circ-\circ} = \phi$
Hence $A \subseteq A^{\circ-\circ}$ (i.e. α-open set).

(ii) Let A = X, so that $X^{\circ-\circ} = X$
Hence $A \subseteq A^{\circ-\circ}$ (i.e. α-open set)

(iii) Let A = {a}, so that $A^{\circ-\circ} = \{a\}^{\circ-\circ} = \phi^{-\circ} = \phi$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

(iv) Let A = {b}, so that $A^{\circ-\circ} = \{b\}^{\circ-\circ} = \phi^{\circ} = \phi$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

(v) Let A = {c}, so that $A^{\circ-\circ} = \{c\}^{\circ-\circ} = \{c\}^{-\circ} = \{c\}^{\circ} = \{c\}$
Hence $A \subseteq A^{\circ-\circ}$ (i.e. α-open set).

(vi) Let A = {d}, so that $A^{\circ-\circ} = \{d\}^{\circ-\circ} = \phi^{\circ} = \phi$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

(vii) Let A = {e}, so that $A^{\circ-\circ} = \{e\}^{\circ-\circ} = \phi^{\circ} = \phi$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

(viii) Let A = {a, b}, so that $A^{\circ-\circ} = \{a, b\}^{\circ-\circ} = \phi^{-\circ} = \phi$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

(ix) Let A = {b, c}, so that $A^{\circ-\circ}$ = $\{b, c\}^{\circ-\circ}$ = $\phi^{-\circ}$ = ϕ
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(x) Let A = {c, d}, so that $A^{\circ-\circ}$ =$\{c, d\}^{\circ-\circ}$ = $\{c\}^{-\circ}$ = $\{c\}^{\circ}$ = {c}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xi) Let A = {d, e}, so that $A^{\circ-\circ}$ = $\{d, e\}^{\circ-\circ}$ = $\{d, e\}^{-\circ}$ = $\{d, e\}^{\circ}$ = {d, e}
Hence A $\subseteq$ $A^{\circ-\circ}$ (i.e. α-open set).

(xii) Let A = {a, e}, so that $A^{\circ-\circ}$ = $\{a, e\}^{\circ-\circ}$ = $\phi^{-\circ}$ = ϕ
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xiii) Let A = {a, c}, so that $A^{\circ-\circ}$ = $\{a, c\}^{\circ-\circ}$ = $\{c\}^{-\circ}$ = $\{a, b, c\}^{\circ}$ = {a, b, c}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. α-open set).

(xiv) Let A = {a, d}, so that $A^{\circ-\circ}$ = $\{a, d\}^{\circ-\circ}$ = $\phi^{-\circ}$ = ϕ° = ϕ
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xv) Let A = {b, d}, so that $A^{\circ-\circ}$ = $\{b, d\}^{\circ-\circ}$ = $\phi^{-\circ}$ = ϕ° = ϕ
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xvi) Let A = {b, e}, so that $A^{\circ-\circ}$ = $\{b, e\}^{\circ-\circ}$ = $\phi^{-\circ}$ = ϕ° = ϕ
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xvii) Let A = {c, e}, so that $A^{\circ-\circ}$ =$\{c, e\}^{\circ-\circ}$ = $\{c\}^{-\circ}$ = $\{a, b, c\}^{\circ}$ = {a, b, c}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xviii) Let A = {a, b, c}, so that $A^{\circ-\circ}$ = $\{a, b, c\}^{\circ-\circ}$ = $\{a, b, c\}^{-\circ}$ = $\{a, b, c\}^{\circ}$ ={a, b, c}
Hence A $\subseteq$ $A^{\circ-\circ}$ (i.e. α-open set).

(xix) Let A = {a, b, d}, so that $A^{\circ-\circ}$ = $\{a, b, d\}^{\circ-\circ}$ = $\phi^{-\circ}$ = ϕ° = ϕ
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xx) Let A = {a, b, e}, so that $A^{\circ-\circ}$ = $\{a, b, e\}^{\circ-\circ}$ = $\phi^{-\circ}$ = ϕ° = ϕ
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxi) Let A = {a, c, d}, so that $A^{\circ-\circ}$ = $\{a, c, d\}^{\circ-\circ}$ = $\{c\}^{-\circ}$ = $\{a, b, c\}^{\circ}$ = {a, b, c}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxii) Let A = {a, c, e}, so that $A^{\circ-\circ}$ =$\{a, c, e\}^{\circ-\circ}$ = $\{c\}^{-\circ}$ = $\{a, b, c\}^{\circ}$ = {a, b, c}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxiii) Let A = {b, c, d}, so that $A^{\circ-\circ}$ = $\{b, c, d\}^{\circ-\circ}$ = $\{c\}^{-\circ}$ = $\{a, b, c\}^{\circ}$ = {a, b, c}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxiv) Let A = {b, c, e}, so that $A^{\circ-\circ}$ = $\{b, c, e\}^{\circ-\circ}$ = $\{c\}^{-\circ}$ = $\{a, b, c\}^{\circ}$ = {a, b, c}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxv) Let A = {c, d, e}, so that $A^{\circ-\circ}$ =$\{c, d, e\}^{\circ-\circ}$ = $\{c, d, e\}^{-\circ}$ = X° = X
Hence A $\subseteq$ $A^{\circ-\circ}$ (i.e. α-open set).

(xxvi) Let A = {d, e, a}, so that $A^{\circ-\circ}$ = $\{d, e, a\}^{\circ-\circ}$ =$\{d, e\}^{-\circ}$ = $\{d, e\}^{\circ}$ = {d, e}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxvii) Let A = {d, e, b}, so that $A^{\circ-\circ}$ = $\{d, e, b\}^{\circ-\circ}$ =$\{d, e\}^{-\circ}$ = $\{d, e\}^{\circ}$ = {d, e}
Hence A $\not\subseteq$ $A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxviii) Let A = {a, b, c, d},

so that $A^{\circ-\circ} = \{a, b, c, d\}^{\circ-\circ} = \{a, b, c\}^{-\circ} = \{a, b, c\}^{\circ} = \{a, b, c\}$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxix) Let A = {a, b, c, e},
so that $A^{\circ-\circ} = \{a, b, c, e\}^{\circ-\circ} = \{a, b, c\}^{-\circ} = \{a, b, c\}^{\circ} = \{a, b, c\}$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

(xxx) Let A = {b, c, d, e},
so that $A^{\circ-\circ} = \{b, c, d, e\}^{\circ-\circ} = \{d, e, c\}^{-\circ} = X^{\circ} = X$
Hence $A \subseteq A^{\circ-\circ}$ (i.e. α-open set).

(xxxi) Let A = {c, e, d, a},
so that $A^{\circ-\circ} = \{c, e, d, a\}^{\circ-\circ} = \{d, e, c\}^{-\circ} = X^{\circ} = X$
Hence $A \subseteq A^{\circ-\circ}$ (i.e. α-open set).

(xxxii) Let A = {d, e, a, b},
so that $A^{\circ-\circ} = \{d, e, a, b\}^{\circ-\circ} = \{d, e\}^{-\circ} = \{d, e\}^{\circ} = \{d, e\}$
Hence $A \not\subseteq A^{\circ-\circ}$ (i.e. NOT α-open set).

Therefore, we have 9, α-open sets, which are the subsets of the set X = {a, b, c, d, e}

V. CONCLUSION:

Here we find 9, α-open sets out of 32 subsets of X = {a, b, c, d, e} with $\mathcal{T}$={ϕ, X, {a, b, c}, {d, e}, {c}, {d, e, c}} as given below:

ϕ, X, {c}, {a, c}, {d, e}, {a, b, c}, {c, d, e} {b, c, d, e}, {c, e, d, a}, Total: 09.

Also, it is easy to understand that other 23 subsets of X, are NOT α-open sets of the topological space (X, $\mathcal{T}$).

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