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COMMON FIXED POINT THEOREMS IN CONE G- METRIC SPACE WITH W- DISTANCE

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ABSTRACT

The purpose of this paper is to introduce the concept of w-distance in Cone G-metric space and then we prove some fixed point theorems for weakly contractive, weakly Caristi type mappings using the concept of w-distance. Also we provide example in support of our theorem.

Mathematics Subject classification: *47H10, 54H25.*

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INTRODUCTION

In 1963, Gahler [3, 4] introduced the concept of 2-metric space akin to the metric space (X, d). But different authors have proved that there is no relation between these two functions. For instance, Ha *et al*. in [5] showed that 2-metric need not be continuous function. Further there is no easy relationship between the results obtained in the two settings. Motivated by the measure of nearness between two or more objects with respect to a specific property or characteristic, also called the parameter of nearness, Bapure Dhage in1992 introduced in his Ph.D. thesis a new class of generalized metric space called *D*-metric space [1, 2]. He claimed that *D*-metrics provide a generalization of ordinary metric functions and presented several fixed point results. Recently, Zead Mustafa and Brailey Sims [9, 10] introduced a new structure of generalized metric spaces, which are called G-metric spaces as generalization of metric space (X, d), to develop and introduce a new fixed point theory for a various mappings in this new structure and demonstrated that most of the claims concerning the fundamental topological structure of *D*-metric space by Dhage [1, 2] are incorrect.

Guang and Xian [6] generalized the concept of metric spaces, replacing the set of real numbers by an ordered Banach space defining in this way a cone metric space. The metric space with w-distance was introduced by Osama Kada, Tomonari Suzuki, Wataru Takahashi, and Naoki Shioji [6, 7, 11]. H. Lakzian and F. Arabyani [8] composed these concepts together and introduced cone metric space with w-distance and proved few fixed point theorems. Motivated by G-metric space and Cone metric space, we define the notion of w-distance cone G-metric space and then prove common fixed point theorems for weakly contractive and weakly Caristi type mappings using w-distance in this newly defined space.

DEFINITIONS AND PRELIMINARIES

Definition: 1.1[6] Let E be a real Banach space and P a subset of E. P is called a cone if and only if
(i) P is closed, non-empty and $P \neq \{0\}$;
(ii) $ax + by \in P$ for all $x, y \in P$ and non-negative real numbers a, b;
(iii) $P \cap (-P) = \{0\}$.

For a given cone $P \subseteq E$, we can define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y-x \in P$. $x < y$ will stand for $x \leq y$ and $x \neq y$, while $x \ll y$ will stand for $y-x \in \text{int } P$, where int P denotes the interior of P. The cone P is called normal if there is a number $M > 0$ such that for all $x, y \in E$, $0 \leq x \leq y$ implies

$$\|x\| \leq M\|y\|.$$

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The least positive number satisfying above is called the normal constant of P ([1]). The cone P is called regular if every increasing sequence which is bounded above is convergent. That is, if $\{x_n\}_{n\geq 1}$ is a sequence such that $x_1 \leq x_2 \leq \ ... \leq y$ for some $y \in$ E, then there is $x \in E$ such that $\lim_{n\to\infty}\|x_n - x\|=0$. Equivalently the cone P is regular if and only if every decreasing sequence which is bounded below is convergent.

Lemma: 1.1. [6] Every regular cone is normal.

Throughout, we denote by E, a Banach space, P a cone in E with int P $\neq \phi$ and $\leq$ partially ordering with respect to P.

Definition: 1.2. Let X be a nonempty set, and let G: $X\times X\times X \rightarrow E$ be a function satisfying:
(G1) G(x, y, z) =0 if x = y = z
(G2) 0<G(x, x, y); for all x, $y \in X$, with $x \neq y$,
(G3) G(x, x, y) $\leq$ G(x, y, z), for all x, y, $z \in X$ with $z \neq y$,
(G4) G(x, y, z) = G(x, z, y) = G(y, z, x) = …., (symmetry in all three variables), and
(G5) G(x, y, z) $\leq$ G(x, a, a) + G(a, y, z), for all x, y, z, $a \in X$, (rectangle inequality).

Then the function G is called a generalized metric, or, more specifically a cone G-metric on X, and the pair (X, G) is a cone G-metric space. A cone G-metric space is generalization of G-metric space [22]. We use the following proposition in cone G- metric space same as in G-metric space.

We use following proposition in Cone G-metric space.

Proposition: 1.1([9]) Let (X, G) be a G-metric space. Then for any x, y, z and $a \in X$ it follows that:
(1) If G(x, y, z) = 0, then x = y = z.
(2) G(x, y, z) $\leq$ G(x, x, y) + G(x, x, z),
(3) G(x, y, y) $\leq$ G(y, x, x),
(4) G(x, y, z) $\leq$ G(x, a, z) +G(a, y, z)
(5) G(x, y, z) $\leq$ 2/3.(G(x, y, a) +G(x, a, z) +G(a, y, z)),
(6) G(x, y, z) $\leq$ (G(x, a, a) +G(y, a, a) +G(z, a, a))

Example: 1.1 Let E = R^2, P = {(x, y) $\in$ E: x, y $\geq$ 0}, X = R and G: $X \times X \times X \rightarrow E$ defined by
G(x, y, z) = $(|x-y|+ |y-z|+|z-x|, \alpha\, [|x-y|+ |y-z|+|z-x|])$ where $\alpha \geq 0$ is constant. Then (X, G) is a cone G- metric space.

Solution:
(i) G(x, y, z) = 0
If $(|x-y|+ |y-z|+|z-x|, \alpha[|x-y|+ |y-z|+|z-x|])$

If $(|x-y|+ |y-z|+|z-x| = 0$ and $\alpha[|x-y|+ |y-z|+|z-x|]= 0$

If x = y = z.

(ii) G(x, x, y) = $(|x-x|+ |x-y|+|y-x|, \alpha[|x-x|+ |x-y|+|y-x|])>0$
$= (2|x-y|, 2\alpha|x-y|)> 0.$

(iii) G(x, x, y) $\leq$ G(x, y, z), for all x, y, $z \in X$ with $z \neq y$,

This is clearly true.

(iv) G(x, y, z) = G(x, z, y) = G(y, z, x) = …., (symmetry in all the three variables)

(v) G(x, y, z) $\leq$ G(x, a, a) + G(a, y, z),

$$G(x, a, a) + G(a, y, z) = (|x-a| + |a-a|+|a-x|, \alpha[|x-a|+ |a-a|+|a-x|]) + (|a-y|+ |y-z|+|z-a|, \alpha[|a-y|+ |y-z|+|z-a|])$$

G(x, a, a) + G(a, y, z) = $(|x-a|+|a-x|, \alpha[|x-a|+|a-x|]) + (|a-y|+|y-z|+|z-a|, \alpha[|a-y|+|y-z|+|z-a|])$

$$
\begin{aligned}
G(x, y, z) &= (|x-y|+|y-z|+|z-x|, \alpha[|x-y|+|y-z|+|z-x|]) \\
&= (|x-a+a-y|+|y-z|+|z-a+a-x|, \alpha[|x-a+a-y|+|y-z|+|z-a+a-x|]) \\
&\leq (|x-a|+|a-y|+|y-z|+|z-a|+|a-x|, \alpha[|x-a|+|a-y|+|y-z|+|z-a|+|a-x|])
\end{aligned}
$$

Therefore, G(x, y, z) $\leq$ G(x, a, a) + G(a, y, z)

Definition: 1.3 Let X be a cone G- metric space with metric G. Then a mapping p: $X \times X \times X \to E$ is called w-distance on X if it satisfies the following:
(a) p(x, y, z) $\geq$ 0 for all x, y, z$\in$X,
(b) p(x, y, z) $\leq$ p(x, a, a) + p(a,y, z) for all x, y, z ,a $\in$ X
(c) p(x, y, .) $\to$ E is lower semi-continuous for all x$\in$X.
(d) for any 0<< α, there exists 0 << β such that p(x, a, a)<< β and p(a, y, z) << β imply G(x, y, z) << α , where α, β$\in$E.

Lemma: 1.2 Let X be a cone G-metric space with metric G, let p be a w−distance on X and let f be a function from X into E that 0 $\leq$ f(x) for any x$\in$ X.

Then a function q from X × X ×X into E given by q(x, y, z) = f(x) +f(y) + p(x, y, z) for each (x, y, z) $\in$ X × X ×X is also a w−distance.

Proof:
(i) q(x, y, z) $\geq$ 0.

(ii) For every x, y, z, a $\in$ X,
$$
\begin{aligned}
q(x, y, z) &= f(x) +f(y) +p(x, y, z) \\
&\leq f(x)+f(a)+p(x, a, a)+f(y)+f(a)+p(a, y, z), \\
&= q(x, a, a) + q(a, y, z).
\end{aligned}
$$

(iii) It is obvious that the function f is lower semi-continuous.

(iv) Let α $\in$ E with 0 << α be fixed <u>then</u> since p is w−distance on X, there exists β$\in$E with 0 <<β such that

p(x, a, a) << β, q(a, y, z)<< β imply G(x, y, z) <<α. So assume

q(x, a, a) << β, q(a, y, z) << β we have p(x, a, a) $\leq$ f(x)+f(a)+p(x, a, a) = q(x, a, a)<< β,

p(a, y, z) $\leq$ f(a)+f(y)+p(a, y, z) = q(a, y, z)<< β. So p(x, a, a) << β, p(a, y, z) <<β and it imply G(x, y, z) << α.

Example: 1.2 Let X = **R** be a cone G-metric space with metric G defined by G(x, y, z) = $(|x-y|+|y-z|+|z-x|)$ $\forall$ x, y, z$\in$**R**. Then a function p: $\mathbf{R}^3 \to [0,\infty)$ defined by p(x, y, z) = $|y-z|$ for all x, y, z$\in$**R** is a w- distance on **R.**

Solution:
(a) Clearly, p(x, y, z) $\geq$ 0 for all x, y, z$\in$X,
(b) Clearly, p(x,y, .) $\to$E is lower semi-continuous for all x$\in$X.
(c) p(x, y, z) $\leq$ p(x, a, a) + p(a,y, z) for all x, y, z ,a $\in$X

As p(x, y, z) = $|y-z|$, p(x, a, a) = $|a-a|$ and p(a, y, z) = $|y-z|$

Therefore, we have above inequality.

(d) for any 0<< α, put $\beta = \frac{\alpha}{2}$ there exists 0 << β such that p(x, a, a)<< β and p(a,y,z) << β imply G(x, y, z) << α , where α, β$\in$E.

If p(x, a, a) << β and p(a, y, z) << β, then $|a-a|$ << β (by remark 1.5), $|y-z|$ << β, this implies G(x, y, z) << β+β = α

Definition: 1.4 Let X be a cone G-metric space with metric G, let p be a w−distance on X, $x \in X$ and $\{x_n\}$ a sequence in X, then
(a) $\{x_n\}$ is called a p-**Cauchy** sequence whenever for every $\alpha \in E$, $0 << \alpha$, there is a positive integer N such that, for all m, n, $l \geq N$, $p(x_n, x_m, x_l) << \alpha$.

(b) A sequence $\{x_n\}$ in X is called a p-**convergent** to a point $x \in X$ whenever for every $\alpha \in E$, $0 << \alpha$, there is a positive integer N such that, for all $n \geq N$, $p(x, x_n, x_m) << \alpha$. Note that by lower semi-continuous p, for all $n \geq N$, $p(x_n, x_n, x) << \alpha$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$.

(c) (X, G) is a complete cone G-metric space with w-distance if every Cauchy sequence is p-convergent.

Lemma: 1.3[6] There is not normal cone with normal constant M<1.

Proposition: 1.1[6] For each k>1, there is a normal cone with normal constant M>k. H. Lakzian and F. Arabyani [8] introduced cone metric space with w-distance and proved the following fixed point theorems:

Theorem: 1.1 Let (X, d) be a cone metric space with w-distance p on X and the mapping $T: X \to X$. Suppose that there exist $r \in [0, 1)$ such that

$$p(Tx, T^2x) \leq r\, p(x, Tx),$$

for every $x \in X$ and that

$$\inf\{p(x,y) + p(x, Tx): x \in X\} > 0,$$

for every $y \in X$ with $y \neq Ty$. Then there is $z \in X$ such that $z = Tz$. Moreover, if P is a normal cone with normal constant M and $v = Tv$ then $p(v, v) = 0$.

Theorem: 1.2 Let (X, d) be a complete cone metric space with w-distance p. Let P be a normal cone on X. Suppose a mapping $T: X \to X$ satisfy the contractive condition

$$p(Tx, Ty) \leq k\, p(x, y)$$

for all $x, y \in X$, where $k \in [0, 1)$ is a constant. Then, T has a unique fixed point in X. For each $x \in X$, the iterative sequence $\{T^n(x)\}_{n\geq 1}$ converges to the fixed point.

Now, we generalize the Theorems 1.1 and 1.2. to Cone G-metric space in the form of following:

Theorem: 1.3 Let X be a complete cone G- metric space with metric G, 'p' a w-distance on X and T a mapping from X into itself. Suppose that there exist $r \in [0, 1)$ such that

$$p(Tx, T^2x, T^3x) \leq r\, p(x, Tx, T^2x),$$

for every $x \in X$ and that

$$\inf\{p(x, x, z) + p(x, Tx, T^2x): x \in X\} > 0,$$

for every $z \in X$ with $z \neq Tz$. Then there is $w \in X$ such that $w = Tw$. Moreover, if P is a normal cone with normal constant M and $v = Tv$ then $p(v, v, v) = 0$.

Proof: Let $u \in X$ and define $u_n = T^n u$ for any $n \in N$. Then we have, for any $n \in N$,

$$\begin{aligned} p(u_n, u_{n+1}, u_{n+2}) &\leq r\, p(u_{n-1}, u_n, u_{n+1}) \leq r\,[p(u_{n-1}, u_n, u_n) + p(u_n, u_n, u_{n+1})] \\ &\leq r\,[p(u_{n-1}, u_n, u_n)] + r[p(u_n, u_{n+1}, u_{n+2})] \end{aligned}$$

Because $p(u_n, u_n, u_{n+1}) \leq p(u_n, u_{n+1}, u_{n+2})$ (1.3.1)

$$p(u_n, u_{n+1}, u_{n+2}) \leq q\,[p(u_{n-1}, u_n, u_n)]$$

Where $\left(\dfrac{r}{1-r} = q\right)$ and $q < 1$, since $0 < r < 1$

Hence (1.3.1) becomes

$p(u_n, u_n, u_{n+1}) \leq q[p(u_{n-1}, u_n, u_n)]$

So if m>n

$$p(u_n, u_n, u_m) \leq p(u_n, u_n, u_{n+1}) + p(u_{n+1}, u_{n+1}, u_{n+2}) + p(u_{n+2}, u_{n+2}, u_{n+3}) + \ldots\ldots + p(u_{m-1}, u_{m-1}, u_m)$$

$$\leq (q^n + q^{n+1} + \ldots\ldots\ldots + q^{m-1})p(u_o, u_1, u_1)$$

$$\leq \frac{q^n}{1-q} p(u_o, u_1, u_1).$$

Thus $\{u_n\}$ is a cauchy sequence because if let $\alpha \in E$ with $0 \ll \alpha$ be given, choose $\beta \in E$ with $0 \ll \beta$ such that $\alpha + N_\beta(0) \subseteq P$ where in it $N_\beta(0) = \{y \in X : \|y\| < \beta\}$, also choose a natural number N_1 such that $\left(\frac{q^n}{1-q}\right)$ $p(u_o, u_1, u_1) \in N_\beta(0)$ for all $n > N_1$.

Then $\left(\frac{q^n}{1-q}\right)$ $p(u_o, u_1, u_1) \ll \alpha$ for all $n > N_1$. Thus $p(u_n, u_n, u_m) \leq \left(\frac{q^n}{1-q}\right) p(u_o, u_1, u_1) \ll \alpha$ for all n>m, therefore $\{u_n\}$ is a cauchy sequence in X. Since X is complete, $\{u_n\}$ converges to some point $w \in X$. Let $n \in N$ be fixed. Then since $\{u_m\}$ converges to w and $p(u_n, ., .)$ is lower semi-continuous, we have,

$$p(u_n, u_n, w) \leq \lim_{n\to\infty} \inf p(u_n, u_n, u_m) \leq \left(\frac{q^n}{1-q}\right) p(u_o, u_1, u_1).$$

Assume that $w \neq Tw$. Then by hypothesis we have,

$$0 < \inf\{p(x, x, w) + p(x, Tx, T^2x) : x \in X\}$$

$$\leq \inf\{p(u_n, u_n, w) + p(u_n, u_{n+1}, u_{n+2}) : n \in N\}$$

$$\leq \inf\{p(u_n, u_n, w) + q\,[p(u_{n-1}, u_n, u_n)]: n \in N\}$$

$$\leq \inf\{\frac{q^n}{1-q} p(u_o, u_1, u_1) + q^n p(u_o, u_1, u_1): n \in N\}$$

$$\leq \inf\{\frac{q^n}{1-q} p(u_o, u_1, u_1) + q^n p(u_o, u_1, u_1): n \in N\}$$

$$= 0.$$

This is a contradiction.

Therefore w = Tw. If v = Tv we have,

$$p(v, v, v) = p(Tv, T^2v, T^3v) \leq r\, p(v, Tv, T^2v) = r\, p(v, v, v).$$

So $p(v, v, v) \leq r^k p(v, v, v)$ where k is a natural number. Since P is the normal cone, there exist M >0 such that,

$$\|p(v,v,v)\| \leq M r^k \|p(v,v,v)\|.$$

Let k is a natural number such that $M r^k < 1$. So $\|p(v,v,v)\| = 0$, therefore $p(v, v, v)=0$.

Example: 1.3 Let X = **R** be a cone G-metric space with metric G defined by $G(x, y, z) = (|x-y| + |y-z| + |z-x|)$ $\forall\, x, y, z \in \mathbf{R}$. Then a function $p: \mathbf{R}^3 \to [0,\infty)$ defined by $p(x, y, z) = |y-z|$ for all $x, y, z \in \mathbf{R}$ is a w- distance on **R.**

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Define a function T: R $\rightarrow$ R as follows:

Tx= x/2, for all x $\in$ R,

Clearly, function p: $\mathbf{R^3} \rightarrow [0,\infty)$ defined by p(x, y, z) = $|y-z|$ for all x, y, z $\in$ **R** is a w- distance on **R.**

then p(Tx, T^2x, T^3x) = $\left|T^2x - T^3x\right| = \left|\frac{x}{4} - \frac{x}{8}\right| = \left|\frac{x}{8}\right|$

p(x, Tx, T^2x)= $\left|Tx - T^2x\right| = \left|\frac{x}{2} - \frac{x}{4}\right| = \left|\frac{x}{4}\right|$

Therefore, we have p(Tx, T^2x, T^3x) $\leq \frac{1}{2}$ p(x,Tx,T^2x).

Thus all the conditions of Theorem 1.3 are satisfied and 0 is a fixed point of T.

Theorem: 1.4 Let X be a complete cone G- metric space with metric G, 'p' a w-distance on X. Let P be a normal cone on X. Suppose T is a mapping from X into itself satisfying the contractive condition

$$p(Tx, Ty, Tz) \leq k\, p(x, y, z)$$

for all x, y, z $\in$ X, where k $\in$ [0, 1) is a constant. Then, T has a unique fixed point in X. For each x $\in$ X, the iterative sequence $\{T^n\ (x)\}_{n\geq 1}$ converges to the fixed point.

Proof: For each $x_o \in$ X and n $\geq$ 1, set $x_1 = Tx_0$ and $x_{n+1} = Tx_n = T^{n+1}x_o$.

Then

$$p(x_{n+1}, x_n, x_n) = p(Tx_n, Tx_{n-1}, Tx_{n-1}) \leq kp(x_n, x_{n-1}, x_{n-1})$$

$$\leq k^2 p(x_{n-1}, x_{n-2}, x_{n-2}) \leq \ldots\ldots \leq k^n\, p(x_1, x_o, x_o).$$

So for n>m,

$$p(x_n, x_m, x_m) \leq p(x_n, x_{n-1}, x_{n-1}) + p(x_{n-1}, x_{n-2}, x_{n-2}) + p(x_{m+1}, x_m, x_m)$$

$$\leq (k^{n-1} + k^{n-2} + \ldots + k^m)\, p(x_1, x_o, x_o).$$

Let c $\in$ E with 0<< c be given. Choose a natural number N_1 such that,

$$\left(\frac{k^n}{1-k}\right) p(x_1, x_o, x_o) << c$$

for all m $\geq N_1$. Thus, p(x_n , x_m , x_m) << c for all n>m. Therefore $\{x_n\}_{n\geq 1}$ is a cauchy sequence in X. Since X is a complete space, there exist $x^* \in$ X such that $x_n \rightarrow x^*$ as n $\rightarrow \infty$. Choose a natural number N_2 such that p(x_n, x^*, x^*)<< $\frac{c}{2}$ and p(x^*, x_n, x_n) << $\frac{c}{2}$ for all n $\geq N_2$. Hence,

$$p(Tx^*, x^*, x^*) \leq p(Tx^*, Tx_n, Tx_n) + p(Tx_n, x^*, x^*)$$

$$\leq k\, p(x^*, x_n, x_n) + p(x_{n+1}, x^*, x^*)$$

$$\leq p(x^*, x_n, x_n) + p(x_{n+1}, x^*, x^*)$$

$$<< \frac{c}{2} + \frac{c}{2}$$

$$= c,$$

for all n $\geq$ N_2. Thus, p $(Tx^*, x^*, x^*) << \frac{c}{m}$, for all m $\geq$ 1. So, $\frac{c}{m}$ - $p(Tx^*, x^*, x^*) \in P$, for all $m \geq 1$. Since $\frac{c}{m} \to 0$ (as $m \to \infty$) and P is closed, - p $(Tx^*, x^*, x^*) \in P$. But $p(Tx^*, x^*, x^*) \in P$. In the same way $p(x^*, x^*, Tx^*) = 0$, and so $Tx^* = x^*$.

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REFERENCES

1. Dhage, B. C., 1992 "Generalized metric space and mapping with fixed point," Bulletin of the Calcutta Mathematical Society; 84:329–336.
2. Dhage, B. C., 2000, "Generalized metric spaces and topological structure. I," Analele Scientific ale Universitatii Al. I. Cuza din Iasi. Serie Noua. Matematica; 46:3–24.
3. Gahler, S., 1963, "2-metrische Raume und ihre topologische Struktur," Mathematische Nachrichten, 26:115–148.
4. Gahler, S., 1966, "Zur geometric 2-metriche raume," Revue Roumaine de Mathematiques Pures et Appliquees; 40: 664–669.
5. Ha, K. S., Cho, Y. J. and White, A., 1988, "Strictly convex and strictly 2-convex 2-normed spaces," Mathematica Japonica; 33: 375–384.
6. Huang, L. G. and Zhang, X., 2007, Cone metric spaces and fixed point theorems of contractive mapping, J. Math. Anal. Appl.; 332:1468-1476.
7. Kada, O., Suzuki, T. and Takahashi, W., 1996, Nonconvex minimization theorems and fixed point theorems in complete metric spaces, Math. Japonica; 44: 381-391.
8. Lakzian, H. and Arabyani, F., 2009,Some fixed point theorems in Cone metric spaces with w- Distance, Int. Jour. of Math. Anal. ; 3(22):1081-1086.
9. Mustafa, Z. and Sims, B., 2006, A New Approach to Generalized Metric Spaces, Journal of Nonlinear and Convex Analysis; 7:289–297.
10. Mustafa, Z. and Sims, B., 2009, Fixed Point Theorems for contractive mappings in G-metric spaces, Fixed Point Theory and Applications; 2009:1-10.
11. Shioji, N., Suzuki, T. and Takahashi, W., 1998, Contractive mappings, Kannan mappings and metric completeness, Bull. Amer. Math. Soc.; 10: 3117-3124.

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