



CONSTRUCTION OF TWO SUMMATION FORMULAE ATTACHED WITH HYPERGEOMETRIC FUNCTION

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ABSTRACT

The main object of present paper is to develop two summation formulae involving the Contiguous relation [1] and derived formula [2]. The results are new and has general character.

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A. INTRODUCTION:

Generalized Gaussian Hypergeometric function of one variable:

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \tag{1}$$

or

$${}_A F_B((a_A); (b_B); z) \equiv {}_A F_B((a_j)_{j=1}^A; (b_j)_{j=1}^B; z) = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{2}$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relations:

[Andrews p.363(9.16), E.D. p.51(10), H.T.F.I. p.103(32)]

$$(a-b) {}_2F_1(a, b; c; z) = a {}_2F_1(a+1, b; c; z) - b {}_2F_1(a, b+1; c; z) \tag{3}$$

[Abramowitz p.558 (15.2.19)]

$$(a-b)(1-z) {}_2F_1(a, b; c; z) = (c-b) {}_2F_1(a, b-1; c; z) + (a-c) {}_2F_1(a-1, b; c; z) \tag{4}$$

A New Summation Formula:

[Ref. [2] p.337 (10)]

$${}_2F_1(a, b; \frac{a+b-1}{2}; \frac{1}{2}) = 2^{b-1} \frac{\Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} \left\{ \frac{b+a-1}{a-1} \right\} + 2 \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})} \right] \tag{5}$$

Recurrence relation:

$$\Gamma(z+1) = z \Gamma(z) \tag{6}$$

B. MAIN SUMMATION FORMULAE:

For both the formulae $a \neq b$

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For $a < 1$ and $a > 19$

$${}_2F_1(a, b; \frac{a+b-19}{2}; \frac{1}{2}) = 2^{b-1} \frac{\Gamma(\frac{a+b-19}{2})}{(a-b)\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-19}{2})} \right.$$

$$\left\{ \frac{654729075a - 1396704420a^2 + 1094071221a^3 - 444647600a^4 + 107494190a^5 - 16486680a^6 + 1646778a^7}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \right.$$

$$+ \frac{-106800a^8 + 4335a^9 - 100a^{10} + a^{11} - 654729075b + 1919582505a^2b - 1779068320a^3b + 8566680570a^4b}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{-187881120a^5b + 33687066a^6b - 2688480a^7b + 216825a^8b - 5600a^9b + 189a^{10}b + 1396704420b^2}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{-1919582505ab^2 + 893172140a^2b^2 - 419849080a^3b^2 + 140368466a^4b^2 - 15938720a^5b^2 + 2201340a^6b^2 - 71820a^7b^2 + 4655a^8b^2 - 1094071221b^3 + 1779068320ab^3 - 893172140a^2b^3 + 116147570a^4b^3}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{-27132000a^9b^3 + 6737780a^{10}b^3 - 310080a^{11}b^3 + 33915a^{12}b^3 + 444647600b^4 - 8566680570ab^4 + 419849080a^2b^4 - 116147570a^3b^4 + 4906370a^4b^4 - 452200a^5b^4 + 87210a^6b^4 - 107494190b^5 + 187881120ab^5 - 140368466a^2b^5}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{27132000a^3b^5 - 4906370a^4b^5 + 58786a^5b^5 + 16486680b^6 - 33687066ab^6 + 15938720a^2b^6 - 6737780a^3b^6}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{432200a^4b^6 - 53786a^5b^6 - 1648773b^7 + 2688480ab^7 - 2201340a^2b^7 + 310080a^3b^7 - 87210a^4b^7 + 106800b^8}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{-216825ab^8 + 71820a^2b^8 - 33915a^3b^8 - 4335b^9 + 5600ab^9 - 4655a^2b^9 + 100b^{10} - 189ab^{10} - b^{11}}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-18}{2})} \left\{ \frac{-995790420a + 1550816724a^2 - 813092208a^3 + 299510128a^4 - 48598200a^5 + 7584696a^6 - 464112a^7 + 34032a^8}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \right.$$

$$+ \frac{-660a^9 + 20a^{10} + 995790420b - 1144830128a^2b + 1026991136a^3b - 282607976a^4b + 74401824a^5b - 6350960a^6b}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+ \frac{765792a^7b - 19404a^8b + 1120a^9b - 1550816724b^2 + 1144830128ab^2 - 259801136a^2b^2 + 155725976a^3b^2}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+ \frac{-21686448a^4b^2 + 4311328a^5b^2 - 150480a^6b^2 + 14364a^7b^2 + 813092208b^3 - 1026991136ab^3 + 259801136a^2b^3}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+ \frac{-16460080a^4b^3 + 7162848a^5b^3 - 397936a^6b^3 + 62016a^7b^3 - 299510128b^4 + 282607976ab^4 - 155725976a^2b^4}{(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+ \frac{16460080a^3b^4 - 271320a^4b^4 + 90440a^5b^4 + 48598200b^5 - 74401824ab^5 + 21686448a^2b^5 - 7152848a^3b^5}{(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+ \frac{271320a^4b^5 - 7584696b^6 + 635096ab^6 - 4311328a^2b^6 + 397936a^3b^6 - 90440a^4b^6 + 464112b^7 - 765792ab^7}{(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+ \frac{1b^{10} + 480a^2b^7 - 63016a^3b^7 - 34032b^8 + 19404ab^8 - 14364a^2b^8 + 660b^9 - 1120ab^9 - 20b^{10}}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \left. \right\} \quad (7)$$

For $a < 1$ and $a > 20$

$${}_2F_1(a, b; \frac{a+b-20}{2}; \frac{1}{2}) = 2^{b-1} \frac{\Gamma(\frac{a+b-20}{2})}{(a-b)\Gamma(b)} \left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-19}{2})} \right.$$

$$\left\{ \frac{3715891200a - 7840972800a^2 + 6660433920a^3 - 2516910592a^4 + 721300608a^5 - 97228320a^6 + 12858384a^7}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \right.$$

$$+ \frac{-686928a^8 + 44352a^9 - 770a^{10} + 21a^{11} - 3715891200b + 2399109120ab + 5369825280a^2b - 6105745408a^3b}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{3366978944a^4b - 713416256a^5b + 150993360a^6b - 10993664a^7b + 1139424a^8b - 25564a^9b + 1309a^{10}b}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{5441363680b^2 - 8765343744ab^2 + 2016699592a^2b^2 + 1689632000a^3b^2 - 1062548704a^4b^2 + 416085712a^5b^2}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{-46381888a^6b^2 + 7570816a^7b^2 - 229482a^8b^2 + 19019a^9b^2 - 3264915456b^3 + 5529540608ab^3 - 3406107904a^2b^3}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+ \frac{363445376a^3b^3 + 168037520a^4b^3 - 54987520a^5b^3 + 16206848a^6b^3 - 739024a^7b^3 + 95931a^8b^3 + 1076416000b^4}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$\begin{aligned}
 & -2153120128ab^4 + 1095107104a^2b^4 - 379926736a^3b^4 + 19173280a^4b^4 + b^788160a^2b^4 - 768740a^2b^4 \\
 & + \frac{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}{177650a^7b^4 - 218683520b^5 + 385775040ab^5 - 395474960a^2b^5 + 61082112a^3b^5 - 14036288a^4b^5 + 271320a^5b^5} \\
 & + \frac{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}{58786a^6b^5 + 28865760b^6 - 60048912ab^6 + 28518336a^2b^6 - 12921216a^3b^6 + 895356a^4b^6 - 149226a^5b^6} \\
 & + \frac{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}{-2524369b^7 + 4131072ab^7 - 3489024a^2b^7 + 491568a^3b^7 - 149226a^4b^7 + 145200b^8 - 297792ab^8 + 98406a^2b^8} \\
 & + \frac{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}{-48279a^5b^8 - 5280a^5b^8 - 5280b^9 + 6820ab^9 - 5775a^2b^9 + 110b^{10} - 209ab^{10} - b^{11}} \\
 & + \frac{(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}{\left. \begin{aligned} & \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-20}{2})} \left\{ \frac{3715891200a - 5441863680a^2 + 3264915456a^3 - 1076416000a^4 + 218683520a^5 - 28865760a^6}{(a-20)(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-2)} \right. \\ & 2524368a^7 - 145200a^8 + 5280a^9 - 110a^{10} + a^{11} - 3715891200b - 2399109120ab + 8765343744a^2b \\ & - 5529540608a^3b + 2153120128a^4b - 385775040a^5b + 60048912a^6b - 4131072a^7b + 297792a^8b \\ & - 6820a^9b + 209a^{10}b + 7840972800b^2 - 5369825280ab^2 - 2016699392a^2b^2 + 3406107904a^3b^2 \\ & - 1095107104a^4b^2 + 305474960a^5b^2 - 28518336a^6b^2 + 3489024a^7b^2 - 98406a^8b^2 + 5775a^9b^2 - 6660433920b^3 \\ & 6105745408ab^3 - 1689632000a^2b^3 - 363445376a^3b^3 + 379926736a^4b^3 - 61082112a^5b^3 + 12921216a^6b^3 \\ & - 491568a^7b^3 + 48279a^8b^3 + 2516910592b^4 - 3366978944ab^4 + 1062548704a^2b^4 - 168037520a^3b^4 \\ & 19173200a^4b^4 + 14036200a^5b^4 + 0953560a^6b^4 + 149226a^7b^4 + 721300600b^5 + 713416256ab^5 \\ & - 416085712a^2b^5 + 54987520a^3b^5 - 5788160a^4b^5 - 271320a^5b^5 + 149226a^6b^5 + 97228320b^6 - 150993360ab^6 \\ & 46381388a^2b^6 - 16206843a^3b^6 + 768740a^4b^6 - 58786a^5b^6 - 12858384b^7 + 10993664ab^7 - 7570816a^2b^7 \\ & 789024a^3b^7 - 177600a^4b^7 + 686928b^8 - 1189424ab^8 + 229482a^2b^8 - 95931a^3b^8 - 44352b^9 + 25564ab^9 \\ & - 19019a^2b^9 + 770b^{10} - 1309ab^{10} - 21b^{11} \end{aligned} \right\}} \\
 & \left. \frac{(a-20)(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)}{177650a^7b^4 - 218683520b^5 + 385775040ab^5 - 395474960a^2b^5 + 61082112a^3b^5 - 14036288a^4b^5 + 271320a^5b^5} \right\} \quad (8)
 \end{aligned}$$

C. DERIVATIONS OF SUMMATION FORMULAE (7) TO (8):

Derivation of (7): Replacing $c = \frac{a+b-19}{2}$ and $z = \frac{1}{2}$ in equation (4), we get

$$(a-b) {}_2F_1(a, b; \frac{a+b-19}{2}; \frac{1}{2}) = (a-b-19) {}_2F_1(a, b-1; \frac{a+b-19}{2}; \frac{1}{2}) + (a-b+19) {}_2F_1(a-1, b; \frac{a+b-19}{2}; \frac{1}{2})$$

Now with the help of the derived result from equation (5), we get

$$\begin{aligned}
 \text{L.H.S} &= 2^{b-2} \frac{\Gamma(\frac{a+b-19}{2})}{(a-b+1)\Gamma(b-1)} \left[\frac{(a-b-19)(b-1)}{(a-b+1)} \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-19}{2})} \right. \\
 & \left\{ \frac{-654729075 + 707515920a + 339870879a^2 - 691535680a^3 + 303914690a^4 - 75012000a^5 + 10891902a^6 - 966720a^7}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \right. \\
 & \frac{51585a^8 - 1520a^9 + 19a^{10} + 1396704420b - 2143943946ab + 625999536a^2b + 321892072a^3b - 237910248a^4b}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & + \frac{66497796a^5b - 9387216a^6b + 826728a^7b - 36252a^8b + 950a^9b - 1094071221b^2 + 1888936416ab^2}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & + \frac{-915094340a^2b^2 + 94605408a^3b^2 + 59742986a^4b^2 - 19690080a^5b^2 + 3308812a^6b^2 - 217056a^7b^2 + 10659a^8b^2}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & + \frac{444647600b^3 - 806337592ab^3 + 453354896a^2b^3 - 96308264a^3b^3 + 2971600a^4b^3 + 3113720a^5b^3 - 387600a^6b^3}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & + \frac{38760a^7b^3 - 107494190b^4 + 196095360ab^4 - 118218190a^2b^4 + 29147520a^3b^4 - 2826250a^4b^4 + 41990a^5b^4 +}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & \left. \frac{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)}{177650a^7b^4 - 218683520b^5 + 385775040ab^5 - 395474960a^2b^5 + 61082112a^3b^5 - 14036288a^4b^5 + 271320a^5b^5} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{16486680b^8 - 30715164ab^7 + 17125880a^2b^6 - 4793320a^3b^5 + 478040a^4b^4 - 15776a^5b^3 - 1646778b^4}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & + \frac{2843424ab^6 - 1775892a^2b^5 + 341088a^3b^4 - 48450a^4b^3 + 106800b^4 - 194808ab^7 + 79344a^2b^7 - 23256a^3b^7}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & - \frac{4335b^8 + 6000ab^8 - 3705a^2b^8 + 100b^9 - 170ab^9 - b^{10}}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)(a-1)} \\
 & + \frac{(a-b-19) \Gamma(\frac{b+1}{2})}{(a-b+1) \Gamma(\frac{a-18}{2})} \left(\frac{-1030249845 + 392832444a + 627306453a^2 - 561551152a^3 + 192296510a^4}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \right. \\
 & - \frac{37417944a^5 + 4196826a^6 - 301488a^7 + 11655a^8 - 260a^9 + a^{10} + 2509369464b - 1975927094ab - 134581388a^2b}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{546845176a^3b - 224599750a^4b + 43828316a^5b - 5010624a^6b + 322104a^7b - 11880a^8b + 170a^9b - 2346020991b^2}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{2332797200ab^2 - 549338716a^2b^2 - 123024848a^3b^2 + 90666670a^4b^2 - 18926736a^5b^2 + 2149812a^6b^2}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & - \frac{119472a^7b^2 + 3705a^8b^2 + 1145841056b^3 - 1247709480ab^3 + 443807168a^2b^3 - 31651416a^3b^3 - 12454380a^4b^3}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{4152438a^5b^3 - 372096a^6b^3 + 23256a^7b^3 - 332112818t^4 + 375631976ab^4 - 148780906a^2b^4 + 23902000a^3b^4}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{113050a^4b^4 - 271320a^5b^4 + 48450a^6b^4 + 60130896b^5 - 70604223ab^5 + 27617792a^2b^5 - 5077560a^3b^5}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{335920a^4b^5 + 16796a^5b^5 - 7475550b^6 + 7824556ab^6 - 3401836a^2b^6 + 490960a^3b^6 - 41990a^4b^6 + 549024b^7}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & - \frac{662568ab^7 + 186048a^2b^7 - 38760a^3b^7 - 31977b^8 + 23484ab^8 - 10659a^2b^8 + 760b^9 - 950ab^9 - 19b^{10}}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \left. \right) \\
 & + 2^{b-1} \frac{\Gamma(\frac{a+b-18}{2})}{\Gamma(b)} \left(\frac{(a-b+19) \Gamma(\frac{b+1}{2})}{(a-b-1) \Gamma(\frac{a-18}{2})} \left(\frac{1030249845 - 2509369464a + 2346020991a^2 - 1145841056a^3}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \right. \right. \\
 & + \frac{332112818a^4 - 60130896a^5 + 7475550a^6 - 549024a^7 + 31977a^8 - 760a^9 + 19a^{10} - 392832444b + 1975927094ab}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & - \frac{2332797200a^2b + 1247709480a^3b - 375631976a^4b + 70604228a^5b - 7824656a^6b + 662568a^7b - 23484a^8b}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{950a^9b - 627306453b^2 + 134581888ab^2 + 549338716a^2b^2 - 443807168a^3b^2 + 148780906a^4b^2 - 27617792a^5b^2}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{3401836a^6b^2 - 186048a^7b^2 + 10659a^8b^2 + 561551152b^3 - 546845176ab^3 + 123024848a^2b^3 + 31651416a^3b^3}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & - \frac{23902000a^4b^3 + 5077560a^5b^3 - 490960a^6b^3 + 38760a^7b^3 - 192296510b^4 + 224599760ab^4 - 90666670a^2b^4}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{12454380a^3b^4 - 113050a^4b^4 - 335920a^5b^4 + 41990a^6b^4 + 37417944b^5 - 43828316ab^5 + 18926736a^2b^5}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & - \frac{4152438a^3b^5 + 271320a^4b^5 - 16796a^5b^5 - 4196826b^6 + 5010624ab^6 - 2149812a^2b^6 + 372096a^3b^6 - 48450a^4b^6}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{301488b^7 - 322104ab^7 + 119472a^2b^7 - 23256a^3b^7 - 11655b^8 + 11380ab^8 - 3705a^2b^8 + 260b^9 - 170ab^9 - b^{10}}{(a-18)(a-16)(a-14)(a-12)(a-10)(a-8)(a-6)(a-4)(a-2)} \\
 & + \frac{(a-b+19) \Gamma(\frac{b}{2})}{(a-b-1) \Gamma(\frac{a-19}{2})} \left(\frac{654729075 - 1396704420a + 1094071221a^2 - 444647600a^3 + 107494190a^4 - 16486580a^5}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \right. \\
 & + \frac{1646778a^6 - 106800a^7 + 4335a^8 - 100a^9 + a^{10} - 707515920b + 2143943946ab - 1888936416a^2b + 806337592a^3b}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \\
 & - \frac{196095360a^4b + 30715164a^5b - 2843424a^6b + 194808a^7b - 6000a^8b + 170a^9b - 339870879b^2 - 625999536ab^2}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \\
 & + \frac{915094340a^2b^2 - 453354896a^3b^2 + 118218190a^4b^2 - 1715840a^5b^2 + 1775892a^6b^2 - 79344a^7b^2 + 3705a^8b^2}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \\
 & + \frac{631535680b^3 - 321892072ab^3 - 94605408a^2b^3 + 96308264a^3b^3 - 27147520a^4b^3 + 4793320a^5b^3 - 341088a^6b^3}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \\
 & + \frac{23256a^7b^3 - 303914690b^4 + 237910248ab^4 - 59942986a^2b^4 - 2971600a^3b^4 + 2826250a^4b^4 - 478040a^5b^4}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \\
 & + \frac{48450a^6b^4 + 75012000b^5 - 6547776ab^5 + 17690080a^2b^5 - 3113720a^3b^5 + 16796a^4b^5 - 10891902b^6}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \\
 & + \frac{9387216ab^6 - 3308812a^2b^6 + 387600a^3b^6 - 41990a^4b^6 + 966720b^7 - 826728ab^7 + 217056a^2b^7 - 38760a^3b^7}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)}
 \end{aligned}$$

$$+ \frac{-51585b^2 + 36252ab^2 - 10657a^2b^2 + 1520b^2 - 950ab^2 - 19b^2}{(a-19)(a-17)(a-15)(a-13)(a-11)(a-9)(a-7)(a-5)(a-3)} \}}]$$

On simplification, we get the formula (7)

Similarly, we can prove the formula (8).

REFERENCES:

- [1] Abramowitz, Milton. A and Stegun, Irene; *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. National Bureau of Standards, 1970.
- [2] Garg, O.P, Salahuddin , Shakeeluddin ; On Certain Summation Formulae Involving Hypergeometric Function, *International Journal of Computational Science and Mathematics.*, 2(2010) , 67-76
- [3] Salahuddin. ; Evaluation of a Summation Formula Involving Recurrence Relation, *Gen. Math. Notes.*2 (2010), 42-59.
- [4] Salahuddin. ; Two Summation Formulae Based on Half Argument Associated to Hypergeometric function, *Global Journal of Science Frontier Reseach.* 10(2010), 08-19.
- [5] Salahuddin. ; Evaluation of Certain Summation Formulae Involving Gauss Theorem, *Global Journal of Mathematical Sciences: Theory and Practical.*, 10(2010), 309-316
- [6] Salahuddin. ; A Summation Formula Related To Bailey Theorem, *Global Journal of Science Frontier Research*, 11(2011), 53-67.
