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COMMON FIXED POINT THEOREMS INTUITIONISTIC FUZZY METRIC SPACE OF INTEGRAL TYPE WITH AND CONTRACTIVE CONDITION

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ABSTRACT

We prove a common fixed point theorems intuitionstic fuzzy metric space with integral type for contraction mapping.

***Keywords**: Contractive Condition, Intuitionistic Fuzzy Metric Space, Common Fixed Point Theorem.*

1. INTRODUCTION

El Naschie was Motivated by the potential applicability of fuzzy topology to quantum particle physics particularly in connection with both string and $e^{(\infty)}$, Park introduced and discussed in [21] a notion of intuitionistic fuzzy metric spaces which is based on the idea of intuitionistic fuzzy sets due to Atanassov [2] and the concept of fuzzy metric space given by George and Veeramani [11]. Actually, Park's notion is useful in modelling some phenomena where it is necessary to study the relationship between two probability functions.

Alaca *et al.* [1] using the idea of intuitionistic fuzzy sets, they defined the notion of intuitionistic fuzzy metric space as Park [21] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [15]. Further, they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces and proved the well known fixed point theorems of Banach [3] and Edelstein [5] extended to intuitionistic fuzzy metric spaces with the help of Grabiec [10]. Turkoglu *et al.* [25] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β). Sharma and Tilwankar [24] and Kutukcu [18] proved fixed point theorems for multivalued mappings in intuitionistic fuzzy metric spaces.

Several authors [12], [13], [15], [23] proved some fixed point theorems for various generalizations of contraction mappings in probabilistic and fuzzy metric space. Branciari [4] obtained a fixed point theorem for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. Sedghi et al. [22] established a common fixed point theorem for weakly compatible mappings in intuitionistic fuzzy metric space satisfying a contractive condition of integral type. Muralisankar *et al.* [20] proved a common fixed point theorem in an intuitionistic fuzzy metric space for pointwise R-weakly commuting mappings using contractive condition of integral type and established a situation in which a collection of maps has
a fixed point which is a point of discontinuity.

2. PRELIMINARIES

Definition: 2.1 ([23]) A binary operation $*$: [0; 1] x [0, 1] $\to$ [0, 1] is continuous t-norm if $*$ is satisfying the following conditions:
(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all a 2 [0, 1],
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, a, b, c, d $\in$ [0, 1].

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Definition: 2.2 ([23]) A binary operation *: [0, 1] x [0, 1] → [0, 1] is continuous t-conorm if ◊ is satisfying the following conditions:
(i) ◊ is commutative and associative,
(ii) ◊ is continuous,
(iii) a ◊ 0 = a for all a ∈ [0, 1],
(iv) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d, a, b, c, d ∈ [0, 1].

Definition: 2.3 ([1]) A 5-tuple (X, M, N, *,◊) is said to be an intuitionistic fuzzy metric spaces if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M , N are fuzzy sets on X^2 x [0, 1) satisfying the following conditions for all x, y, z ∈ X and t, s > 0,
(i) M (x, y, t) + N (x; y; t) ≤ 1,
(ii) M (x, y, 0) = 0,
(iii) M (x, y, t) = 1 for all t > 0 if and only if x = y,
(iv) M (x, y, t) = M (y, x, t),
(v) M (x, y, t) * M (y, z, s) ≤ M (x, z, t + s),
(vi) M (x, y, .) : [0, ∞) → [0, 1] is left continuous,
(vii) $\lim_{t\to\infty}$ M (x, y, t) = 1 for all x, y in X,
(viii) N (x, y, 0) = 1,
(ix) N (x, y, t) = 0 for all t > 0 if and only if x = y,
(x) N (x, y, t) = N (y, x, t),
(xi) N (x, y, t) ◊ N (y, z, s) ≥ N (x, z, t + s),
(xii) N (x, y, ·) : [0, ∞) → [0, 1] is right continuous,
(xiii) Limt→∞N (x, y, t) = 0 for all x, y ∈ X.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x; y; t) and N (x; y; t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark: 1 Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *,◊) such that t-norm * and t-conorm ◊ are associated, i.e., x ◊ y = 1-((1-x) * (1-y)) for all x, y ∈ X.

Example: 1 Let (X, d) be a metric space. Define t-norm a*b = min {a, b} and t-conorm a ◊ b = max{a, b} and for all x, y ∈ X and t >0,

$$(2a) \quad M_d\left(x,y,t\right)=\frac{t}{t+d\left(x,y\right)}, N_d\left(x,y,t\right)=\frac{d\left(x,y\right)}{t+d\left(x,y\right)},$$

Then (X, M, N, *,◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric. On the other hand, note that there exists no metric on X satisfying (2a).

Remark: 2 In intuitionistic fuzzy metric space (X, M, N, *,◊), M (x, y, ·) is non -decreasing and N (x, y;) is non - increasing for all x, y ∈ X.

Definition: 2.4 ([1]) Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then
(i) A sequence $\{x_n\}$ in X is said to be convergent to a point x ∈ X (denoted by $\lim_{n\to\infty} x_n = x$) if, for all t >0,

$$\lim_{n\to\infty} M\left(x_n,x,t\right)=1, \lim_{n\to\infty} N\left(x_n,x,t\right)=0$$

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,

$$\lim_{n\to\infty} M\left(x_{n+p},x_n,t\right)=1, \quad \lim_{n\to\infty} N\left(x_{n+p},x_n,t\right)=0$$

Remark: 3 Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi), respectively.

Definition: 2.5 ([1]) An intuitionistic fuzzy metric space (X, M, N, *,◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma: 1 ([1]) Let (X, M, N, *,◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be sequence in X. If there exists a number k ∈ (0, 1) such that
M (y_{n+2}, y_{n+1}, kt) ≥ M (y_{n+1}, y_n, t), N (y_{n+2}, y_{n+1}, kt) ≤ N (y_{n+1}, y_n, t)
for all t >0 and n = 1, 2, ………, then $\{y_n\}$ is a Cauchy sequence in X.

Lemma: 2 ([1]) Let (X, M, N, *◊) be an intuitionistic fuzzy metric space and for all x, y $\in$ X, t > 0 and if for a number k $\in$ (0, 1),
M (x, y, kt) ≥ M (x, y, t) and N (x, y, kt) ≤ N (x, y, t),
then x = y.

Definition: 6 ([14]) Two self mappings S and T are said to be weakly compatible if they commute at their coincidence points; i.e., if T u = Su for some u $\in$ X, then T Su = ST u.

3. MAIN RESULTS

Theorem: 3. 1 Let (X.M.N.*◊) be an Intuitionistic Fuzzy Metric Space with continuous t-norm * and continuous t-conorm ◊ defined by t * t ≥ t and (1 – t) ◊ (1 – t) ≤ (1 – t) for all t ε [0, 1]. Let A.B.S.T.P and q be mapping from x into itself such that

(a) $P(X) \subseteq ST(X)$ and $Q(X) \subseteq AB(X)$

(b) There exists a constant k ε (0, 1) such that

$$\int_0^{\left[M\left(Pu,Qv,(Kx)\right)\right]^2} \phi(t)\,dt \geq \int_0^{M(u.v.t)} \phi(t)\,dt \text{ and}$$

$$\int_0^{\left[N\left(Pu,Qv,(Kx)\right)\right]^2} \phi(t)\,dt \leq \int_0^{N(u.v.t)} \phi(t)\,dt$$

where φ: $R^+ \to R^+$ is a Lebesque – Integrable mapping which is summable non-negative and such that

$$\int_0^{\in} \phi(t)\,dt > 0 \text{ for each } \varepsilon > 0.$$

for u.v ε x and x ≥ 0 and

(c) If one of P(x). AB(x). ST(x) or Q(x) is a complete subspace of X. Then
(i) P and AB have a coincidence point and (ii) Q and St have coincidence point

The paid {P.AB} is weakly compatible.

(d) AB = BA, QB = BQ, QA = AQ, PT = TP, ST = TS and

(e) The paid {P.ST} is weakly compatible

Then
A.B.S.T.P and Q have a unique common fixed point in X.

Proof: For any point x_0 in X, there exists a point $x_1 \in$ X, such that $Px_0 = STx_1$. For this point x_1, we can choose a point x_2 in X, such that $Qx_1 = ABx_2$ and so on, in this manner we can define a sequence $\{y_n\}$ in X such that $y_{2n} = Px_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Qx_{2n+1} = Ax_{2n+2}$, for n = 0, 1, 2,……

Now we shall prove Fy_{2n}, $y_{2n+1}(kx) \geq Fy_{2n-1}$, $y_{2n}(x)$ for all x > 0, where k$\in$ (0, 1). Suppose that Fy_{2n}, $y_{2n+1}(kx) < Fy_{2n-1}$, $y_{2n}(x)$. Fy_{2n}, $y_{2n+1}(kx) \leq Fy_{2n}$, $y_{2n+1}(x)$, we have

$$m(u,v,t) = \min \int_0^{\left[MPu,Qk(kx)\right]^2} \phi(t)\,dt$$

$$\geq \min \int_0^{\left[MAu,Tvk(x)\right]^2,Mau,PUk(x)} \phi(t)\,dt$$

$$\int_0^{MTv,Qvk(x),MAu,Tvk(x).MAu,PUk(x),MAu,Tvk(x)} \phi(t)\,dt$$

$$\int_0^{MTv,Qvk(x),MAu,Tvk(x).MAu,Qvk(2x),MAu,Tvk(x)} \phi(t)\,dt$$

$$\int_0^{MTv,PUk(x),MAu,Qvk(2x).MTv,PUk(x),MAu,PUk(x)} \phi(t)\,dt$$

$$\int_0^{MTv,PUk(x),MAu,Qvk(2x).MTv,Qvk(x)} \phi(t)\,dt$$

$$n(u,v,t)=\max\int_0^{\left[NPu,Qk(kx)\right]^2}\phi(t)\,dt$$

$$\le \max\int_0^{\left[NAu,Tvk(x)\right]^2,NAu,PUk(x)}\phi(t)\,dt$$
$$\int_0^{NTv,Qvk(x),NAu,Tvk(x).NAu,PUk(x),NAu,Tvk(x)}\phi(t)\,dt$$
$$\int_0^{NTv,Qvk(x),NAu,Tvk(x).NAu,Qvk(2x),NAu,Tvk(x)}\phi(t)\,dt$$
$$\int_0^{NTv,PUk(x),NAu,Qvk(2x).NTv,PUk(x),NAu,PUk(x)}\phi(t)\,dt$$
$$\int_0^{NTv,PUk(x),NAu,Qvk(2x).NTv,Qvk(x)}\phi(t)\,dt$$

$$\int_0^{\left[M(y_{2n},y_{2n+1},(kx))\right]^2}\phi(t)\,dt=\int_0^{\left[M(Px_{2n},Qx_{2n+1},(kx))\right]^2}\phi(t)\,dt$$
$$=\int_0^{\left[M(Qx_{2n+1},Px_{2n},(kx))\right]^2}\phi(t)\,dt$$
$$\ge\int_0^{\left[M(x_{2n+1},x_{2n},(x))\right]^2}\phi(t)\,dt \qquad 1(a)$$

$$\int_0^{\left[N(y_{2n},y_{2n+1},(kx))\right]^2}\phi(t)\,dt=\int_0^{\left[N(Px_{2n},Qx_{2n+1},(kx))\right]^2}\phi(t)\,dt$$
$$=\int_0^{\left[N(Qx_{2n+1},Px_{2n},(kx))\right]^2}\phi(t)\,dt$$
$$\le\int_0^{\left[N(x_{2n+1},x_{2n},(x))\right]^2}\phi(t)\,dt \qquad 1(b)$$

$$\left[M\left(x_{2n+1}x_{2n}(x)\right)\right]^2=\max\int_0^{\left[M(y_{2n},y_{2n+1},(kx))\right]^2}\phi(t)\,dt$$
$$=\int_0^{\left[M(Px_{2n},Qx_{2n+1},(kx))\right]^2}\phi(t)\,dt$$
$$=\int_0^{\left[M(Qx_{2n+1},Px_{2n},(kx))\right]^2}\phi(t)\,dt$$
$$\ge\int_0^{\left[M(x_{2n+1},x_{2n},(x))\right]^2}\phi(t)\,dt$$

$$\left[N\left(x_{2n+1},x_{2n}(x)\right)\right]^2=\max\int_0^{\left[N(y_{2n},y_{2n+1},(kx))\right]^2}\phi(t)\,dt$$
$$=\int_0^{\left[N(Px_{2n},Qx_{2n+1},(kx))\right]^2}\phi(t)\,dt$$
$$=\int_0^{\left[N(Qx_{2n+1},Px_{2n},(kx))\right]^2}\phi(t)\,dt$$
$$\le\int_0^{\left[N(x_{2n+1},x_{2n},(x))\right]^2}\phi(t)\,dt$$
$$\ge\min\int_0^{\min\{[M(y_{2n-1},\,y_{2n}k(x)]^2}\phi(t)\,dt,\int_0^{M(y2n-1,\,y_{2n}k(x).M(y_{2n},\,y_{2n+1}k(x)}\phi(t)\,dt,$$
$$\int_0^{My_{2n-1},\,y_{2n}k(x).My_{2n-1}}\phi(t)\,dt,\int_0^{y_{2n}(x),\,My_{2n-1},\,y_{2n}(x)\,My_{2n},\,y_{2n+1}(x)}\phi(t)\,dt,$$
$$\int_0^{My_{2n-1},y_{2n}(x).My_{2n-1},y_{2n+1}(2x),My_{2n-1},y_{2n}(x)My_{2n},y_{2n+1}(x)}\phi(t)\,dt,$$

$$\int_0^{My_{2n-1},y_{2n+1}(2x).My_{2n},y_{2n}(x),My_{2n-1},y_{2n}(x)My_{2n},y_{2n+1}(x)} \phi(t)\,dt,$$
$$\int_0^{My_{2n-1},y_{2n+1}(2x).My_{2n},y_{2n+1}(x)} \phi(t)\,dt\}$$

$$\geq \min \int_0^{M[y_{2n-1},y_{2n}(x)]^2,\,My_{2n-1},y_{2n}(x).My_{2n},y_{2n+1}(x)} \phi(t)\,dt,$$
$$\int_0^{\left[My_{2n-1},y_{2n}(x)^2,My_{2n-1}y_{2n}(x).My_{2n},y_{2n+1}(x)\right]} \phi(t)\,dt,$$
$$\int_0^{My_{2n-1},y_{2n}(x).t(My_{2n-1}y_{2n}(x),My_{2n},y_{2n+1}(x)),My_{2n-1},y_{2n}(x)} \phi(t)\,dt$$
$$t(My_{2n-1}y_{2n}(x),My_{2n},y_{2n+1}(x)),\ My_{2n-1},y_{2n}(x),$$
$$t(My_{2n-1}y_{2n}(x),My_{2n},y_{2n+1}(x)),\ My_{2n},y_{2n+1}(x)\}$$

$$\geq \min \int_0^{\{[My_{2n},y_{2n+1}(kx)]^2,\,[My_{2n},y_{2n+1}(kx)],\,[My_{2n},y_{2n+1}(kx)]} \phi(t)\,dt,$$
$$\int_0^{[My_{2n},y_{2n+1}(kx)]^2,\,[My_{2n},y_{2n+1}(kx)]^2,My_{2n},y_{2n+1}(kx)} \phi(t)\,dt,$$

$$\int_0^{[My_{2n},y_{2n+1}(kx)],\,[My_{2n},y_{2n+1}(kx)],\left[My_{2n},y_{2n+1}(kx)\right]^2} \phi(t)\,dt = \{[My_{2n},y_{2n+1}(kx)]^2\}$$

which is a contradiction. Thus we have

$$\int_0^{\left[M(y_{2n},y_{2n+1}(kx))\right]} \phi(t)\,dt \geq \int_0^{\left[M(y_{2n-1},y_{2n}(x))\right]} \phi(t)\,dt$$

and

$$\int_0^{\left[N(y_{2n},y_{2n+1}(kx))\right]^2} \phi(t)\,dt \leq \int_0^{\left[N(y_{2n-1},y_{2n}(x))\right]} \phi(t)\,dt$$

Similarly we have $\int_0^{\left[M(y_{2n+1},y_{2n+2}(kx))\right]} \phi(t)\,dt \geq \int_0^{\left[M(y_{2n},y_{2n+1}(x))\right]} \phi(t)\,dt$

and $\int_0^{\left[N(y_{2n+1},y_{2n+2}(kx))\right]} \phi(t)\,dt \leq \int_0^{\left[N(y_{2n},y_{2n+1}(x))\right]} \phi(t)\,dt$

$\{y_n\}$ is a Cauchy sequence in X. Since $\{y_n\}$ converges to a point z in X, and the subsequences $\{Px_{2n}\}$, $\{Qx_{2n+1}\}$, $\{Ax_{2n}\}$ and $\{STx_{2n+1}\}$ of $\{y_{2n}\}$ also converges to z.

Now suppose that P is continuous, since P and AB are weak compatible of type (α), it follows from

$(AB)Px_{2n} \to Pz$ and $PPx_{2n} \to Pz$ as $n \to \infty$.

Now putting $u = Px_{2n}$ and $v = x_{2n+1}$ in the given equation, we have

$$\int_0^{\left[MPPx_{2n},Qx_{2n+1}(kx)\right]^2} \phi(t)\,dt \geq \min \int_0^{\left[M(A)Px_{2n},Tx_{2n+1}(x)\right]^2} \phi(t)\,dt$$
$$\int_0^{M(A)Px_{2n},PPx_{2n}(x).MTx_{2n+1},Qx_{2n+1}(x)} \phi(t)\,dt$$
$$\int_0^{M(A)Px_{2n},Tx_{2n+1}(x).M(A)Px_{2n},PPx_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{M(A)Px_{2n},Tx_{2n+1}(x).MTPx_{2n+1},Qx_{2n+1}(x)} \phi(t)\,dt$$

$$\int_0^{M(A)Px_{2n},Tx_{2n+1}(x).M(A)Px_{2n},Qx_{2n+1}(2x)} \phi(t)\,dt$$
$$\int_0^{M(A)Px_{2n},Qx_{2n+1}(2x).MTx_{2n+1},PPx_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{M(A)Px_{2n},PPx_{2n}(x).MTx_{2n+1},PPx_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{M(A)Px_{2n},Qx_{2n+1}(2x).MTx_{2n+1},Qx_{2n+1}(x)} \phi(t)\,dt$$

$$\int_0^{\left[NPPx_{2n},Qx_{2n+1}(kx)\right]^2} \phi(t)\,dt \le \max \int_0^{\left[N(A)Px_{2n},Tx_{2n+1}(x)\right]^2} \phi(t)\,dt$$
$$\int_0^{N(A)Px_{2n},PPx_{2n}(x).NTx_{2n+1},Qx_{2n+1}(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Px_{2n},Tx_{2n+1}(x).N(A)Px_{2n},PPx_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Px_{2n},Tx_{2n+1}(x).NTPx_{2n+1},Qx_{2n+1}(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Px_{2n},Tx_{2n+1}(x).N(A)Px_{2n},Qx_{2n+1}(2x)} \phi(t)\,dt$$
$$\int_0^{N(A)Px_{2n},Qx_{2n+1}(2x).NTx_{2n+1},PPx_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Px_{2n},PPx_{2n}(x).NTx_{2n+1},PPx_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Px_{2n},Qx_{2n+1}(2x).NTx_{2n+1},Qx_{2n+1}(x)} \phi(t)\,dt$$

Taking the limit $n \to \infty$, we have

$$\int_0^{\left[MPz,z(kx)\right]^2} \phi(t)\,dt \ge \min \int_0^{\left\{\left[MPz,z(x)\right]^2,\left[MPz,Pz(x)\right]^2\right\}.Mz,z(x)} \phi(t)\,dt$$
$$\int_0^{\left[MPz,z(x)\right].MPz,Pz(x),MPz,z(x).Mz,z(x)} \phi(t)\,dt$$
$$\int_0^{\left[MPz,z(2x)\right].Mz,Pz(x),MPz,Pz(x).Mz,Pz(x)} \phi(t)\,dt$$

$$\int_0^{\left[MPz,z(2x)\right].Mz,z(x)} \varphi(t)dt = \int_0^{\left[MPz,z(x)\right]^2}$$

$$\int_0^{\left[NPz,z(kx)\right]^2} \phi(t)\,dt \le \max \int_0^{\left\{\left[NPz,z(x)\right]^2,\left[NPz,Pz(x)\right]^2\right\}.Nz,z(x)} \phi(t)\,dt$$
$$\int_0^{\left[NPz,z(x)\right].NPz,Pz(x),NPz,z(x).Nz,z(x)} \phi(t)\,dt$$
$$\int_0^{\left[NPz,z(2x)\right].Nz,Pz(x),NPz,Pz(x).Nz,Pz(x)} \phi(t)\,dt$$

$$\int_0^{\left[NPz,z(2x)\right].Nz,z(x)} \phi(t)\,dt = \int_0^{\left[NPz,z(x)\right]^2} \phi(t)\,dt$$

which is a contradiction. Thus we have Pz = z. Since $P(X) \subset ST(X)$, there exists a point $u \in X$ such that z = Pz = Tp. Again putting $u = Px_{2n}$ and v = p in, given equation

$$\int_0^{\left[MPPx_{2n},Qpk(kx)\right]^2} \phi(t)\,dt \ge \min \int_0^{\left[M(A)Px_{2n},Tpk(x)\right]^2} \phi(t)\,dt$$

$$\int_0^{[M(A)]Px_{2n},PPx_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{MTP,Qp(x),M(A)Px_{2n},Tp(x).M(A)Px_{2n},PPx_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{[M(A)]Px_{2n},TP(x).MPp,Qp(x),M(A)Px_{2n},Tp(x)}\phi(t)\,dt$$
$$\int_0^{[M(A)]Px_{2n},QP(2x),M(A)Px_{2n},Tp(x),MTp,PPx_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{[M(A)]Px_{2n},QP(2x),MTp,PPx_{2n}(x),F(A)Px_{2n},PPx_{2n}}\phi(t)\,dt$$
$$\int_0^{MTp,PPx_{2n},M(A)Px_{2n},Qp(2x).MTp,Qp(x)}\phi(t)\,dt$$

$$\int_0^{[NPPx_{2n},Qpk(kx)]^2}\phi(t)\,dt \le \max\int_0^{[N(A)Px_{2n},Tpk(x)]^2}\phi(t)\,dt$$
$$\int_0^{[N(A)]Px_{2n},PPx_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{NTP,Qp(x),N(A)Px_{2n},Tp(x).N(A)Px_{2n},PPx_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{[N(A)]Px_{2n},TP(x).NPp,Qp(x),N(A)Px_{2n},Tp(x)}\phi(t)\,dt$$
$$\int_0^{[N(A)]Px_{2n},QP(2x),N(A)Px_{2n},Tp(x),NTp,PPx_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{[N(A)]Px_{2n},QP(2x),NTp,PPx_{2n}(x),F(A)Px_{2n},PPx_{2n}}\phi(t)\,dt$$
$$\int_0^{NTp,PPx_{2n},N(A)Px_{2n},Qp(2x).NTp,Qp(x)}\phi(t)\,dt$$

Taking the limit n $\rightarrow \infty$, we have

$$\int_0^{[Mz,Qp(kx)]^2 \ge [Mz,Qp(x)]^2}\phi(t)\,dt \text{ and } \int_0^{[Nz,Qp(kx)]^2 \le [Nz,Qp(x)]^2}\phi(t)\,dt$$

which is a contradiction, therefore z = Qp. Since Q and T are weak compatible of type (α) and Tp = Qp = z, (T)Qp = Q(T)p and hence STz = (T)Qp = Q(T)p = Qz. Again by putting u = x_{2n} and v = z in we have

$$\int_0^{[MPx_{2n},Qz(kx)]^2}\phi(t)\,dt \ge \min\int_0^{[MAx_{2n},Tz(x)]^2,MAx_{2n},Px_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{MTz,Qz(x),MAx_{2n},Tz(x).MAx_{2n},Px_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{MAx_{2n},Tz(x),MTz,Qz(x).MAx_{2n},Tz(x)}\phi(t)\,dt$$
$$\int_0^{MAx_{2n},Qz(2x),MAx_{2n},Tz(x),MTz,Px_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{MAx_{2n},Qz(2x),MTz,Px_{2n}(x),MAx_{2n},Px_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{MTz,Px_{2n},MAx_{2n},Qz(2x).Mtz,Qz(x)}\phi(t)\,dt$$

$$\int_0^{[NPx_{2n},Qz(kx)]^2}\phi(t)\,dt \le \max\int_0^{[NAx_{2n},Tz(x)]^2,NAx_{2n},Px_{2n}(x)}\phi(t)\,dt$$
$$\int_0^{NTz,Qz(x),NAx_{2n},Tz(x).NAx_{2n},Px_{2n}(x)}\phi(t)\,dt$$

$$\int_0^{NAx_{2n},Tz(x),NTz,Qz(x).NAx_{2n},Tz(x)} \phi(t)\,dt$$
$$\int_0^{NAx_{2n},Qz(2x),NAx_{2n},Tz(x),NTz,Px_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{NAx_{2n},Qz(2x),NTz,Px_{2n}(x),NAx_{2n},Px_{2n}(x)} \phi(t)\,dt$$
$$\int_0^{NTz,Px_{2n},NAx_{2n},Qz(2x).Ntz,Qz(x)} \phi(t)\,dt$$

Letting n $\to \infty$, we have

$$\int_0^{[Mz,Qz(kx)]^2} \phi(t)\,dt \ge \int_0^{[Mz,Qz(x)]^2} \phi(t)\,dt \text{ and } \int_0^{[Nz,Qz(kx)]^2} \phi(t)\,dt \le \int_0^{[Nz,Qz(x)]^2} \phi(t)\,dt$$

which is a contradiction, therefore we have Qz = z. Thus Qz = STz = z. Similarly since P and AB are weak compatible of type (α), we have ABz = Pz = z. Now we prove Az = z. Suppose that $A \ne z$ then by putting u = Az and v = z in given equation

$$\int_0^{[MPAz,Qz(kx)]^2} \phi(t)\,dt \ge \min \int_0^{[M(A)Az,Tz(x)]^2,M(A)Az,PAz(x)} \phi(t)\,dt$$
$$\int_0^{M(A)Az,Tz(x).MTz,Qz(x).M(A)Az,Tz(x)} \phi(t)\,dt$$
$$\int_0^{M(A)Az,Qz(2x).M(A)Az,Tz(x).MTz,PAz(x)} \phi(t)\,dt$$
$$\int_0^{M(A)Az,Qz(2x).MTz,PAz(x).M(A)Az,PAz(x)} \phi(t)\,dt$$
$$\int_0^{MTz,PAz(x),M(A)Az,Qz(2x).MTz,Qz(x)} \phi(t)\,dt$$
$$\int_0^{MTz,Qz(x),M(A)Az,Tz(x)} \phi(t)\,dt$$

$$\int_0^{[NPAz,Qz(kx)]^2} \phi(t)\,dt \le \max \int_0^{[N(A)Az,Tz(x)]^2,N(A)Az,PAz(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Az,Tz(x).NTz,Qz(x).N(A)Az,Tz(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Az,Qz(2x).N(A)Az,Tz(x).NTz,PAz(x)} \phi(t)\,dt$$
$$\int_0^{N(A)Az,Qz(2x).NTz,PAz(x).N(A)Az,PAz(x)} \phi(t)\,dt$$
$$\int_0^{NTz,PAz(x),N(A)Az,Qz(2x).NTz,Qz(x)} \phi(t)\,dt$$
$$\int_0^{NTz,Qz(x),N(A)Az,Tz(x)} \phi(t)\,dt$$

which yields

$$\int_0^{[MAz,z(kx)]^2} \phi(t)\,dt \ge \int_0^{[MAz,z(x)]^2} \phi(t)\,dt \text{ and } \int_0^{[NAz,z(kx)]^2} \phi(t)\,dt \le \int_0^{[NAz,z(x)]^2} \phi(t)\,dt$$

which is a contradiction, there fore we have Az = z. Similarly if we put u= Bz and y=z in given equation, we have

$$\int_0^{[MPz,Qz(kx)]^2} \phi(t)\,dt \ge \min \int_0^{[M(A)z,Tz(x)]^2,M(A)z,PBz(x)} \phi(t)\,dt$$

$$\int_0^{MTz,Qz(x),M(A)z,Tz(x).M(A)z,Pz(x)} \phi(t)\,dt$$

$$\int_0^{M(A)z,Tz(x).MTz,Qz(x),M(A)z,Tz(x)} \phi(t)\,dt$$
$$\int_0^{M(A)z,Qz(2x),M(A)z,Tz(x).MTz,PBz(x)} \phi(t)\,dt$$
$$\int_0^{M(A)z,Qz(2x),MTz,Pz(x).M(A)z,PBz(x)} \phi(t)\,dt$$
$$\int_0^{MTz,Pz(x),M(A)z,Qz(2x).MTz,Qz(x)} \phi(t)\,dt$$

$$\int_0^{[NPz,Qz(kx)]^2} \phi(t)\,dt \le \max \int_0^{[N(A)z,Tz(x)]^2,N(A)z,PBz(x)} \phi(t)\,dt$$
$$\int_0^{NTz,Qz(x),N(A)z,Tz(x).N(A)z,Pz(x)} \phi(t)\,dt$$
$$\int_0^{N(A)z,Tz(x).NTz,Qz(x),N(A)z,Tz(x)} \phi(t)\,dt$$
$$\int_0^{N(A)z,Qz(2x),N(A)z,Tz(x).NTz,PBz(x)} \phi(t)\,dt$$
$$\int_0^{N(A)z,Qz(2x),NTz,Pz(x).N(A)z,PBz(x)} \phi(t)\,dt$$
$$\int_0^{NTz,Pz(x),N(A)z,Qz(2x).NTz,Qz(x)} \phi(t)\,dt$$

which gives
$$\int_0^{[MBz,z(kx)]^2} \phi(t)\,dt \ge \int_0^{[MBz,z(x)]^2} \phi(t)\,dt \text{ and } \int_0^{[NBz,z(kx)]^2} \phi(t)\,dt \le \int_0^{[NBz,z(x)]^2} \phi(t)\,dt$$

which is a contradiction, therefore we have z = z. So Az = z = z. Finally we show that Tz = z. By using given equation

$$\int_0^{[Mz,Qz(kx)]^2} \phi(t)\,dt \ge \min \int_0^{[Mz,(T)z(x)]^2,Mz,z(x),Mz,(T)z(x)} \phi(t)\,dt$$
$$\int_0^{M(T),Qz(x),Mz,(T)z(x).Mz,Qz(2x)} \phi(t)\,dt$$
$$\int_0^{Mz,(T)z(x).M(T)z,z(x),Mz,Qz(2x)} \phi(t)\,dt$$
$$\int_0^{M(T)z,z(x),Mz,z(x).M(T)z,z(x)} \phi(t)\,dt$$
$$\int_0^{Mz,Qz(2x).M(T)z,Qz(x)} \phi(t)\,dt$$

$$\int_0^{[Nz,Qz(kx)]^2} \phi(t)\,dt \le \max \int_0^{[Nz,(T)z(x)]^2,Nz,z(x),Nz,(T)z(x)} \phi(t)\,dt$$
$$\int_0^{N(T),Qz(x),Nz,(T)z(x).Nz,Qz(2x)} \phi(t)\,dt$$
$$\int_0^{Nz,(T)z(x).N(T)z,z(x),Nz,Qz(2x)} \phi(t)\,dt$$
$$\int_0^{N(T)z,z(x),Nz,z(x).N(T)z,z(x)} \phi(t)\,dt$$
$$\int_0^{Nz,Qz(2x).N(T)z,Qz(x)} \phi(t)\,dt$$

which gives
$$\int_0^{[Mz,z(kx)]^2} \phi(t)\,dt \ge \int_0^{[Mz,z(x)]^2} \phi(t)\,dt \text{ and } \int_0^{[Nz,z(kx)]^2} \phi(t)\,dt \le \int_0^{[Nz,z(x)]^2} \phi(t)\,dt$$

which is a contradiction, therefore we have Sz = z. So Sz = Tz = z. Thus combining the results, we have Pz = Qz = Az = Tz = z. Thus z is a common fixed point of A, B, S, T, P and Q.

For uniqueness let w (z ≠ w) be another common fixed point of A, T, P and Q, then by, we have

$$\int_0^{\left[Mz,w(kx)\right]^2}\phi(t)\,dt=\int_0^{\left[MPz,Qw(kx)\right]^2}\phi(t)\,dt$$

$$\geq \min\int_0^{\left[Mz,w(x)\right]^2,Mz,z(x).Mw,w(x)}\phi(t)\,dt$$

$$\int_0^{Mz,(x).Mz,z(x),Mz,w(x).Mw,w(x)}\phi(t)\,dt$$

$$\int_0^{Mz,w(x).Mz,w(2x),Mz,w(x).Mw,z(x)}\phi(t)\,dt$$

$$\int_0^{Mz,(x).Mz,z(x),Mz,w(x).Mw,w(x)}\phi(t)\,dt$$

$$\int_0^{Mz,w(2x).Mw,z(x),Mz,z(x).Mw,z(x)}\phi(t)\,dt$$

$$\int_0^{Mz,w(2x).Mw,w(x)g}\phi(t)\,dt=\int_0^{\left[Mz,w(x)\right]^2}\phi(t)\,dt$$

$$\int_0^{\left[Nz,w(kx)\right]^2}\phi(t)\,dt=\int_0^{\left[NPz,Qw(kx)\right]^2}\phi(t)\,dt$$

$$\leq \max\int_0^{\left[Nz,w(x)\right]^2,Nz,z(x).Nw,w(x)}\phi(t)\,dt$$

$$\int_0^{Nz,(x).Nz,z(x),Nz,w(x).Nw,w(x)}\phi(t)\,dt$$

$$\int_0^{Nz,w(x).Nz,w(2x),Nz,w(x).Nw,z(x)}\phi(t)\,dt$$

$$\int_0^{Nz,(x).Nz,z(x),Nz,w(x).Nw,w(x)}\phi(t)\,dt$$

$$\int_0^{Nz,w(2x).Nw,z(x),Nz,z(x).Nw,z(x)}\phi(t)\,dt$$

$$\int_0^{Nz,w(2x).Nw,w(x)g}\phi(t)\,dt=\int_0^{\left[Nz,w(x)\right]^2}\phi(t)\,dt$$

which is a contradiction, therefore z = w. Hence z is a unique common fixed point of A, T, P and Q.

If we put T = I (I is identity mapping on X) in Theorem 4.1., we obtain the following result due to Pathak *et al.* [17].

Corollary: 3.1 Let (X, F, t) be a complete Menger space with t(x; y) = min{x,y} for all x, y ∈ [0, 1] and P, Q, A and T be mappings from X into itself such that

(a) P (X) ⊂ T(X) and Q(X) ⊂ A(X),
(b) the pairs {P, A} and {Q, S} are weak compatible of type (α)
(c) P is continuous,
(d) $\int_0^{\left[MPu,Qv(kx)\right]^2}\phi(t)\,dt \geq \min\int_0^{\left[MAu,Tv(x)\right]^2,MAu,Pu(x).MTv,Qv(x)}\phi(t)\,dt$

$$\int_0^{MAu,Tv(x).MAu,Pu(x),MAu,Tv(x)}\phi(t)\,dt$$

$$\int_0^{MTv,Qv(x),MAu,Tv(x).MAu,Qv(2x),MAu,Tv(x)}\phi(t)\,dt$$

$$\int_0^{MTv,Pu(x),MAu,Qv(2x).MTv,Pu(x),MAu,Pu(x)}\phi(t)\,dt$$

$$\int_0^{MTv,Pu(x),MAu,Qv(2x).MTv,PQv(x)}\phi(t)\,dt$$

$$\int_0^{[NPu,Qv(kx)]^2} \phi(t)\,dt \le \max \int_0^{[NAu,Tv(x)]^2, NAu,Pu(x).NTv,Qv(x)} \phi(t)\,dt$$
$$\int_0^{NAu,Tv(x).NAu,Pu(x),NAu,Tv(x)} \phi(t)\,dt$$
$$\int_0^{NTv,Qv(x),NAu,Tv(x).NAu,Qv(2x),NAu,Tv(x)} \phi(t)\,dt$$
$$\int_0^{NTv,Pu(x),NAu,Qv(2x).NTv,Pu(x),NAu,Pu(x)} \phi(t)\,dt$$
$$\int_0^{NTv,Pu(x),NAu,Qv(2x).NTv,PQv(x)} \phi(t)\,dt$$

for all u, v ∈ X and x ≥ 0, where k ∈ (0, 1). Then P, Q, A and T have a unique common fixed point.

If we put A = B = T = I (I is identity mapping on X) in Theorem 4.1, we have the following:

Corollary: 3.2 Let (X, F, t) be a complete Menger space with t(x, y) = min{x, y} for all x, y ∈ [0, 1] and P and Q be mappings from X into itself such that

(a) $P(X) \subset Q(X)$

(b) P is continuous;

(c) $$\int_0^{[MPu,Qv(kx)]^2} \phi(t)\,dt \ge \min \int_0^{[Mu,v(x)]^2 Mu,Pu(x).Mv,Qv(x)} \phi(t)\,dt$$
$$\int_0^{Mu,v(x).Mu,Pu(x),Mu,v(x).Mv,Qv(x)} \phi(t)\,dt$$
$$\int_0^{Mu,v(x).Mu,Qv(2x),Mu,v(x).Mv,Pu(x)} \phi(t)\,dt$$
$$\int_0^{Mu,Qv(2x).Mv,Pu(x),Mu,Pu(x).Mv,Pu(x)} \phi(t)\,dt$$
$$\int_0^{Mu,Qv(2x),Mv,Qv(x)} \phi(t)\,dt$$

$$\int_0^{[NPu,Qv(kx)]^2} \phi(t)\,dt \le \max \int_0^{[Nu,v(x)]^2 Nu,Pu(x).Nv,Qv(x)} \phi(t)\,dt$$
$$\int_0^{Nu,v(x).Nu,Pu(x),Nu,v(x).Nv,Qv(x)} \phi(t)\,dt$$
$$\int_0^{Nu,v(x).Nu,Qv(2x),Nu,v(x).Nv,Pu(x)} \phi(t)\,dt$$
$$\int_0^{Nu,Qv(2x).Nv,Pu(x),Nu,Pu(x).Nv,Pu(x)} \phi(t)\,dt$$
$$\int_0^{Nu,Qv(2x),Nv,Qv(x)} \phi(t)\,dt$$

for all u, v ∈ X and x ≥ 0, where k ∈ (0;,1). Then P and Q have a unique common fixed point.

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