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$\hat{\alpha}$g Interior and $\hat{\alpha}$g Closure in Topological Spaces

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ABSTRACT

In this paper we introduce $\hat{\alpha}$g interior, $\hat{\alpha}$g closure and study some of its properties.

Key words: *$\hat{\alpha}$g open, $\hat{\alpha}$g closed, $\hat{\alpha}$g int A, $\hat{\alpha}$g cl A.*

AMS subject classification: *54C10, 54C08, 54C05.*

1. INTRODUCTION

Levine [6] introduced generalized closed sets in topology as a generalization of closed sets. Many authors like Arya *et al* [2], Balachandran *et.al* [3], Bhattacharya *et al* [4], Arokiarani[1], Ganambal[5], Malghan[7] and Nagaveni[8] have worked on generalized closed sets. Palaniappan *et al* [9] introduced regular $\hat{\alpha}$ generalized beta ($\hat{\alpha}$g) closed sets and worked on them. In this paper, $\hat{\alpha}$g interior, $\hat{\alpha}$g closure are introduced and their properties are investigated.

Throughout this paper X denote the topological space (X, τ) on which no separation axioms are assumed unless otherwise stated.

2. PRELIMINARIES

Definition: 2.1 A subset A of a topological space X is called
1) A pre open set if A $\subset$ int cl A and a preclosed set if cl int A $\subset$ A.
2) A regular open set if A = int cl A and a regular closed set if A = cl int A.
3) A α open set if A $\subset$ int cl int A and a α closed set if cl int cl A$\subset$A.

The intersection of all α closed subsets of X containing A is called the α closure of A and is denoted by αcl A. αcl A is a α closed set.

Definition: 2.2A subset A of a topological space X is called a $\hat{\alpha}$ generalized closed set ($\hat{\alpha}$g–closed set) if int cl int A$\subset$U whenever A$\subset$U and U is open in X.

The complement of $\hat{\alpha}$g closed set in X is $\hat{\alpha}$g open in X.

The intersection of all $\hat{\alpha}$g closed sets in X containing A is called $\hat{\alpha}$ generalized closure of A and is denoted by $\hat{\alpha}$g cl A. In general $\hat{\alpha}$g cl A is not $\hat{\alpha}$g closed.

The union of all $\hat{\alpha}$g open sets contained in A is denoted by $\hat{\alpha}$g int A. In general $\hat{\alpha}$g int A is not $\hat{\alpha}$g open.

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In what follows we assume that X is a topological space in which arbitrary intersections of âg closed sets of X be âg closed in X. Then âg cl A will be a âg closed set in X and âg int A will be a âg open set in X, for $A\subset X$.

Definition: 2.3 Let X be a topological space and let $x \in X$. A subset N of X is said to be a âg neighbourhood of x if and only if there exists a âg open set G such that $x \in G \subset N$

Definition: 2.4 Let X be a topological space and $A \subset X$. A point $x \in X$ is called a âg limit point of X if and only if every âg neighbourhood of x contains point of A other than x. The set of all âg limit points of A is called the âg derived set of A and shall be denoted by $D_{\hat{a}g}$ (A).

Thus x will be a âg limit point of A if and only if $(N-\{x\}) \cap A = \phi$, for every âg neighbourhood N of x.

Definition: 2.5 Let A be a sub set of a topological space X and let $x \in X$. Then x is called an âg adherent point of A if and only if every âg neighbourhood of x contains point of A. The set of all âg adherent points of A is called the âg adherence of A and shall be denoted by âg Adh A.

Definition: 2.6 A point x is said to be an âg isolated point of a subset A of a topological space X if and only if $x\in A$ but is not a âg limit point of A. That is, there exists some âg neighbourhood N of x such that N contains no point of A other than x. A âg closed set which has no âg isolated points is said to be âg perfect.

Remarks: 2.7 An âg adherent point is either âg limit point or âg isolated point.

3. Properties of âg Limit Points

Theorem: 3.1 A set is âg closed in X if and only if it contains all its âg limit pts.

Proof: Let A be âg closed in X. Then A' is âg open. For each $x\in A'$, there exists âg neighbourhood Nx of x such that $N_x \subset A'$. $A\cap A' = \phi$ implies N_X contains no point of A. So, x is not a âg limit point of A. A' contains no âg limit point of A. Hence $D_{\hat{a}g}$ (A) $\subset$ A.

Conversely, let $D_{\hat{a}g}$ (A) $\subset$ A. Let $x\in A'$. Since $x\notin A$, $x\notin D_{\hat{a}g}$ (A). Therefore, there exises some âg neighbourhood Nx of x such that $Nx\cap A=\varphi$. So $Nx\subset A'$. Hence A' contains a âg neighbourhood of each of its points. That is A' is âg open. So A is âg closed.

Theorem: 3.2 Let X be a topological space and $A\subset X$. Then A= $D_{\hat{a}g}$ (A) if and only if A is âg perfect.

Proof: Let A be âg perfect. Then A has no âg isolated point. $x\in A \Rightarrow$x is not an âg isolated point$\Rightarrow$x is a âg limit point. $\Rightarrow A\subset D_{\hat{a}g}$ (A)

Since A is âg closed, $D_{\hat{a}g}$ (A) $\subset$ A

Hence A= $D_{\hat{a}g}$(A)

Conversely, let A= $D_{\hat{a}g}$ (A). Let $x\in X$. $x\in X-A$ implies $x\notin A$. That is $x\notin D_{\hat{a}g}$ (A). This implies there exists âg neighbourhood N of x such that $N\subset X-A$. X-A contains a âg neighbourhood of each of its points. So, X-A is âg open. That is, A is âg closed. Let $y\in A$. So $y \in D_{\hat{a}g}$ (A). Hence y is a âg limit point of A. This implies y is not an âg isolated point of A. That is, no point of A is an âg isolated point of A. A is a âg closed set having no âg isolated point. Hence A is âg perfect.

Let X be any discrete topological space and $A\subset X$. If $x\in X$, {x} is âg open which contains no point of {x} other than x. So x is not a âg limit point of A. Hence $D_{\hat{a}g}$ (A) = φ.

Let X be any indiscrete topological space. Let $A\subset X$ containing two or more points. $x\in A$ is a âg limit point of A, since the only âg open set containing x is X, which contains all points of A, other than x. Hence $D_{\hat{a}g}$(A)=X.

Theorem: 3.3 Let A and B be subsets of a topological space X. Then

i) $D_{\hat{a}g}(\varphi)=\varphi$
ii) $A\subset B \Rightarrow D_{\hat{a}g}(A) \subset D_{\hat{a}g}(B)$
iii) $D_{\hat{a}g}(A\cap B) \subset D_{\hat{a}g}(A) \cap D_{\hat{a}g}(B)$

iv) $D_{\hat{\alpha}g}(A \cup B) = D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B)$

Proof:

i) φ is closed. So $D_{\hat{\alpha}g}(\varphi) \subset \varphi$. But $\varphi \subset D_{\hat{\alpha}g}(\varphi)$. Hence $D_{\hat{\alpha}g}(\varphi) = \varphi$.

ii) Let $p \in D_{\hat{\alpha}g}$ (A). Every α̂g neighbourhood of p contains a point of A, other than p. Since $A \subset B$, every α̂g neighbourhood of p contains a point of B, other than p. Hence p is a α̂g limit point of B. So, $D_{\hat{\alpha}g}(A) \subset D_{\hat{\alpha}g}$ (B). $A \cap B \subset A$. Hence $D_{\hat{\alpha}g}(A \cap B) \subset D_{\hat{\alpha}g}(A)$. Similarly, $D_{\hat{\alpha}g}(A \cap B) \subset D_{\hat{\alpha}g}(B)$.

iii) $A \cap B \subset A$. Hence $D_{\hat{\alpha}g}(A \cap B) \subset D_{\hat{\alpha}g}(A)$. Similarly, $D_{\hat{\alpha}g}(A \cap B) \subset D_{\hat{\alpha}g}(B)$. So $D_{\hat{\alpha}g}(A \cap B) \subset D_{\hat{\alpha}g}(A) \cap D_{\hat{\alpha}g}(B)$.

iv) $A \subset A \cup B$. Hence $D_{\hat{\alpha}g}(A) \subset D_{\hat{\alpha}g}(A \cup B)$. Similarly, $D_{\hat{\alpha}g}(B) \subset D_{\hat{\alpha}g}(A \cup B)$.
So $D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B) \subset D_{\hat{\alpha}g}(A \cup B)$.
To prove the other way, we prove the contra positive.
$x \notin D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B) \Rightarrow x \notin D_{\hat{\alpha}g}(A \cup B)$

If $x \notin D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B)$, then $x \notin D_{\hat{\alpha}g}(A)$ and $x \notin D_{\hat{\alpha}g}(B)$. That is, x is neither a α̂g limit point of A nor a α̂g limit point of B. Hence, there exist α̂g neighbourhoods N_1 and N_2 of x such that $(N_1 - \{x\}) \cap A = \varphi$ and $(N_2 - \{x\}) \cap B = \varphi$. $N = N_1 \cap N_2$ is a α̂g neighbourhood of x which contains no point of $A \cup B$ other than (possibly) x. So it follows that $x \notin D_{\hat{\alpha}g}(A \cup B)$ as required.

Theorem: 3.4 α̂g cl A = $A \cup D_{\hat{\alpha}g}(A)$

Proof: Let us prove $A \cup D_{\hat{\alpha}g}(A)$ is α̂g closed. That is, $(A \cup D_{\hat{\alpha}g}(A))' = A' \cap D'_{\hat{\alpha}g}(A)$ is α̂g open. Let $x \in A' \cap D'_{\hat{\alpha}g}(A)$. Then $x \in A'$ and $x \in D'_{\hat{\alpha}g}(A)$. So $x \notin A$ and $x \notin D_{\hat{\alpha}g}(A)$. That is, x is not a α̂g limit point of A. Hence, there exists a α̂g neighbourhood N_x of x which contains no point of A. Hence $N_x \subset D'_{\hat{\alpha}g}(A)$. But $N_x \subset A'$. So $N_x \subset A' \cap D'_{\hat{\alpha}g}(A)$ $A' \cap D'_{\hat{\alpha}g}(A)$ contains a α̂g neighbourhood of each of its points and hence α̂g open. We now show that α̂g cl A = $A \cup D_{\hat{\alpha}g}(A)$. $A \cup D_{\hat{\alpha}g}(A)$ is a α̂g closed set containing A.

Hence α̂g cl A $\subset A \cup D_{\hat{\alpha}g}(A)$. α̂g cl A is rgβ closed. Hence $D_{\hat{\alpha}g}(A) \subset A$. But $A \subset$ α̂g cl A

So $D\alpha_g(A) \subset$ α̂g cl A.

Hence $A \cup D_{\hat{\alpha}g}(A) \subset$ α̂g cl A. This completes the proof.

Theorem: 3.5 α̂g cl A = α̂g Adh A.

Proof: $x \in$ α̂g Adh A ⇔ every α̂g neighbourhood of x intersects A ⇔ $x \in A$ or every α̂g neighbourhood of x intersects A in a point other than x ⇔ $x \in A$ or $x \in D_{\hat{\alpha}g}(A)$ ⇔ $x \in A \cup D_{\hat{\alpha}g}(A)$ ⇔ $x \in$ α̂g cl A.

Theorem: 3.6 Let X be a topological space and let G be an α̂g open subset of X and $A \subset X$. Then G is disjoint from A if and only if G is disjoint from the α̂g closure of A.

Proof: Let $G \cap$ α̂g cl A = φ.
As $A \subset$ α̂g cl A, $G \cap A = \varphi$.

Conversely, let $G \cap A = \varphi$. Let $x \in G \cap$ α̂g cl A
α̂g cl A = $A \cup D_{\hat{\alpha}g}(A)$. Hence $x \in D_{\hat{\alpha}g}(A)$.

As G is α̂g neighbourhood of x, it intersects A, a contradiction. This completes the proof.

4. PROPERTIES OF α̂g CLOSURE

Theorem: 4.1 Let X be a topological space and A and B be subsets of X.

i) α̂g cl φ = φ
ii) $A \subset$ α̂g cl A.
iii) $A \subset B \Rightarrow$ α̂g cl A $\subset$ α̂g cl B
iv) α̂g cl ($A \cup B$) = α̂g cl A $\cup$ α̂g cl B.
v) α̂g cl ($A \cap B$) $\subset$ α̂g cl A $\cap$ α̂g cl B.
vi) α̂g cl (α̂g cl A) = α̂g cl A.

Proof:

i) Since φ is α̂g closed, α̂g cl φ = φ
ii) By definition of α̂g cl A, A⊂ α̂g cl A.
iii) A ⊂ B ⊂ α̂g cl B. Hence α̂g cl A ⊂ α̂g cl B.
iv) A ⊂ A ∪ B. Hence α̂g cl A ⊂ α̂g cl (A ∪ B)
Similarly α̂g cl B ⊂ α̂g cl (A ∪ B)
So α̂g cl A ∪ α̂g cl B ⊂ α̂g cl (A ∪ B)
α̂g cl A ∪ α̂g cl B is a α̂g closed set containing A ∪ B.
Hence α̂g cl (A ∪ B) ⊂ α̂g cl A ∪ α̂g cl B.This completes the proof.
v) A∩B ⊂ A, A∩B ⊂ B
Hence α̂g cl (A∩B) ⊂ α̂g cl A ∩ α̂g cl B.
vi) α̂g cl A is α̂g closed.
Hence α̂g cl (α̂g cl A) = α̂g cl A.

5. α̂g interior points and α̂g interior of a set

Definition: 5.1 Let X be a topological space and A⊂X. A point x∈A is said to be α̂g interior point of A if and only if A is a α̂g neighbourhood of x. That is, there exists an α̂g open set G such that x ∈ G ⊂ A. The set of all α̂g interior points of A is called the α̂g interior A and is denoted by α̂g int A.

Theorem: 5.2 α̂g int A = ∪ {G: G is α̂g open, G ⊂ A}

Proof: x∈ α̂g int A ⇔ A is a α̂g neighbourhood of x. ⇔ there exists an α̂g open set G such that x ∈ G ⊂ A ⇔ x ∈ ∪{ G:G is α̂g open, G⊂ A}. Thus A = ∪ {G: G is α̂g open, G⊂ A}

Theorem: 5.3 Let X be a topological space and A⊂ X. Then

i) α̂g int A is α̂g open
ii) α̂g int A is the largest α̂g open set contained in A.

Proof:

i) Let x∈ α̂g int A. So there exists a α̂g open set G such that x∈ G⊂ A. Since G is α̂g open, it is a α̂g neighbourhood of each of its points. So A is also a α̂g neighbourhood of each of the points of G. It follows that every point of G is a α̂g interior point of A. Hence G ⊂ α̂g int A. α̂g int A contains a α̂g neighbourhood of each of its points. Hence α̂g int A is α̂g open.

ii) Let G be any α̂g open set such that G ⊂A. Let x∈ G. A is α̂g neighbourhood of x. Therefore x ∈ α̂g int A. Hence G ⊂ α̂g int A. So α̂g int A is the largest α̂g open set contained in A.

Remark: 5.4 If X be any discrete topological space, then every subset of X coincides with its α̂g interior.

Theorem: 5.5 Let X be a topological space. Then α̂g int A equals the set of all points of A which are not α̂g limit points of A’

Proof: Let x∈A, which is not a α̂g limit point of A’ Then, there exists a α̂g neighbourhood N of x, which contains no point of A’. So N ⊂ A. This implies A is also α̂g neighbourhood of x. Hence x ∈ α̂g int A. Let x ∈ α̂g int A. Since α̂g int A is α̂g open, α̂g int A is a α̂g neighbourhood of x. Also α̂g int A contains no point of A’. It follows x is not a α̂g limit point of A’. Thus no point of α̂g int A can be a α̂g limit point of A’. So α̂g int A consists precisely those points of A which are not α̂g limit points of A’.

6. Properties of α̂g interior

Theorem: 6.1 Let X be a topological space and let A,B be subsets of X

i) α̂g int X=X, α̂g int φ = φ
ii) α̂g int A⊂ A
iii) A⊂B ⇒ α̂g int A⊂ α̂g int B
iv) α̂ g int (A∩B) = α̂g int A∩ α̂g int B
v) α̂g int A ∪ α̂g int B ⊂ α̂g int (A∪B)
vi) α̂g int (α̂g int A) = α̂g int A

Proof:

i) obvious

ii) obvious

iii) Let x ∈ α̂g int A. A is a α̂g neighbourhood of x. As A⊂ B, B is a α̂g neighbourhood of x.
This implies x∈ α̂g int B.
Hence α̂g int A ⊂ α̂g int B.

iv) A∩B⊂A, A∩B⊂B.
Hence α̂g int (A∩B) ⊂ α̂g int A ∩ α̂g int B.
Let x ∈ α̂g int A ∩ α̂g int B.
x ∈ α̂g int A and x ∈ α̂g int B
A and B are α̂g neighbourhoods of x. Hence A∩B is a α̂g neighbourhood of x.
So x∈ α̂g int (A∩B).
Therefore α̂g int A∩ α̂g int B ⊂ α̂g int (A∩B). This completes the proof.

v) A ⊂ A∪B, B ⊂ A∪B
Hence α̂g int A∪ α̂g int B⊂ α̂g int (A∪B).

vi) α̂g int A is α̂g open.
Hence α̂g int (α̂g int A) = α̂g int A.

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