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SOME RESULTS ON ONE MODULO THREE HARMONIC MEAN GRAPHS

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ABSTRACT

A graph G with p vertices and q edges is said to be an one modulo three harmonic mean graph if there is a function φ from the vertex set of G to $\{1, 3, 4, \ldots, 3q-2, 3q\}$ with φ is one-one and φ induces a bijection φ^* from the edge set of G to {1, 4, …, 3q – 2}, where $\varphi^*(e = uv) = \left\lceil\frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)}\right\rceil$ or $\left\lfloor\frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)}\right\rfloor$ and the function φ is called as an one modulo three harmonic mean labeling of G. In this paper, we investicate one modulo three harmonic mean labeling of some graphs.

***Key words**: One modulo three harmonic mean labeling, one modulo three harmonic mean graphs.*

1. INTRODUCTION

We begin with simple, finite, connected and undirected graph G(V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian[1]. For all other standared terminology and notations we follow Harary [2]. V. Swaminathan and C. Sekar introduce the concept of an one modulo three graceful labeling in [3]. S. Somasundaram and R. Ponraj introduced mean labeling of graphs in [4]. S. Somasundaram and S.S Sandhya introduced the concept of harmonic mean labeling in [5] and studied their behavior in [7], [8] and [9]. C. David Raj, S.S Sandhya and C. Jayasekaran introduced the concept of an one modulo three harmonic mean graphs in [6]. In this paper, we investigate one modulo three harmonic mean labeling of some graphs.

We will provide brief summary of definitions and other information which are necessary for the present investigation.

Definition: 1.1 A graph G is said to be an one modulo three harmonic mean graph if there is a function φ from the vertex set of G to $\{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ with φ is one-one and φ induces a bijection φ* from the edge set of G to {1, 4, …, 3q – 2}, where $\varphi^*(e = uv) = \left\lceil\frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)}\right\rceil$ or $\left\lfloor\frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)}\right\rfloor$ and the function φ is called as an one modulo three harmonic mean labeling of G.

Definition: 1.2 The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.

Definition: 1.3 The graph $P_n \odot K_1$ is called Comb.

Definition: 1.4 The graph $C_n \odot K_1$ is called crown.

Definition: 1.5 The product $P_m \times P_n$ is called a planar grid and $C_m \times P_n$ is called a prism. The product $P_2 \times P_n$ is called a ladder, and it is denoted by L_n.

Definition: 1.6 Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by identifying (fusing) two vertices u and v by a single vertex x is such that every edge which was incident with either u or v in G is now incident with x in G_1.

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Remark: 1.7 If G is an one modulo three harmonic mean graph, then 1 must be a label of one of the vertices of G, since an edge must get the label 1.

2. MAIN RESULTS

Theorem: 2.1 nP_m is an one modulo three harmonic mean Graph.

Proof: Let $v_{i,1}v_{i,2}\ldots v_{i,m}$ be the i^{th} P_m of nP_m, $1 \le i \le n$. Then $V = \{v_{i,j} / 1 \le i \le n, 1 \le j \le m\}$ is the vertex set and $E = \{v_{i,j}v_{i,j+1} / 1 \le i \le n, 1 \le j \le m-1\}$ is the edge set of nP_m. Define a function φ: $V(G) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(v_{1,1}) = 1$; $\varphi(v_{1,j}) = 3(j-1)$, $2 \le j \le m-1$;
$\varphi(v_{i,j}) = 3(m-1)(i-1) + 3(j-1)$, $2 \le i \le n$, $1 \le j \le m-1$;
$\varphi(v_{i,m}) = 3im - 3(i+1) + 1$, $1 \le i \le n$.

Then φ induces a bijective function φ^*: $E(G) \to \{1, 4, 7, \ldots, 3q-2\}$, where

$$\varphi^*(v_{i,j}v_{i,j+1}) = 3(m-1)(i-1) + 3(j-1) + 1,\ 1 \le i \le n,\ 1 \le j \le m-1.$$

Thus φ provides an one modulo three harmonic mean labeling for nP_m. Hence nP_m is an one modulo three harmonic mean graph.

Example: 2.2 An one modulo three harmonic mean labeling of $4P_7$ is shown in figure 2.1.

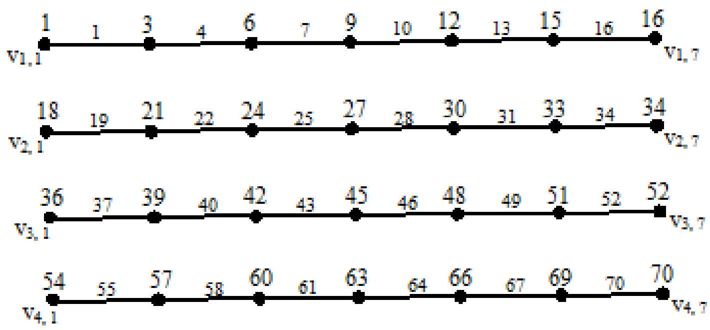

Figure: 2.1

Theorem: 2.3 The graph $P_n \odot \overline{K}_3$ is an one modulo three harmonic mean graph.

Proof: Let P_n be the path $u_1u_2\ldots u_n$. Let x_i, y_i, z_i be the vertices of i^{th} copy of $\overline{K}_3$ which are adjacent to the vertex u_i of P_n, $1 \le i \le n$. The resultant graph is $P_n \odot \overline{K}_3$ whose edge set is $E = \{u_iu_{i+1} / 1 \le i \le n-1\} \cup \{u_ix_i, u_iy_i, u_iz_i / 1 \le i \le n\}$. Define a function φ: $V(P_n \odot \overline{K}_3) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 7$; $\varphi(u_2) = 18$; $\varphi(u_3) = 30$; $\varphi(u_i) = 12i - 8$, $4 \le i \le n$;
$\varphi(x_1) = 1$; $\varphi(x_2) = 10$; $\varphi(x_i) = 12(i-1) - 3$, $3 \le i \le 4$;
$\varphi(x_i) = 12(i-1) - 2$, $5 \le i \le n$;
$\varphi(y_1) = 3$; $\varphi(y_i) = 12(i-1) + 3$, $2 \le i \le n$;
$\varphi(z_1) = 6$; $\varphi(z_i) = 12i - 5$, $2 \le i \le 3$; $\varphi(z_i) = 12i - 3$, $4 \le i \le n$.

Then φ induces a bijective function φ^*: $E(P_n \odot \overline{K}_3) \to \{1, 4, 7, \ldots, 3q-2\}$, where

$\varphi^*(u_iu_{i+1}) = 12i - 2$, $1 \le i \le n-1$;
$\varphi^*(u_ix_i) = 12(i-1) + 1$, $1 \le i \le n$;
$\varphi^*(u_iy_i) = 12(i-1) + 4$, $1 \le i \le n$;
$\varphi^*(u_iz_i) = 12(i-1) + 7$, $1 \le i \le n$.

In the view of the above labeling pattern, φ provides an one modulo three harmonic mean labeling for $P_n \odot \overline{K}_3$. Hence $P_n \odot \overline{K}_3$ is an one modulo three harmonic mean graph.

Example: 2.4 An one modulo three harmonic mean labeling of $P_6 \odot \overline{K}_3$ is given in figure 2.2.

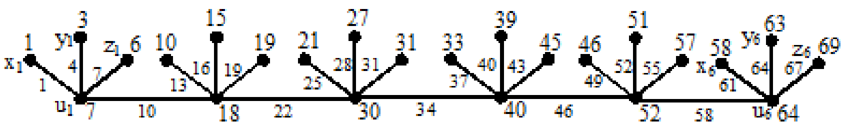

Figure: 2.2

Theorem: 2.5 A graph obtained by identifying the central vertex of $K_{1,2}$ at each pendent vertex of a comb $P_n \odot K_1$ is an one modulo three harmonic mean graph.

Proof: Let P_n be the path $u_1u_2\ldots u_n$. Let v_i be a vertex adjacent to u_i, $1 \leq i \leq n$. The resultant graph is $P_n \odot K_1$. Let x_i, w_i, y_i be the vertices of i^{th} copy of $K_{1,2}$ with w_i is the central vertex. Identify the vertex w_i with v_i, $1 \leq i \leq n$. We get the required graph G whose edge set is $E = \{u_iu_{i+1} / 1 \leq i \leq n-1\} \cup \{u_iv_i, v_ix_i, v_iy_i / 1 \leq i \leq n\}$. Define a function $\varphi: V(G) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 7$; $\varphi(u_i) = 12(i-1) + 6$; $2 \leq i \leq 3$; $\varphi(u_i) = 12(i-1) + 4$, $4 \leq i \leq n$;
$\varphi(v_1) = 6$; $\varphi(v_2) = 19$; $\varphi(v_i) = 12i - 3$; $3 \leq i \leq 4$; $\varphi(v_i) = 12i - 3$, $5 \leq i \leq n$;
$\varphi(x_1) = 1$; $\varphi(x_2) = 9$; $\varphi(x_3) = 21$; $\varphi(x_4) = 31$; $\varphi(x_i) = 12(i-1) - 6$, $5 \leq i \leq n$;
$\varphi(y_1) = 3$; $\varphi(y_2) = 13$; $\varphi(y_i) = 12(i-1)$, $3 \leq i \leq n$.

Then φ induces a bijective function $\varphi^*: E(G) \rightarrow \{1, 4, 7, \ldots, 3q-2\}$, where

$\varphi^*(u_iv_i) = 12i - 5$, $1 \leq i \leq n$.;
$\varphi^*(u_iu_{i+1}) = 12i - 2$, $1 \leq i \leq n-1$;
$\varphi^*(v_ix_i) = 12(i-1) + 1$, $1 \leq i \leq n$;
$\varphi^*(v_iy_i) = 12(i-1) + 4$, $1 \leq i \leq n$;

In the view of the above labeling pattern, φ provides an one modulo three harmonic mean labeling for G. Hence G is an one modulo three harmonic mean graph.

Example: 2.6 An one modulo three harmonic mean labeling of G when n = 7 is given in figure 2.3.

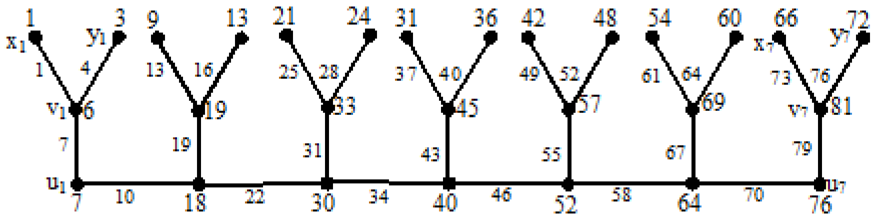

Figure: 2.3

Theorem: 2.7 The graph $C_n \odot \overline{K}_3$ is an one modulo three harmonic mean graph.

Proof: Let C_n be the cycle $u_1u_2\ldots u_nu_1$. Let x_i, y_i, z_i be the vertices of i^{th} copy of $\overline{K}_3$ which are adjacent to the vertex u_i of C_n, $1 \leq i \leq n$. The resultant graph is $C_n \odot \overline{K}_3$ whose edge set is $E = \{u_iu_{i+1}, u_nu_1 / 1 \leq i \leq n-1\} \cup \{u_ix_i, u_iy_i, u_iz_i / 1 \leq i \leq n\}$.

Case- 1. n = 3.

One modulo three harmonic mean labeling of $C_3 \odot \overline{K}_3$ is shown in figure 2.4.

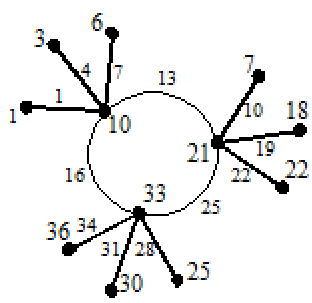

Figure: 2.4

Case- 2. $n \geq 4$.
Define a function $\varphi: V(C_n \odot \overline{K}_3) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 7$; $\varphi(u_2) = 21$; $\varphi(u_i) = 12i - 5$, $3 \leq i \leq n$;
$\varphi(x_i) = 12i - 11$, $1 \leq i \leq n$;
$\varphi(y_1) = 3$; $\varphi(y_i) = 12i - 6$, $2 \leq i \leq n$;
$\varphi(z_1) = 6$; $\varphi(z_i) = 12i$, $2 \leq i \leq n$.

Then φ induces a bijective function $\varphi^*: E(C_n \odot \overline{K}_3) \to \{1, 4, \ldots, 3q-2\}$, where

$\varphi^*(u_1u_2) = 10$; $\varphi^*(u_iu_{i+1}) = 12i + 1$, $2 \le i \le n-1$; $\varphi^*(u_1u_n) = 13$;
$\varphi^*(u_1x_1) = 1$; $\varphi^*(u_ix_i) = 12i - 8$, $2 \le i \le n$;
$\varphi^*(u_1y_1) = 4$; $\varphi^*(u_iy_i) = 12i - 5$, $2 \le i \le n$;
$\varphi^*(u_1z_1) = 7$; $\varphi^*(u_iz_i) = 12i - 2$, $2 \le i \le n$.

In the view of the above labeling pattern, φ provides an one modulo three harmonic mean labeling for $C_n \odot \overline{K}_3$. Hence $C_n \odot \overline{K}_3$ is an one modulo three harmonic mean graph.

Example: 2.8 An one modulo three harmonic mean labeling of $C_5 \odot \overline{K}_3$ is given in figure 2.5.

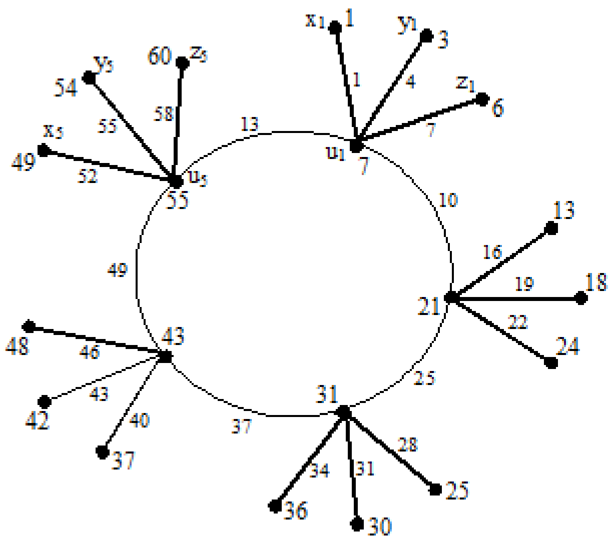

Figure: 2.5

Theorem: 2.9 A graph obtained by attaching $K_{1,3}$ at each vertex of a cycle C_n is an one modulo three harmonic mean graph.

Proof: Let C_n be the cycle $u_1u_2\ldots u_nu_1$. Let v_i, x_i, y_i, z_i be the vertices of i^{th} copy of $K_{1,3}$ in which v_i is the central vertex. Identify z_i with u_i, $1 \le i \le n$. Let the resultant graph be G.

Case- 1. n = 3.
An one modulo three harmonic mean labeling of G when n = 3 is shown in figure 2.6.

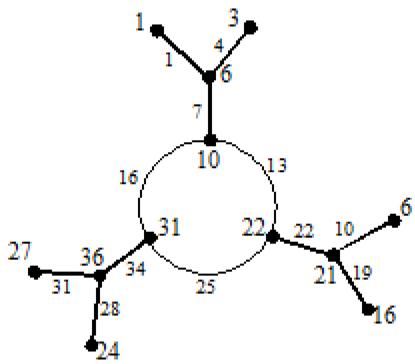

Figure: 2.6

Case: 2. $n \ge 4$.
Define a function $\varphi: V(G) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 7$; $\varphi(u_2) = 21$; $\varphi(u_i) = 12i - 5$, $3 \le i \le n$;
$\varphi(v_1) = 6$; $\varphi(v_2) = 24$; $\varphi(v_3) = 37$; $\varphi(v_i) = 12i$, $4 \le i \le n$;
$\varphi(x_1) = 1$; $\varphi(x_2) = 12$; $\varphi(x_3) = 22$; $\varphi(x_i) = 12(i-1) - 3$, $4 \le i \le n$;
$\varphi(y_1) = 4$; $\varphi(y_2) = 16$; $\varphi(y_i) = 12(i-1) + 3$, $3 \le i \le n$.

Then φ induces a bijective function $\varphi^*: E(G) \to \{1, 4, \ldots, 3q-2\}$, where

$\varphi^*(u_1u_2) = 10$; $\varphi^*(u_iu_{i+1}) = 12i + 1, 2 \le i \le n-1$; $\varphi^*(u_nu_1) = 13$;
$\varphi^*(u_1v_1) = 7$; $\varphi^*(u_2v_2) = 22$; $\varphi^*(u_iv_i) = 12i - 2, 3 \le i \le n$;
$\varphi^*(v_1x_1) = 1$; $\varphi^*(v_ix_i) = 12(i-1) + 4, 2 \le i \le n$;
$\varphi^*(v_1y_1) = 4$; $\varphi^*(v_2y_2) = 19$; $\varphi^*(v_iy_i) = 12i - 5, 3 \le i \le n$.

Thus φ provides an one modulo three harmonic mean labeling for G. Hence G is an one modulo three harmonic mean graph.

Example: 2.10 An one modulo three harmonic mean labeling of G, when n = 7 is shown in figure 2.7.

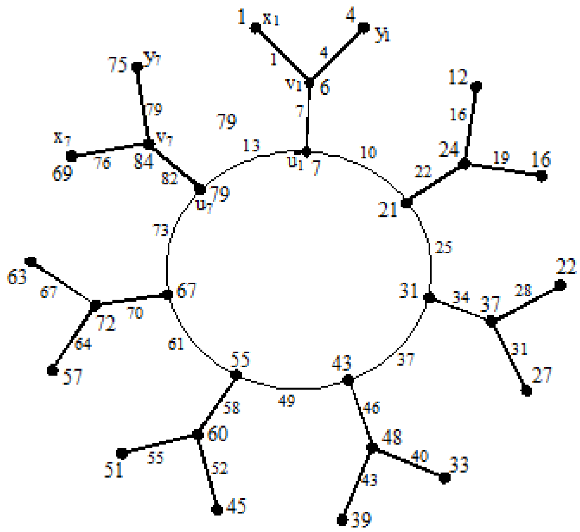

Figure: 2.7

Theorem: 2.11 $L_n \odot K_1$ is an one modulo three harmonic mean graph.

Proof: Let $u_1u_2\ldots u_n$, $v_1v_2\ldots v_n$ be two paths of equal length. Join u_i and v_i, $1 \le i \le n$. The resultant graph is L_n. Add two new vertices x_i, y_i and join these with u_i and v_i respectively, $1 \le i \le n$. The resultant graph is $L_n \odot K_1$. Define a function $\varphi: V(L_n \odot K_1) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 7$; $\varphi(u_2) = 21$; $\varphi(u_i) = 15(i-1) + 3, 3 \le i \le n$;
$\varphi(x_1) = 1$; $\varphi(x_i) = 15(i-1) - 2, 2 \le i \le n$;
$\varphi(v_1) = 9$; $\varphi(v_2) = 22$; $\varphi(v_i) = 15(i-1) + 6, 3 \le i \le n$;
$\varphi(y_1) = 3$; $\varphi(y_2) = 16$; $\varphi(y_i) = 15(i-1) + 9, 3 \le i \le n$.

Then φ induces a bijective function $\varphi^*: E(L_n \odot K_1) \to \{1, 4, 7, \ldots, 3q-2\}$, where

$\varphi^*(u_iu_{i+1}) = 15i - 5, 1 \le i \le n-1$;
$\varphi^*(u_ix_i) = 15i - 14, 1 \le i \le n$;
$\varphi^*(u_iv_i) = 15i - 8, 1 \le i \le 2$; $\varphi^*(u_iv_i) = 15i - 11, 3 \le i \le n$;
$\varphi^*(v_iv_{i+1}) = 15i - 2, 1 \le i \le n-1$;
$\varphi^*(v_iy_i) = 15i - 11, 1 \le i \le 2$.
$\varphi^*(v_iy_i) = 15i - 8, 3 \le i \le n$.

In the view of the above labeling pattern, φ provides an one modulo three harmonic mean labeling for $L_n \odot K_1$. Hence $L_n \odot K_1$ is an one modulo three harmonic mean graph.

Example: 2.12 An one modulo three harmonic mean labeling for $L_8 \odot K_1$ is given in figure 2.8.

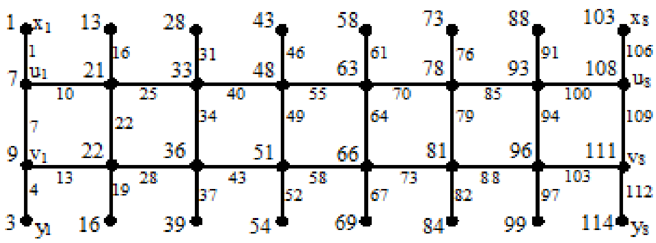

Figure: 2.8

3. CONCLUSION

As all graphs are not an one modulo three harmonic mean graphs, it is very interesting to investigate graphs which admits an one modulo three harmonic mean graphs. It is possible to investigate similar results for several other graphs in the context of different labeling techniques.

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