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b-OPEN SETS AND t-OPEN SETS IN BITOPOLOGICAL SPACES

***P. Thangavelu and G. Thamizharasi**

Department of Mathematics, Aditanar College, Tiruchendur-628216, India

Department of Mathematics, RMD Engineering College, Chennai-601206, India

**E-mail: ptvelu12@gmail.com, pthangavelu_2004@yahoo.co.in*

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ABSTRACT

The purpose of this paper is to characterize b-open sets in bitopological spaces. The concepts of b_t-open sets and t-open sets are also introduced in bitopological spaces and they are studied with existing concepts in bitopological spaces.

Keywords: *Bitopology, b-open sets, t-open sets, p-set, q-set etc.*

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1. introduction and preliminaries:

Abo Khadra and Nasef [1] discussed b-open sets in bitopological spaces. In this paper we further characterize b-open sets in bitopological spaces. We also introduce the notions of b_t-open sets and t-open sets in bitopological spaces and investigate their basic properties. Throughout this paper (X, τ_1, τ_2) denotes a bitopological space, i, j=1, 2 and $i \neq j$. Let A be a subset of X. We use the following notations.

(i) $i\text{-}cl$A = the closure of A with respect to the topology τ_i.

(ii) $i\text{-}int$A = the interior of A with respect to the topology τ_i.

(iii) A is open with respect to τ_i if and only if A is i-open in (X, τ_1, τ_2).

(iv) A is closed with respect to τ_i if and only if A is i-closed in (X, τ_1, τ_2).

Definition: 1.1

A is called
(i) ij-semi-open in (X, τ_1, τ_2) if there exists an i-open set U with $U \subseteq A \subseteq j\text{-}cl U$, [8]

(ii) ij-pre-open in(X, τ_1, τ_2) if there exists an i-open set U with $A \subseteq U \subseteq j\text{-}cl A$, [7]

(iii) ij-b-open in (X, τ_1, τ_2) if $A \subseteq j\text{-}cl(i\text{-}int A) \cup i\text{-}int(j\text{-}cl A)$, [1]

(iv) an i-p-set if $i\text{-}cl(i\text{-}int A) \subseteq i\text{-}int(i\text{-}cl A)$, [11]

(v) an ij-p-set if $i\text{-}cl(j\text{-}int A) \subseteq i\text{-}int(j\text{-}cl A)$, [6]

(vi) a contra ij-p-set in (X, τ_1, τ_2) if $i\text{-}cl(j\text{-}int A) \subseteq j\text{-}int(i\text{-}cl A)$, [13]

**Corresponding author: *P. Thangavelu , *E-mail: ptvelu12@gmail.com*

(vii) an *i*-q-set if *i-int*(i-*cl*A) $\subseteq$ *i-cl*(i-*int*A), [12]

(viii) an *ij*-q-set if *i-int*(*j-cl*A) $\subseteq$ *i-cl*(*j-int*A), [13]

(ix) a pair wise contra p-set in (X, τ_1, τ_2) if it is a contra 12-p-set and a contra 21-p-set, [13]

(x) a contra *ij*-q-set in (X, τ_1, τ_2) if *i-int*(*j-cl*A) $\subseteq$ *j-cl*(*i-int*A), [13]

(xi) a pair wise contra q-set in (X, τ_1, τ_2) if it is a contra 12-q-set and a contra 21-q-set. [13]

The complement of an *ij*-b-open set is *ij*-b-closed. Also *ij*-semi-closed and *ij*-pre-closed sets can be analogously defined. The results in the following lemma follow immediately from the definitions.

Lemma: 1.2
Let A be a subset of (X, τ_1, τ_2). Then A is

(a) *ij*-semi-open if and only if A $\subseteq$ *j-cl* (*i-int*A),

(b) *ij*-pre-open if and only if A $\subseteq$ *i -int*(*j-cl*A),

(c) *ij*-b-closed if and only if *j-int*(*i-cl*A) $\cap$ *i-cl*(*j-int*A) $\subseteq$ A.

The concepts of *i-sint* A, *i- pint*A, *ij-sint*A, *ij-scl*A and *ij*-p*cl*A can be defined in a usual way.

Lemma: 1.3
Let A be a subset of (X, τ_1, τ_2). Then

(i) *i-sint* A = A $\cap$ *i-cl* (*i- int* A), (ii) *i- pint*A = A $\cap$ *i-int*(*i- cl*A). [2]

Lemma: 1.4
Let A be a subset of (X, τ_1, τ_2). Then

(i) *ij-sint*A = A $\cap$ *j-cl*(*i-int*A), (ii) *ij-scl*A = A $\cup$ *j-int*(*i-cl*A). [10]

Definition: 1.5
Let (X, τ) be a topological space. Let A and B be any two subsets of X. We say that (i) A is near to B in (X, τ) if *int*A = *int*B and (ii) A is closer to B in (X, τ) if *cl*A = *cl*B. [9]

Lemma: 1.6
If A is closer to A $\cap$ *i-int*(*j-cl*A) in (X, τ_j) then A $\cap$ *i-int*(*j-cl*A) = *ij-pint*A. [9]

Lemma: 1.7
If A is near to A $\cup$ *i-cl*(*j-int*A) in (X, τ_j) then A $\cup$ *i-cl*(*j-int*A)= *ij-pcl*A. [9]

Definition: 1.8
Let (X, τ_1, τ_2) be a bitopological space. Then

(i) τ_1 is coupled to τ_2 if 1-*cl*U $\subseteq$ 2-*cl*U for every U$\in\tau_1$, [14]

(ii) τ_1 is near τ_2 if 1-*cl*U $\subseteq$ 2-*cl*U for every U $\in\tau_2$. [4 ,5]

Lemma: 1.9
In a bitopological space (X, τ_1, τ_2), the following are equivalent.

(i) τ_1 is coupled to τ_2.

(ii) 2-*int*A $\subseteq$ 1-*int*A for every 1-closed set A in (X, τ_1, τ_2),

(iii) 1-*cl*(1-*int*A) $\subseteq$ 2-*cl*(1-*int*A) for every subset A of X,

(iv) 2-*int*(1-*cl*A) $\subseteq$ 1-*int*(1-*cl*A) for every subset A of X. [13]

Lemma: 1.10
In a bitopological space (X, τ_1, τ_2), the following are equivalent.

(i) τ_1 is near τ_2.

(ii) 2-*int*A $\subseteq$ 1-*int*A for every 2-closed set A in (X, τ_1, τ_2),

(iii) 1-*cl*(2-*int*A) $\subseteq$ 2-*cl*(2-*int*A) for every subset A of X,

(iv) 2-*int*(2-*cl*A) $\subseteq$ 1-*int*(2-*cl*A) for every subset A of X. [13]

Lemma: 1.11
Let B be a subset of (X, τ_1, τ_2). Then B is a contra *ij*-q-set in (X, τ_1, τ_2) if and only if X\B is a contra *ji*-q-set in (X, τ_1, τ_2). [13]

Lemma: 1.12
If A is a contra *ij*-p-set and an *ij*-q- set then *i*-*int*(*j*-*cl*A) $\subseteq$ *j*-*int*(*i*-*cl*A). [13]

Lemma: 1.13.
If A is both an *ij*-p-set and a contra *ji*-q-set then *j*-*int*(*i*-*cl*A) $\subseteq$ *i*-*int*(*j*-*cl*A). [13]

2. *ij*-b-open sets:

Andrijevic [3] introduced the concept of b-open sets in unital topological spaces and Abo Khadra and Nasef [1] extended this notion to bitopological spaces. In this section we characterize *ij*-b-open sets using contra *ij*-p-sets, contra *ij*-q-sets and the corresponding pair wise sets. The concept of pair wise b-open sets is also introduced and studied in this section.

Proposition: 2.1
Let A be *ij*-b-open and a contra *ji*-p-set in (X, τ_1, τ_2). Then it is *ij*-pre-open.

Proof: Since A is *ij*-b-open in (X, τ_1, τ_2), by Definition **1**.1(iii), A $\subseteq$ *j*-*cl*(*i*-*int*A) $\cup$ *i*-*int*(*j*-*cl*A). Since A is a contra *ji*-p-set, using Definition 1.1(vi), *j*-*cl*(*i*-*int*A) $\subseteq$ *i*-*int*(*j*-*cl*A). This implies that A $\subseteq$ *i*-*int*(*j*-*cl*A) so that A is *ij*-pre-open.

Corollary: 2.2
If A is *ij*-b-closed and a contra *ij*-p-set in (X, τ_1, τ_2) then it is *ij*-pre-closed.

Proof: Suppose A is *ij*-b-closed and a contra *ij*-p-set in (X, τ_1, τ_2). Then X\A is *ij*-b-open and is a contra *ji*-p-set. Then using Proposition 2.1, X\A is *ij*-pre-open that implies A is *ij*-pre-closed.

Corollary: 2.3
If A is *ij*-b-clopen and a pair wise contra p-set in (X, τ_1, τ_2) then it is *ij*-pre-clopen.

Proof: Follows from Proposition 2.1 and Corollary 2.2.

Proposition: 2.4
If A is *ij*-b-open and a contra *ij*-q-set in (X, τ_1, τ_2) then it is a *ij*-semi-open set.

Proof: Since A is *ij*-b-open in (X, τ_1, τ_2), by Definition 1.1(iii), A $\subseteq$ *j*-*cl*(*i*-*int*A) $\cup$ *i*-*int*(*j*-*cl*A). Since A is a contra *ij*-q-set, by Definition 1.1(x), *i*-*int*(*j*-*cl*A) $\subseteq$ *j*-*cl*(*i*-*int*A). This implies that A $\subseteq$ *j*-*cl*(*i*-*int*A). Therefore A is *ij*-semi-open.

Corollary: 2.5
If A is *ij*-b-closed and a contra *ji*-q-set in (X, τ_1, τ_2) then it is a *ij*-semi-closed.

Proof: Follows from Proposition 2.4 and Lemma 1.11.

Corollary: 2.6
If A is *ij*-b-clopen and a pair wise contra-q-set in (X, τ_1, τ_2) then it is a *ij*-semi-clopen.
Proof: Follows from Proposition 2.4 and Corollary 2.5.

Proposition: 2.7
If A is *ij*-b-open in (X, τ_1, τ_2) and A is closer to $A \cap$ *i-int*(*j-cl*A) in (X, τ_j) then
A = *ij-sint*A $\cup$ *ij-pint*A.

Proof: Suppose A is *ij*-b-open in (X, τ_1, τ_2). Then A $\subseteq$ *j-cl*(*i-int*A) $\cup$ *i-int*(*j-cl*A) so that A = A $\cap$ (*j-cl*(*i-int*A) $\cup$ *i-int*(*j-cl*A))= (A $\cap$ *j-cl*(*i-int*A)) $\cup$ (A$\cap$ *i-int*(*j-cl*A)).
Then by using Proposition 1.4 (i) and lemma 1.6 we see that A = *ij-sint*A $\cup$ *ij-pint*A,

Proposition: 2.8
If A is *ij*-b-closed in (X, τ_1, τ_2) and A is near to A $\cup$ *i-cl*(*j-int*A) in (X, τ_j) then A = *ij-scl*A $\cap$ *ij-pcl*A.

Proof: Suppose A is *ij*-b-closed in (X, τ_1, τ_2). Then *j-int*(*i-cl*A) $\cap$ *i-cl*(*j-int*A) $\subseteq$ A. Therefore
A = A $\cup$ (*j-int*(*i-cl*A) $\cap$ *i-cl*(*j-int*A)) = (A $\cup$ *j-int*(*i-cl*A)) $\cap$ (A $\cup$ *i-cl*(*j-int*A)). Then by using Lemma 1.4(ii) and Lemma 1.7 we have A = *ij-scl*A $\cup$ *ij-pcl*A,

Definition: 2.9
A subset B of a bitopological space (X, τ_1, τ_2) is called pair wise b-open in (X, τ_1, τ_2) if B is 12-b-open and 21-b-open.

The next proposition and the subsequent corollaries follow respectively from Proposition 2.4, Corollary 2.5 and Corollary 2.6.

Proposition: 2.10
If A is pair wise b-open and a pair wise contra q-set in (X, τ_1, τ_2) then it is pair wise semi-open.

Corollary: 2.11
If A is pair wise b-closed and a pair wise contra q-set in (X, τ_1, τ_2) then it is pair wise semi-closed.

Corollary: 2.12
If A is pair wise b-clopen and a pair wise contra-q-set in (X, τ_1, τ_2) then it is pair wise semi-clopen.

3. *ij*-b_t-open sets:

In this section the concepts of b_t-open sets and pair wise b_t-open sets in bitopological spaces are introduced and their properties are investigated.

Definition: 3.1
A subset B of a bitopological space (X, τ_1, τ_2) is called ij-b_t-open in (X, τ_1, τ_2) if B $\subseteq$ *j-cl*(*i-int*B) $\cup$ *j-int*(*i-cl*B).
The next proposition follows from Lemma 1.2 and Definition 3.1.

Proposition: 3.2

(i) Every *ij*-semi-open set is *ij*-b_t-open.

(ii) Every *ji*-pre-open set is *ij*-b_t-open.

The next lemma can be easily proved.

Lemma: 3.3
B $\subseteq$ *j-cl*(*i-int*B) $\cup$ *j-int*(*i-cl*B) if and only if *j-cl*(*i-int*(X\B)) $\cap$ *j-int*(*i-cl*(X\B)) $\subseteq$ X\B.

Proposition: 3.4
If A is a *ji*-q-set and *ij*-b_t-open then it is *ij*-semi-open.

Proof: Suppose A is a *ji*-q-set and *ij*-b_t-open. Then by Definition 1.1(viii) and by Definition 3.1, we get $j\text{-}int(i\text{-}cl A) \subseteq j\text{-}cl(i\text{-}int A)$ and $A \subseteq j\text{-}cl(i\text{-}int A) \cup j\text{-}int(i\text{-}cl A)$. This implies that $A \subseteq j\text{-}cl(i\text{-}int A)$. Therefore A is *ij*-semi-open.

Proposition: 3.5
If A is a *ji*-p-set and *ij*-b_t-open then it is *ji*-pre-open.

Proof: Suppose A is a *ji*-p-set and *ij*-b_t-open. Then by Definition 1.1(v) and by Definition 3.1, we get $j\text{-}int(i\text{-}cl A) \supseteq j\text{-}cl(i\text{-}int A)$ and $A \subseteq j\text{-}cl(i\text{-}int A) \cup j\text{-}int(i\text{-}cl A)$. This implies that $A \subseteq j\text{-}int(i\text{-}cl A)$. Therefore A is *ji*-pre-open

Proposition: 3.6
Let B be *ij*-b_t-open in (X, τ_1, τ_2) and let B be closer to $B \cap j\text{-}int(i\text{-}cl B))$ in (X, τ_i). Then $B = ij\text{-}sint B \cup ji\text{-}pint B$.

Proof: $B = B \cap (j\text{-}cl(i\text{-}int B) \cup j\text{-}int(i\text{-}cl B)) = (B \cap j\text{-}cl(i\text{-}int B)) \cup (B \cap j\text{-}int(i\text{-}cl B))$. Then by using Lemma 1.4(i) and Lemma 1.6 we see that $B = ij\text{-}sint B \cup ji\text{-}pint B$,

Proposition: 3.7
Suppose A is *ij*-b-open, a contra *ij*-p-set and an *ij*-q-set. Then A is *ij*-b_t-open.

Proof: Since A is *ij*-b-open , by Definition 1.1(iii), $A \subseteq j\text{-}cl(i\text{-}int A) \cup i\text{-}int(j\text{-}cl A)$.Since A is a contra *ij*-p-set and an *ij*-q-set, by Lemma 1.12, $i\text{-}int(j\text{-}cl A) \subseteq j\text{-}int(i\text{-}cl A)$ that gives $A \subseteq j\text{-}cl(i\text{-}int A) \cup j\text{-}int(i\text{-}cl A)$.

Then by Definition 3.1, A is *ij*-b_t-open.

Proposition: 3.8
Suppose A is *ji*-b-open, an *ij*-p-set and a contra *ji*-q-set. Then A is *ji*-b_t-open.

Proof: Since A is *ji*-b-open , by Definition 1.1(iii), $A \subseteq i\text{-}cl(j\text{-}int A) \cup j\text{-}int(i\text{-}cl A)$. Since A is an *ij*-p-set and a contra *ji*-q-set, by Lemma 1.13. $j\text{-}int(i\text{-}cl A) \subseteq i\text{-}int(j\text{-}cl A)$ that gives $A \subseteq i\text{-}cl(j\text{-}int A) \cup i\text{-}int(j\text{-}cl A)$.

Then by Definition 3.1, A is *ji*-b_t-open.

The complement of an *ij*-b_t-open set is *ij*-b_t-closed. It follows from Lemma 3.3 that B is *ij*-b_t-closed if and only if the relation $j\text{-}cl(i\text{-}int B) \cap j\text{-}int(i\text{-}cl B) \subseteq B$ holds. The next proposition follows from Proposition 3.2, Proposition 3.4 and Proposition 3.5.

Proposition: 3.9
(i) Every *ij*-semi-closed set is *ij*-b_t-closed.

(ii) Every *ji*-pre-closed set is *ij*-b_t-closed.

(iii) If A is a *ji*-q-set and *ij*-b_t-closed then it is *ij*-semi-closed.

(iv) If A is a *ji*-p-set and *ij*-b_t-closed then it is *ji*-pre-closed.

Proposition: 3.10
Suppose B is *ij*-b_t-closed, *ij*-semi-open and *ji*-pre-open in (X, τ_1, τ_2). Then $B = j\text{-}cl(i\text{-}int B) \cap j\text{-}int(i\text{-}cl B)$.

Proof: Since B is *ij*-b_t-closed, $j\text{-}cl(i\text{-}int B) \cap j\text{-}int(i\text{-}cl B) \subseteq B$. Therefore $B = B \cup (j\text{-}cl(i\text{-}int B) \cap j\text{-}int(i\text{-}cl B)) = (B \cup j\text{-}cl(i\text{-}int B)) \cap (B \cup j\text{-}int(i\text{-}cl B)) = j\text{-}cl(i\text{-}int B) \cap j\text{-}int(i\text{-}cl B)$, using Lemma 1.2 (a) and Lemma 1.2(b).

Definition: 3.11
A subset B of a bitopological space (X, τ_1, τ_2) is called pair wise b_t-open in (X, τ_1, τ_2) if B is 12-b_t-open and 21-b_t-open.

The following proposition follows from Proposition 3.2 and Proposition 3.4.

Proposition: 3.12

(i) Every pair wise semi-open set is pair wise b_t-open.

(ii) Every pair wise pre-open set is pair wise b_t-open.
(iii) If A is a pair wise q-set and pair wise b_t-open then it is pair wise semi-open.

Proposition: 3.13
If A is a pair wise p-set and pair wise b_t-open then it is pair wise pre-open.

Proof: Suppose A is a pair wise p-set and pair wise b_t-open set.
Since A is a *ji*-p-set and an *ij*-b_t-open set, by Proposition 3.5, A is *ji*-pre-open. Since A is an *ij*-p-set and a *ji*-b_t-open set, again by Proposition 3.5, A is *ij*-pre-open. Therefore A is pair wise pre-open.

4. *ij*-t-open sets:

In this section the notion of *ij*-t-open sets is introduced in bitopological spaces and their relationships with p-sets, q-sets, b-open sets, semi-open sets and pre-open sets are studied.

Definition: 4.1
A subset A of a bitopological space (X, τ_1, τ_2) is called *ij*-t-open if A $\subseteq$ *i-cl*(*i-int*A) $\cup$ *j-int*(*j-cl*A).

Proposition: 4.2
If A is an *ij*-t-open set then A is a union of an *i*-semi-open set and a *j*-pre-open set.

Proof: Suppose A is an *ij*-t-open set. Then by Definition 4.1, A $\subseteq$ *i-cl*(*i-int*A) $\cup$ *j-int*(*j-cl*A).
Therefore A = A $\cap$ (*i-cl*(*i-int*A) $\cup$ *j-int*(*j-cl*A)) = (A $\cap$ *i-cl*(*i-int*A)) $\cup$ (A $\cap$ *j-int*(*j-cl*A)= *i-sint*A $\cup$ *j-pint*A , by Lemma 1.3(i) and Lemma 1.3(ii).

Proposition: 4.3
(i) Every *i*-semi-open set is *ij*-t-open,

(ii) Every *j*-pre-open set is *ij*-t-open.

Proof: Let A be *i*-semi-open. Then A $\subseteq$ *i-cl*(*i-int*A).Therefore A $\subseteq$ *i-cl*(*i-int*A) $\cup$ *j-int*(*j-cl*A). Then by Definition 4.1, A is *ij*-t-open. Now let A be *j*-pre-open. Then A $\subseteq$ *j-int*(*j-cl*A). Therefore A $\subseteq$ *i-cl*(*i-int*A) $\cup$ *j-int*(*j-cl*A). Then by Definition 4.1, A is *ij*-t-open.

Proposition: 4.4
Suppose A is an *i*-p-set and a *j*-q-set. Then if A is *ij*-t-open then it is *ji*-t-open.

Proof: Since A is an *i*-p-set, using Definition 1.1(iv), *i-cl*(*i-int*A) $\subseteq$ *i-int*(*i-cl*A). Since A is a *j*-q-set, using Definition 1.1(vii), *j-int*(*j-cl*A) $\subseteq$ *j-cl*(*j-int*A). If A is *ij*-t-open, by Definition 4.1, A $\subseteq$ *i-cl*(*i-int*A) $\cup$ *j-int*(*j-cl*A) $\subseteq$ *i-int*(*i-cl*A)$\cup$ *j-cl*(*j-int*A) that implies A is *ji*-t-open.

Proposition: 4.5
Suppose τ_i is coupled to τ_j and τ_i is near τ_j then every *ij*-t-open set is *ij*-b-open.

Proof: Let A be *ij*-t-open. Then A $\subseteq$ *i-cl*(*i-int*A) $\cup$ *j-int*(*j-cl*A). Then by using Lemma 1.9 and by Lemma 1.10 we see that A $\subseteq$ *j-cl*(*i-int*A)$\cup$ *j-int*(*j-cl*A)$\subseteq$ *j-cl*(*i-int*A)$\cup$ *i-int*(*j-cl*A)),

Now by Definition 1.1(iii)**,** A is *ij*-b-open.

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