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EFFECT OF FIRST ORDER CHEMICAL REACTION IN A VERTICAL DOULBE PASSAGE CHANNEL

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ABSTRACT

This work is aimed at describing free convective heat and mass transfer in a vertical channel in the presence of chemical reaction. The considered channel is divided into two passages by means of a thin plane baffle. Each stream has its own pressure gradient and hence the velocity, temperature and concentration will be individual in each stream. After placing the baffle, the fluid in one of the passage is concentrated. The first order chemical reaction is used as an example of calculation to obtain the analytical solutions. The coupled non-linear ordinary differential equations governing the fluid motion is solved using regular perturbation method. The temperature and concentration fields are observed to be governed by complex interactions among dispersions and natural convection mechanisms. Results are drawn for varying physical parameters, such as ratio of Grashof number to Reynolds number, modified Grashof number to Reynolds number, Brinkman number, and chemical reaction parameter on the flow field at different positions of the baffle. It is found that the ratio of Grashof number to Reynolds number, modified Grashof number to Reynolds number, Brinkman number enhances the flow where as first order chemical reaction parameter suppresses the flow at all the baffle positions in both the streams.

Keywords: *Baffle, first order chemical reaction, free convection, perturbation method*

1. INTRODUCTION

The phenomenon of natural convection heat transfer plays an important role, both in nature and in engineering systems. Many investigations have been performed for cavities both theoretically and experimentally [1-2]. Recently, heat transfer in partially divided enclosures has received attention primarily due to its application in the design of energy efficient buildings and reduction of heat loss from flat plate solar collectors. Most works about cavities of complex geometry deal with partitions fitted to insulated walls. Cavities with baffles on their active walls have been less studied.

When the channel is divided into several passages by means of plane baffle, as usually occurs in heat exchangers or electronic equipment, it is quite possible to enhance the heat transfer performance between the walls and fluid by the adjustments of each baffle position and strengths of the separate flow streams. In such configurations, perfectly conductive and thin baffles may be used to avoid significant increase of the transverse thermal resistance. For a number of fluids, the density-temperature relation exhibits an extreme. Because the coefficient of thermal expansion changes signs at this extreme, simple linear relations for density as a function of temperature are inadequate near the extreme.

The natural convection heat transfer from an isothermal vertical wavy surface was first studied by Yao [3, 4], and using an extended Prandtl's transposition theorem and a finite difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surface. Moulic and Yao [5] analyzed mixed convection with thermal diffusion. Heat transfer enhancement in a heat exchanger tube by installing a baffle was reported by Nasiruddin and Siddiqui [6]. The effect of baffle size and orientation on the heat transfer enhancement was studied in detail. Three different baffle arrangements were considered. Zhou et al. [7] applied numerical modeling to study the performance of circular clarifiers with reaction baffles under various ranges of suspended solid concentrations and hydraulic loadings.

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Recently several studies by Rathish Kumar *et al*. [8, 9], Murthy *et al.* [10] and Kumar and Shalini [11] have been reported and were concerned with natural convection heat transfer in wavy vertical porous enclosures. The rate of heat transfer in a vertical channel could be enhanced by using special inserts. These inserts can be specially designed to increase the included angle between the velocity vector and the temperature gradient vector, rather than to promote turbulence. This increases the rate of heat transfer without a considerable drop in the pressure (Guo and Wang [12]). A plane baffle may be used as an insert to enhance the rate of heat transfer in the channel. A thin and perfectly conductive baffle is used so as to avoid a considerable increase in the transfer aspects of a laminar fully developed forced–convection within an asymmetrically heated horizontal double–passage channel and concluded that the thermal characteristics of the fully developed flow could be significantly affected by the position of the baffle, the pressure gradient ratio and the thermal boundary conditions. Similar mixed convection problem in a vertical double-passage channel has been investigated analytically by Salah El-Din [13]. Salah El-Din [14] investigated numerically mixed convection in a vertical double-passage channel, taking into account the effect of viscous dissipation. He drew the conclusion that the increase in Brinkman number decreases the Nusselt number on the hot wall and increases that on the cold wall especially when the baffle becomes near the hot wall or the cold wall, respectively.

The aim of this paper is to investigate effect of chemical reaction, heat and mass transfer of viscous fluid with in vertical channel for asymmetric wall temperature distribution with different location of the baffle. After inserting the baffle, the fluid in stream-1 is concentrated. Analytical solutions are found using regular perturbation method, using Brinkman number as the perturbation parameter.

2. MATHEMATICAL FORMULATION

Consider a steady, two-dimensional laminar fully developed free convective flow in an open ended vertical channel filled with purely viscous fluid. The *X*-axis is taken vertically upward, and parallel to the direction of buoyancy, and the *Y*-axis is normal to it (Fig. 1). Walls are maintained at a constant temperature and the fluid properties are assumed to be constant. The channel is divided into two passages by means of thin, perfectly conducting plane baffle and each stream will have its own pressure gradient and hence the velocity will be individual in each stream.

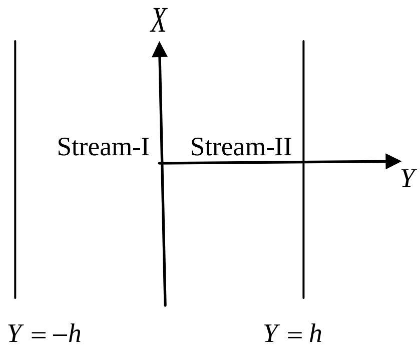


Fig. 1: Physical Configuration.

The governing equations for velocity, temperature and concentration are

Stream-I

$$\rho g \beta_T \left(T_1 - T_{W_2}\right) + \rho g \beta_C (C - C_0) - \frac{dP}{dX} + \mu \frac{d^2 U_1}{dY^2} = 0 \tag{1}$$

$$\frac{d^2 T_1}{dY^2} + \frac{\nu}{\alpha C_P}\left(\frac{dU_1}{dY}\right)^2 = 0 \tag{2}$$

$$D\frac{d^2 C}{dY^2} - k\, C = 0 \tag{3}$$

Stream-II

$$\rho g \beta_T \left(T_2 - T_{W_2}\right) - \frac{dP}{dX} + \mu \frac{d^2 U_2}{dY^2} = 0 \tag{4}$$

$$\frac{d^2 T_2}{dY^2} + \frac{\nu}{\alpha C_P}\left(\frac{dU_2}{dY}\right)^2 = 0 \tag{5}$$

Subject to the boundary conditions on velocity, temperature and concentration as

$U_1 = 0$, $T_1 = T_{W_1}$, $C = C_1$, at $Y = -h$

$U_2 = 0$, $T_2 = T_{W_2}$, at $Y = h$

$$U_1 = 0 \ , \ U_2 = 0 \ , \ T_1 = T_2 \ , \ \frac{dT_1}{dY} = \frac{dT_2}{dY} \ , \ C = C_2 \ , \text{ at } \ Y = h^* \tag{6}$$

Introducing the following non-dimensional variables in the governing equations for velocity, temperature and concentration as,

$$u_i = \frac{U_i}{U_1} \ , \quad \theta_i = \frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}} \ , \quad Gr = \frac{g\beta_T \Delta T h^3}{\upsilon^2} \ , \quad G_C = \frac{g\beta_C \Delta C h^3}{\upsilon^2} \ , \quad \mathrm{Re} = \frac{\bar{U}_1 h}{\upsilon} \ , \quad Br = \frac{\bar{U}_1^2 \mu}{k\Delta T} \ , \quad p = \frac{h^2}{\mu \bar{U}_1}\frac{dp}{dX} \ ,$$

$$\phi = \frac{C - C_0}{C_1 - C_0} \ , \quad \Delta T = T_{W_2} - T_{W_1} \ , \ \Delta C = C_1 - C_0 \ , \ Y^* = \frac{y^*}{h} \ , \quad Y = \frac{y}{h} \tag{7}$$

one obtains the momentum, energy and concentration equations corresponding to stream-I and stream-II as

Stream-I

$$\frac{d^2u_1}{dy^2} + GR_T \ \theta_1 + GR_c \ \phi - p = 0 \tag{8}$$

$$\frac{d^2\theta_1}{dy^2} + Br\left(\frac{du_1}{dy}\right)^2 = 0 \tag{9}$$

$$\frac{d^2\phi}{dy^2} - \alpha^2\phi = 0 \tag{10}$$

Stream-II

$$\frac{d^2u_2}{dy^2} + GR_T \ \theta_2 - p = 0 \tag{11}$$

$$\frac{d^2\theta_2}{dy^2} + Br\left(\frac{du_2}{dy}\right)^2 = 0 \tag{12}$$

Subject to the boundary conditions,

$u_1 = 0$, $\theta_1 = 1$, $\phi = 1$, at $y = -1$

$u_2 = 0$, $\theta_2 = 0$, at $y = 1$

$$u_1 = 0 \ , \quad u_2 = 0 \ , \ \theta_1 = \theta_2 \ , \ \frac{d\theta_1}{dy} = \frac{d\theta_2}{dy} \ , \ \phi = n \ , \text{ at } \ y = y^* \tag{13}$$

where $\alpha = \frac{kh^2}{D}$, $n = \frac{C_2 - C_0}{C_1 - C_0}$

3. SOLUTIONS

The solution of equation (10) using boundary condition (13) become

$$\phi = B_1 Cosh(\alpha y) + B_2 Sinh(\alpha y) \tag{14}$$

Equations (8), (9), (11) and (12) are coupled non-linear ordinary differential equations and hence closed form solutions can not be found. However approximate solutions can be found by using the regular perturbation method. The perturbation parameter Br is usually small and hence regular perturbation method can be strongly justified. Adopting this technique, solutions for velocity and temperature are assumed in the form

$$u_i(y) = u_{i0}(y) + Br \ u_{i1}(y) + Br^2 \ u_{i2}(y) + \ldots \tag{15}$$

$$\theta_i(y) = \theta_{i0}(y) + Br \ \theta_{i1}(y) + Br^2 \ \theta_{i2}(y) + \ldots \tag{16}$$

Substituting equations (15) and **(**16) in equations (8), (9), (11) and (12) and equating the coefficients of like power of Br to zero and one, we obtain the zero and first order equations as

Stream-I Zeroth order equations

$$\frac{d^2\theta_{10}}{dy^2}=0 \tag{17}$$

$$\frac{d^2u_{10}}{dy^2}+GR_T\ \theta_{10}+GR_c\ \phi-p=0 \tag{18}$$

First order equations

$$\frac{d^2\theta_{11}}{dy^2}+\left(\frac{du_{10}}{dy}\right)^2=0 \tag{19}$$

$$\frac{d^2u_{11}}{dy^2}+GR_T\ \theta_{11}=0 \tag{20}$$

Stream-II Zeroth order equations

$$\frac{d^2\theta_{20}}{dy^2}=0 \tag{21}$$

$$\frac{d^2u_{20}}{dy^2}+GR_T\ \theta_{20}-p=0 \tag{22}$$

First order equations

$$\frac{d^2\theta_{21}}{dy^2}+\left(\frac{du_{20}}{dy}\right)^2=0 \tag{23}$$

$$\frac{d^2u_{21}}{dy^2}+GR_T\ \theta_{21}=0 \tag{24}$$

The corresponding boundary conditions reduces to Zeroth-order

$u_{10}=0$, $\theta_{10}=1$, $\phi=1$, at $y=-1$

$u_{20}=0$, $\theta_{20}=0$, at $y=1$

$$u_{10}=0,\quad u_{20}=0,\quad \theta_{10}=\theta_{20},\quad \frac{d\theta_{10}}{dy}=\frac{d\theta_{20}}{dy},\quad \phi=n,\ \text{at}\ y=y^* \tag{25}$$

First order

$u_{11}=0$, $\theta_{11}=0$ at $y=-1$

$u_{21}=0$, $\theta_{21}=0$ at $y=1$

$$u_{11}=0,\quad u_{21}=0,\quad \theta_{11}=\theta_{21},\quad \frac{d\theta_{11}}{dy}=\frac{d\theta_{21}}{dy}\quad \text{at}\ y=y^* \tag{26}$$

The solutions of zeroth and first order equations (17) to (24) using the boundary conditions as given in equations (25) and (26) are Zeroth-order

Stream-I

$$\theta_{10}=z_1y+z_2 \tag{27}$$

$$u_{10}=A_2+A_1y+r_1y^2+r_2y^3+r_4Cosh\left(\alpha y\right)+r_5Sinh\left(\alpha y\right) \tag{28}$$

Stream-II

$$\theta_{20}=z_3y+z_4 \tag{29}$$

$$u_{20}=A_4+A_3y+r_5y^2+r_6y^3 \tag{30}$$

First order

Stream-I

$$\begin{aligned}\theta_{11} &= G_2 + G_1 y + p_1 y^2 + p_2 y^3 + p_3 y^4 + p_4 y^5 + p_5 y^6 + p_6 Cosh(2\alpha y) \\ &+ p_7 Sinh(2\alpha y) + p_8 Cosh(\alpha y) + p_9 Sinh(\alpha y) + p_{10} y Cosh(\alpha y) \\ &+ p_{11} y Sinh(\alpha y) + p_{12} y^2 Cosh(\alpha y) + p_{13} y^2 Sinh(\alpha y)\end{aligned} \quad (31)$$

$$\begin{aligned}u_{11} &= G_6 + G_5 y + R_1 y^2 + R_2 y^3 + R_3 y^4 + R_4 y^5 + R_5 y^6 + R_6 y^7 + R_7 y^8 \\ &+ R_8 Cosh(2\alpha y) + R_9 Sinh(2\alpha y) + R_{10} Cosh(\alpha y) + R_{11} Sinh(\alpha y) \\ &+ R_{12} y Cosh(\alpha y) + R_{13} y Sinh(\alpha y) + R_{14} y^2 Cosh(\alpha y) + R_{15} y^2 Sinh(\alpha y)\end{aligned} \quad (32)$$

Stream-II

$$\theta_{21} = G_4 + G_3 y + q_1 y^2 + q_2 y^3 + q_3 y^4 + q_4 y^5 + q_5 y^6 \quad (33)$$

$$u_{21} = G_8 + G_7 y + S_1 y^2 + S_2 y^3 + S_3 y^4 + S_4 y^5 + S_5 y^6 + S_6 y^7 + S_7 y^8 \quad (34)$$

The constants appeared in all the above equations are presented the section Appendix

4. RESULTS AND DISCUSSIONS

The purpose of this study is to bring out the effect of position of the baffle on the flow and heat transfer in a vertical channel. Nonlinear coupled ordinary differential equations governing the flow have been solved by regular perturbation method. The solutions are found up to the order of one of the perturbation parameter. The ratio of Grashof number to Reynolds number GR_T, modified Grashof number to Reynolds number GR_C, pressure gradient p, and chemical reaction parameter α, are fixed as 5, 5, -5 and 0.5 respectively, for all the graphs except the varying one.

The effect of ratio of Grashof number to Reynolds number GR_T on the velocity and temperature is shown in figure 2a, b, c and figure 3 a, b, c at all the three different baffle positions $(y^* = -0.8,\ 0,\ 0.8)$. As the ratio of Grashof number to Reynolds number increases, the velocity and temperature increases at the hot (left) wall and cold (right) wall at all the baffle position. It is seen from figure 2b and figure 3b that the maximum point of velocity and temperature is in stream-I, however the maximum velocity and temperature should be equal in both the streams when the baffle position is in the center of the channel. For the present problem this is not true because the left wall is at higher temperature compared to the right wall, hence the convection is high in stream-I when compared to stream-II. Further it is well-known that since Grashof number is the ratio of buoyancy force to viscous force, increase in Grashof number increase the buoyancy force and hence increases the flow. Therefore as the ratio of Grashof number to Reynolds number increases velocity and temperature increases at all baffle position in both the streams.

The effect of ratio of modified Grashof number to Reynolds number GR_C on the flow field is shown in figure 4 a, b, c for velocity field and in figure 5 a, b, c for the temperature field. Here also the effect of GR_C is to increase the velocity and temperature field in both the streams. It is seen from figure 4a and 5a that there no much effect of GR_C on the velocity and temperature fields. This is due to the fact that the baffle position is near the left wall and the fluid is concentrated only in stream-I. However magnifying the graphs one can see that velocity increases as GR_C increases where as there is no effect of GR_C in stream-II. The flow field is enhanced as GR_C increases in both the streams for the baffle position at $y^* = 0$ and 0.8. This is an expected result for, the ratio of modified Grashof number to Reynolds number, is the ratio of concentration buoyancy force to viscous force. Hence increase in GR_C implies increase in concentration buoyancy force which increases the flow field. The similar result was also observed by Fasogbon [15] for irregular channel. It is seen from figures 4 and 5 that the optimum values for velocity and temperature is seen in stream-I for the baffle position at $y^* = 0$ and 0.8. When the baffle position is at the centre of the channel the maximum velocity is in stream-I due to the fact that the fluid in the stream-I is concentrated and also left wall is at higher temperature.

The effect of Brinkman number *Br* on the velocity and temperature fields are shown in figures 6 a, b, c and figures 7 a, b, c respectively. As the Brinkman number increases both the velocity and temperature increases in both the streams at all baffle positions. One can see from temperature equation that increase in Brinkman number increases the viscous dissipation and hence the temperature is enhanced.

The effect of first order chemical reaction parameter α , on velocity, temperature and concentration fields are seen in figures 8 a, b, c, 9 a, b, c and 10 a, b, c respectively. As α increases the velocity and temperature decreases in stream-I, and remains constant in stream-II .The similar result was also obtained by Srinivas and Muturajan [16] for mixed convective flow in a vertical channel. This is due to the fact that the fluid in stream-I is concentrated. The maximum value of velocity and temperature is seen in stream-II for the baffle position at $y^* = -0.8$ and in stream-I for the baffle position at $y^* = 0$ and 0.8.

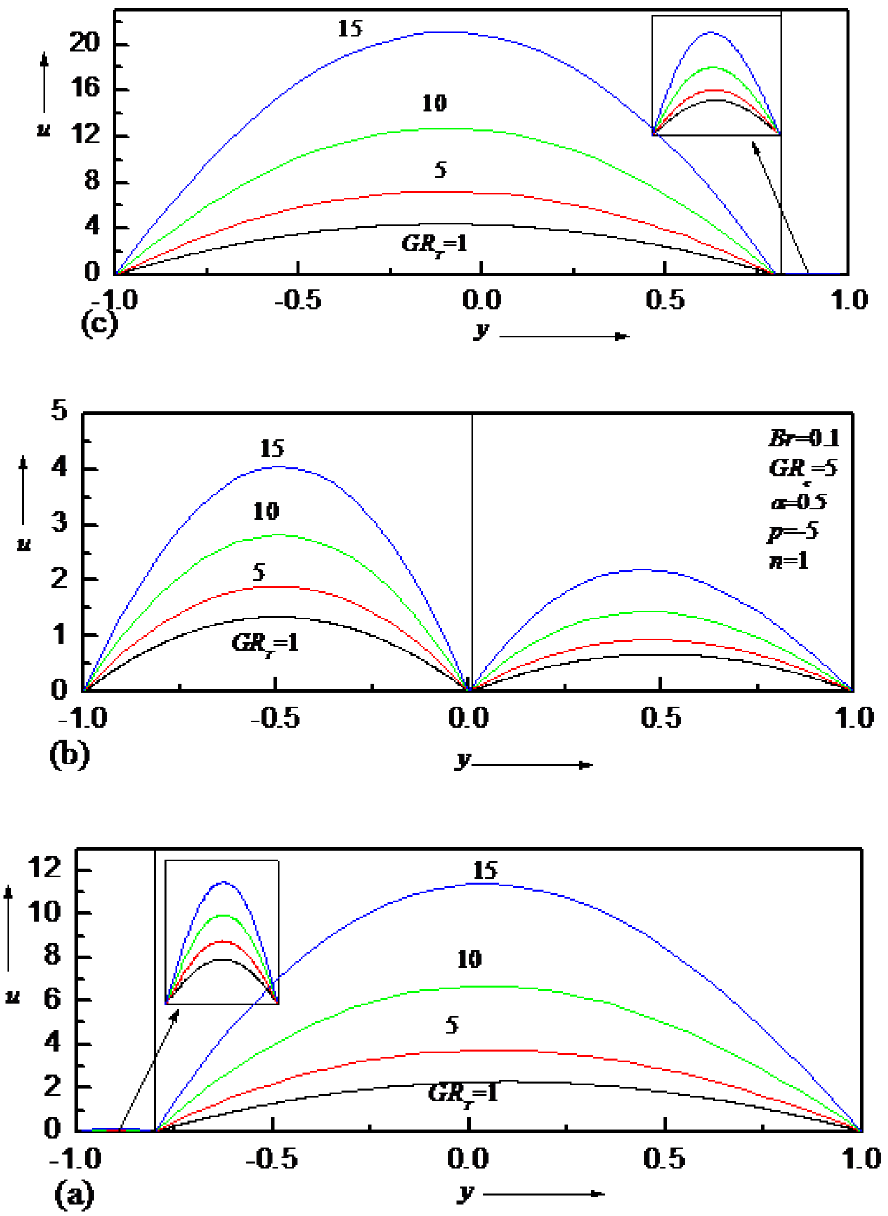


Fig.2: velocity profiles for different values of ratio of Grashof number to Reynolds number GR_T at (a) $y^* = -0.8$ (b) $y^* = 0.0$ (c) $y^* = 0.8$

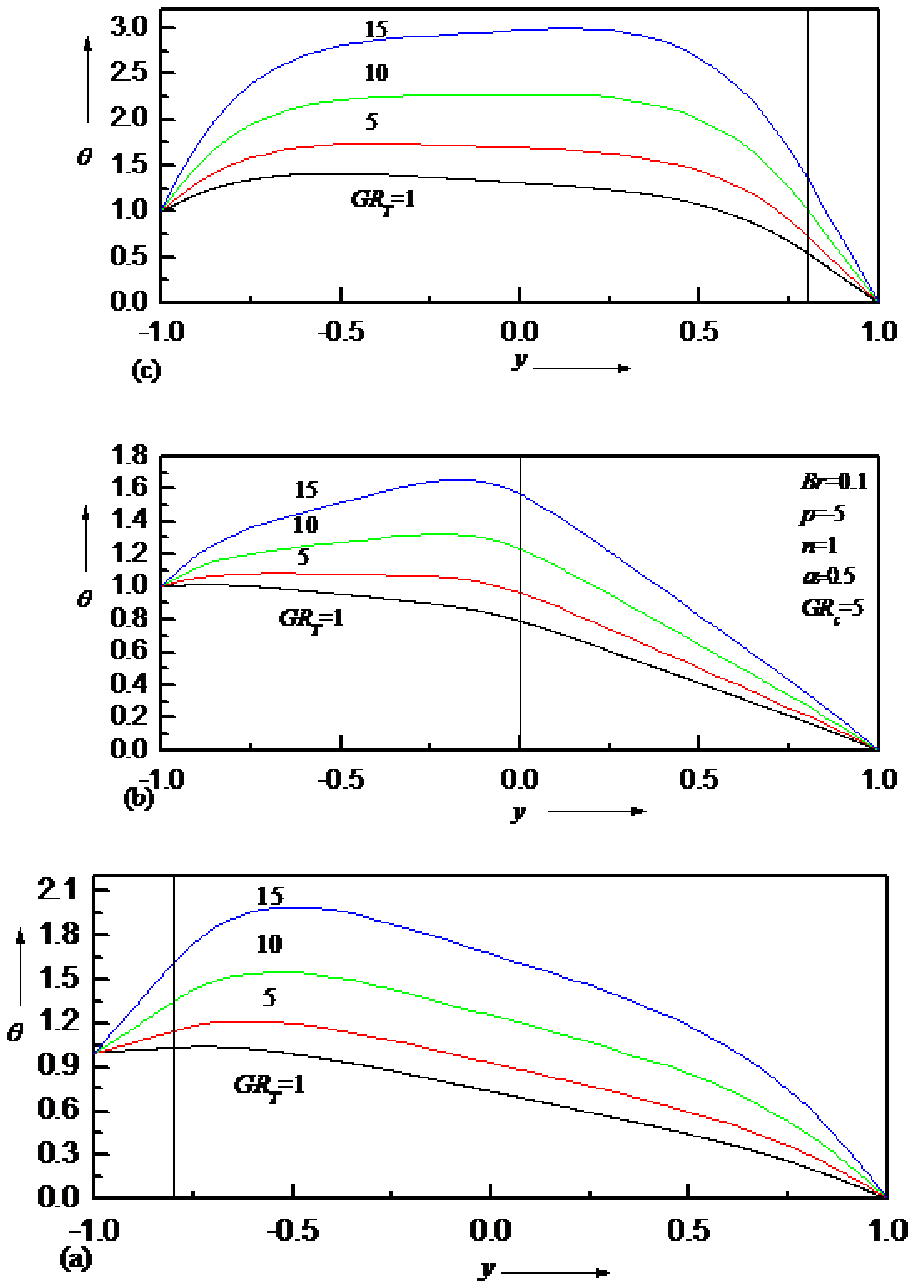
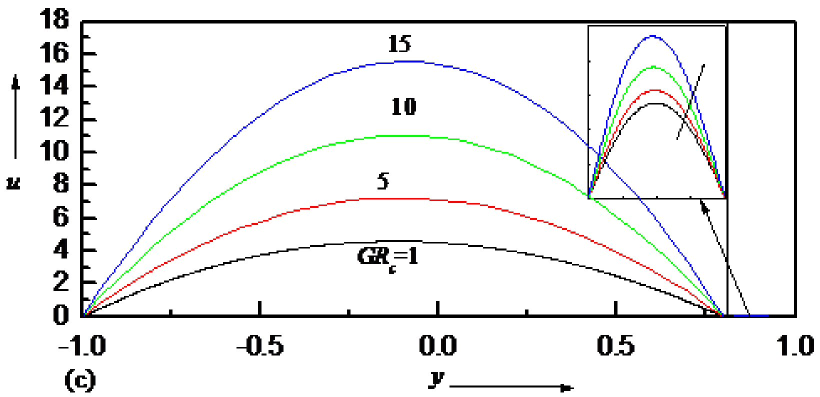


Fig. 3: Temperature profiles for different values of ratio of Grashof number to Reynolds number GR_T at (a) y^* = 0.8 (b) y^* = 0 (c) y^* = 0.8



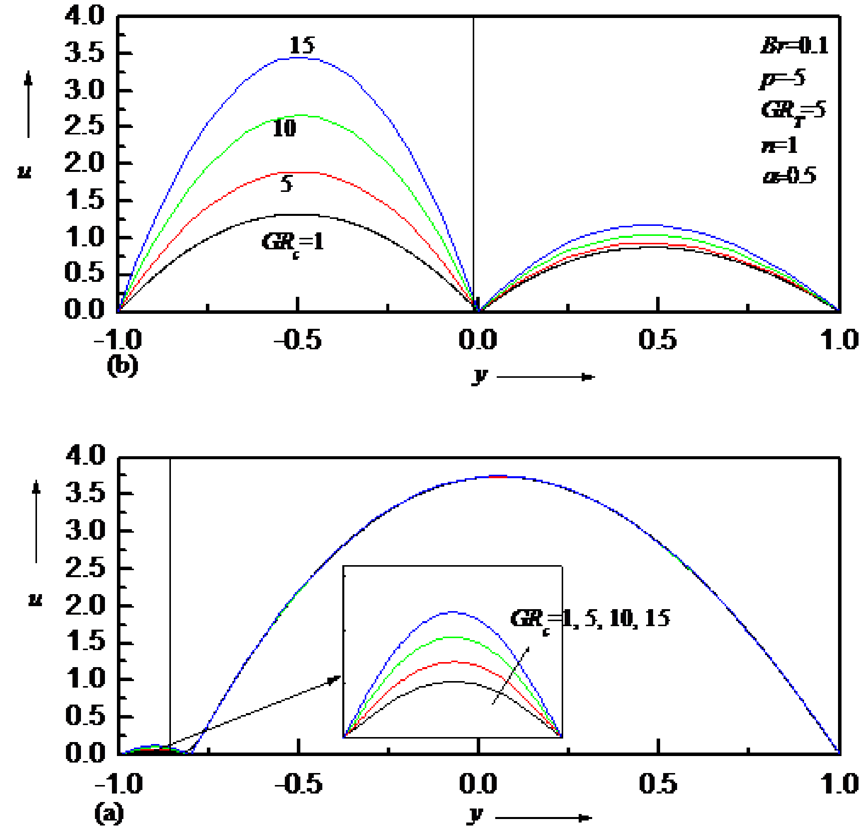
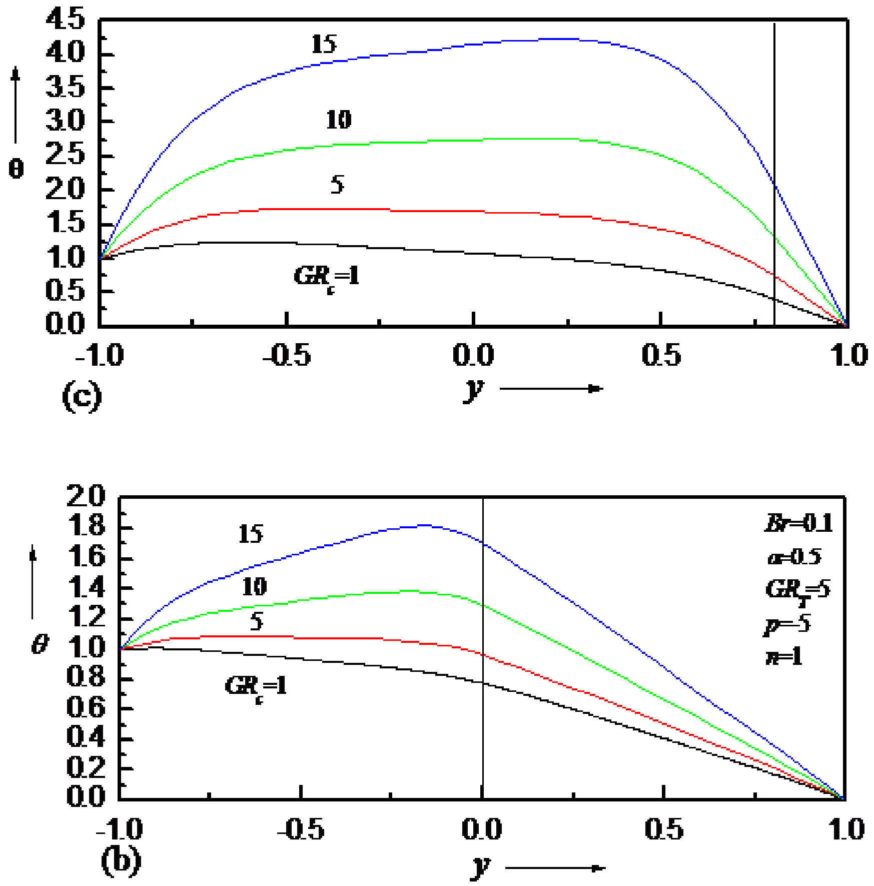


Fig.4: Velocity profiles for different values of ratio of modified Grashof number to Reynolds number GR_c at (a)y^*=-0.8 (b)y^*=0 (c)y^*=0.8



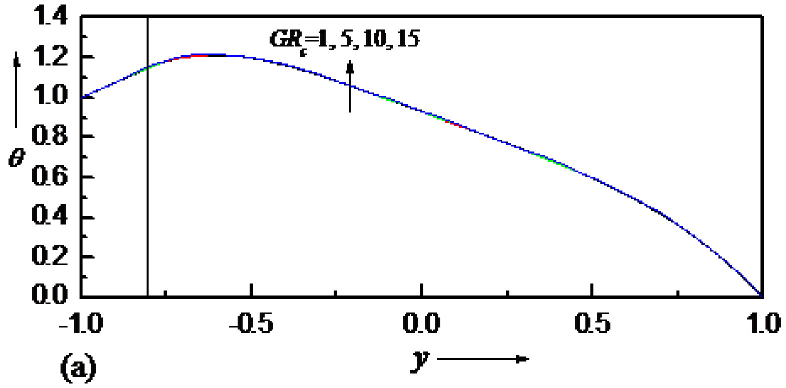
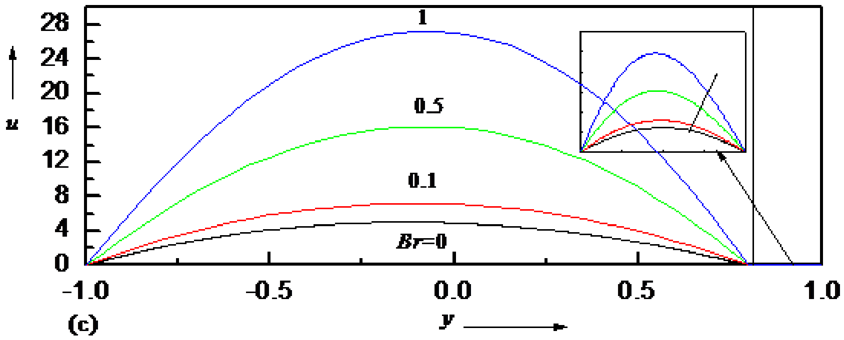
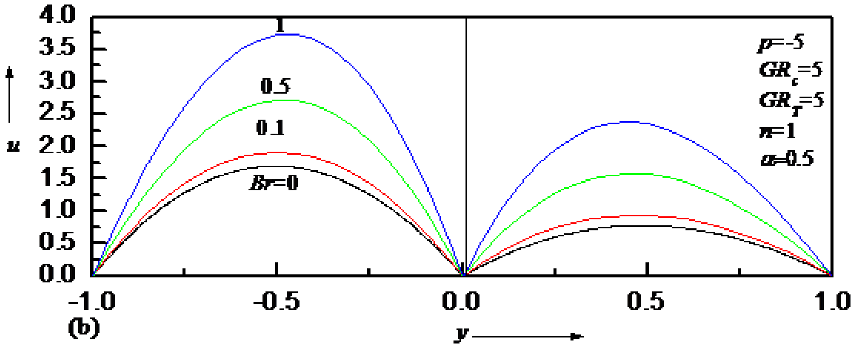


Fig.5: Temperature profiles for different values of ratio of modified Grashof number to Reynolds number GR_c at (a)y*=-0.8 (b)y*=0 (c)y*=0.8





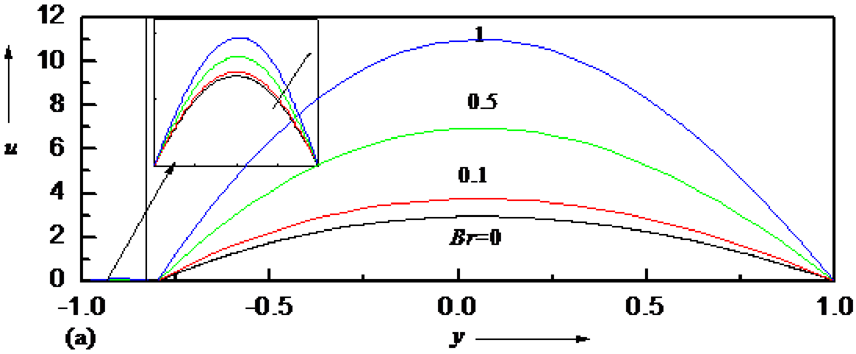


Fig.6: Velocity profiles for different values of Brinkman number Br (a)y*=-0.8 (b)y*=0 (c)y*=0.8

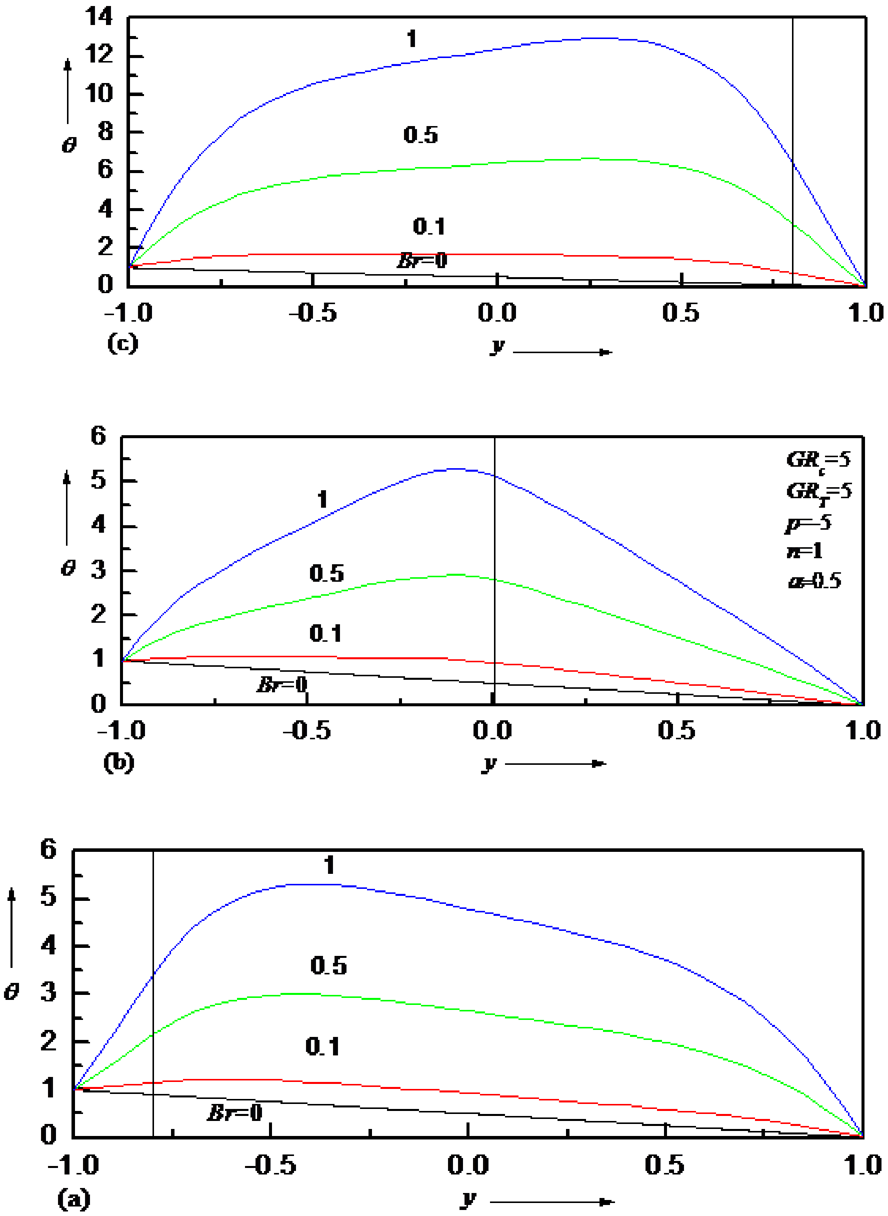
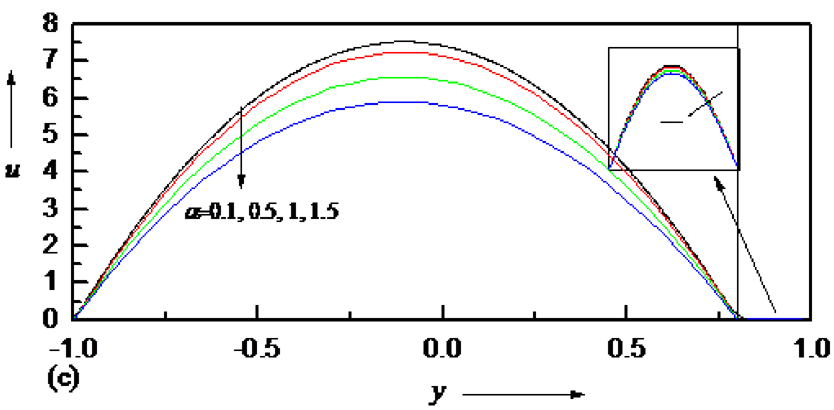


Fig.7: Temperature profiles for different values of Brinkman number Br at (a) y*=-0.8 (b)y*=0 (c) y*=0.8



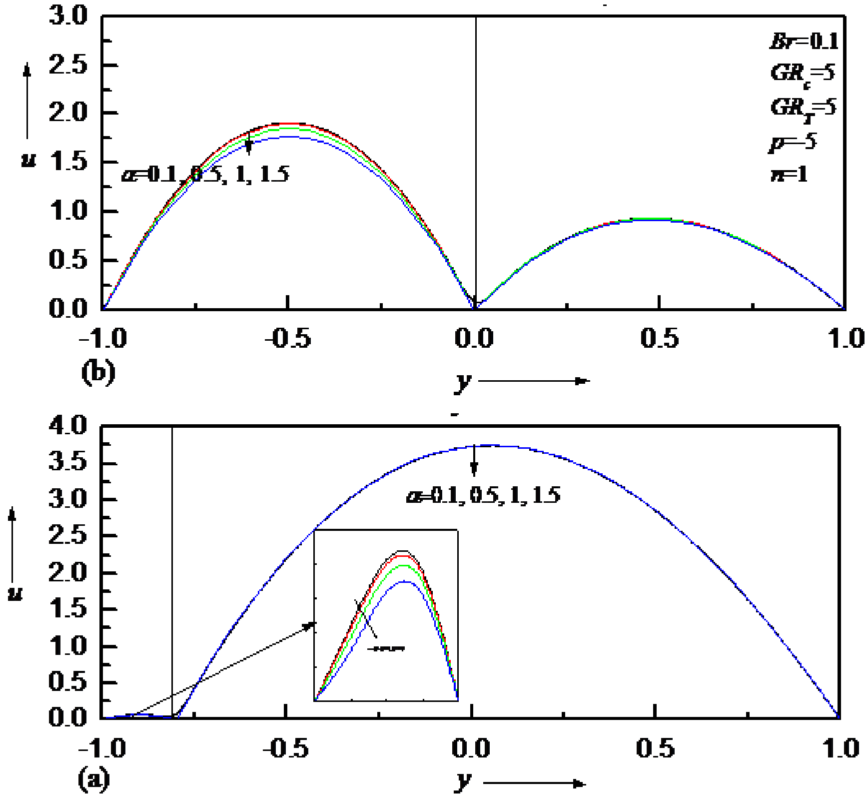
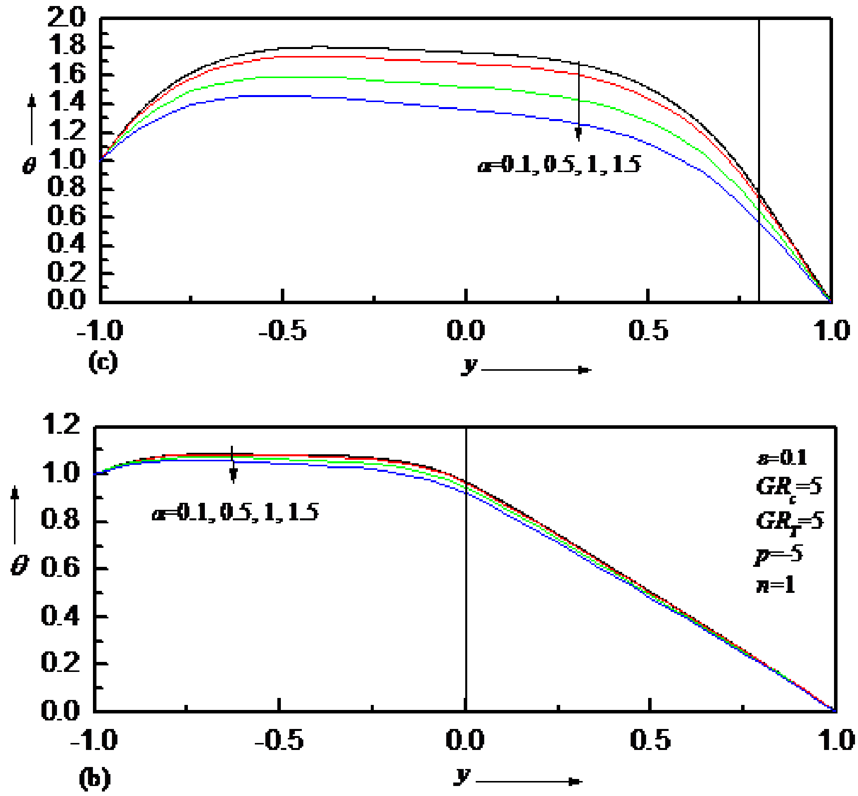


Fig.8: Velocity profiles for different values of chemical reaction parameter α at (a) y*=-0.8 (b)y*=0 (c) y*=0.8



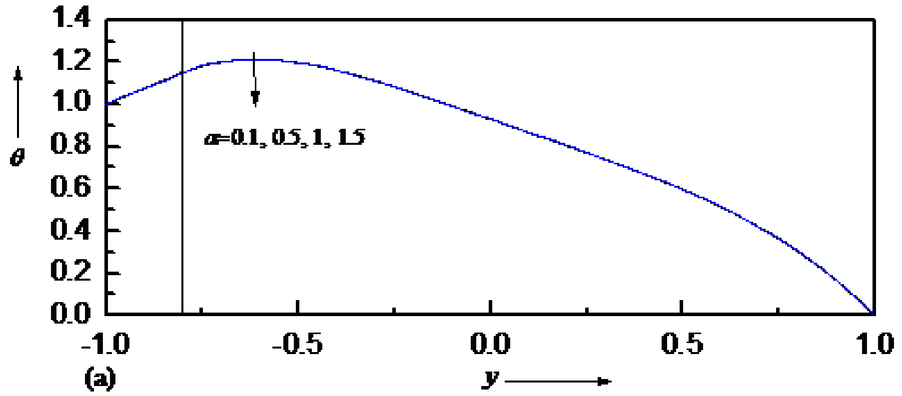


Fig.9: Temperature profile for different values of chemical reaction parameter α at (a) y*=-0.8 (b)y*=0 (c) y*=0.8

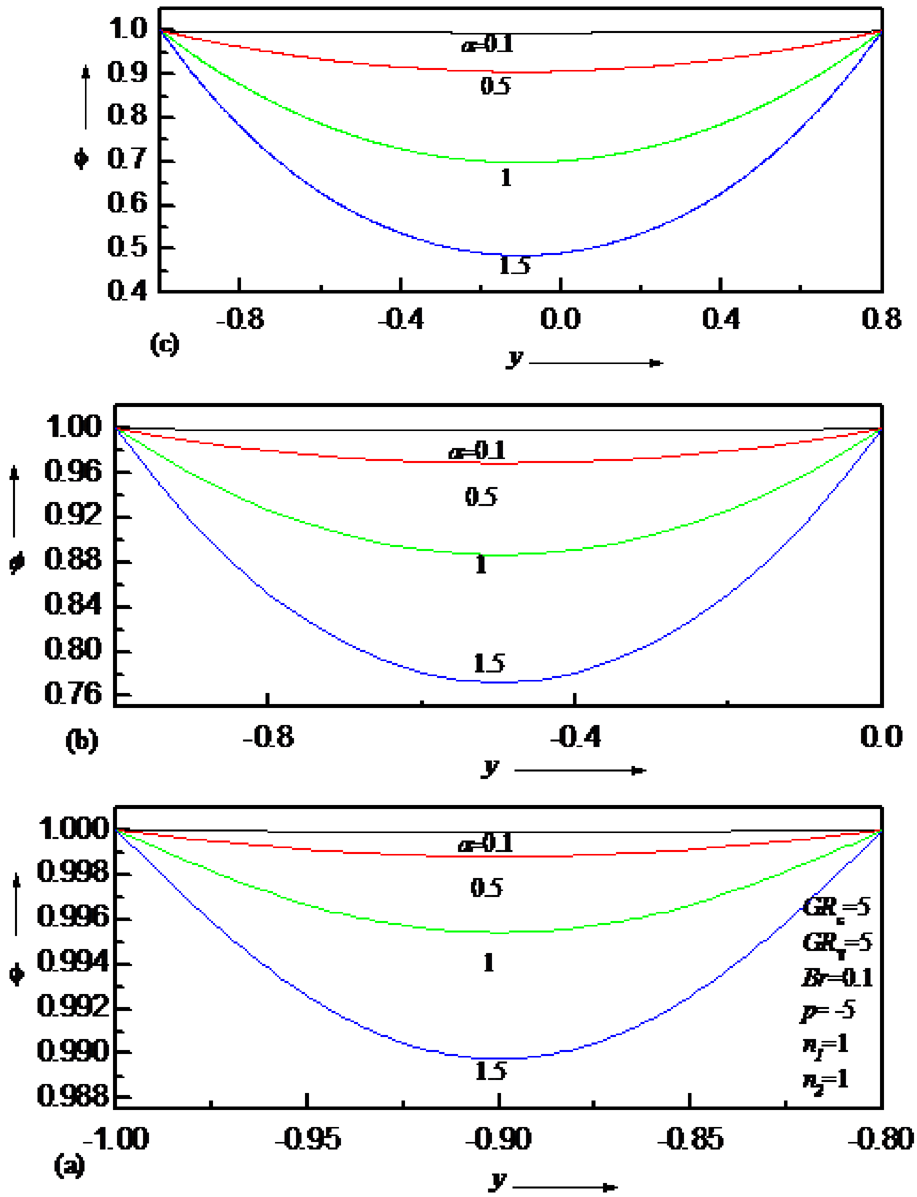


Fig.10.Concentration profiles for different values of chemical reaction parameter α at (a) y*=-0.8 (b)y*=0 (c) y*=0.8

5. CONCLUSION

The effect of first order chemical reaction in a vertical double passage channel filled with purely viscous fluid is investigated. According to the results, the following conclusions are drawn:

1. Increasing the values of the ratio of Grashof number to Reynolds number, ratio of modified Grashof number to Reynolds number increases the velocity and temperature in both the streams at different baffle position.
2. Increase in the Brinkman number, enhances the velocity and temperate in both the streams.
3. Increase in the chemical reaction parameter suppresses the velocity, temperate and concentration in stream-I and remains invariant in stream-II at different positions of the baffle.

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NOMENCLATURE

h channel width
h^* width of passage
y^* baffle position
c_p dimensionless specific heat at constant pressure
g acceleration due to gravity
Gr Grashof number $\left(\frac{h^3 g\beta\Delta T}{\nu^2}\right)$
β_T coefficient of thermal expansion
β_C coefficient of concentration expansion
C concentration in Stream-I
k thermal conductivity of fluid
Br Brinkman number $\left(\frac{\overline{u_1}^2\mu_1}{K\Delta T}\right)$
p non-dimensional pressure gradient $\left(\frac{h^2}{\overline{U_1}\mu}\frac{\partial p}{\partial X}\right)$
GR_T dimensionless parameter $\left(GR_T = \frac{Gr}{\text{Re}}\right)$
GR_C dimensionless parameter $\left(GR_C = \frac{Gc}{\text{Re}}\right)$
Re Reynolds number $\left(\frac{\overline{U_1}h}{\gamma}\right)$
C_p specific heat at constant pressure
C_0 reference concentration
$\overline{U_1}$ reference velocity
D diffusion coefficient
T_1, T_2 dimensional temperature
T_{w_1}, T_{w_2} temperatures at the boundaries
U_1, U_2 dimensional velocities
u_1, u_2 non-dimensional velocities
Gc Modified Grashof Number $\left(\frac{g\,\beta c\,\Delta C\,h^3}{\upsilon^2}\right)$
α chemical reaction parameter
ΔT difference in temperature
ΔC difference in concentration
Br perturbation parameter
θ_i non-dimensional temperature $\left(\frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}}\right)$
υ kinematic viscosity
ϕ non-dimensional concentrations
ρ density
μ viscosity

SUBSCRIPTS

i refer quantities for the fluids in stream-I and stream-II, respectively.

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APPENDIX

$$B_1=\frac{Sinh(\alpha y^*)+n\,Sinh(\alpha)}{Sinh(\alpha y^*)Cosh(\alpha)+Sinh(\alpha)Cosh(\alpha y^*)},\ B_2=\frac{n\,Cosh(\alpha)-Cosh(\alpha y^*)}{Sinh(\alpha y^*)Cosh(\alpha)+Sinh(\alpha)Cosh(\alpha y^*)},\ z_1=-\frac{1}{2},\ z_2=\frac{1}{2}$$

$$z_3=-\frac{1}{2},\ z_4=\frac{1}{2},\ r_1=\frac{(p-GR_T\,C_2)}{2},\ r_2=-\frac{GR_T C_1}{6},\ r_3=-\frac{GR_c\,B_1}{\alpha^2},\ r_4=-\frac{GR_c\,B_2}{\alpha^2},\ r_5=\frac{(p-GR_T\,C_4)}{2}$$

$$r_6=-\frac{GR_T\,C_3}{6},\ A_1=-\frac{\left(r_1\left(y^{*2}-1\right)+r_2\left(y^{*3}+1\right)+r_3\left(Cosh(\alpha y^*)-Cosh(\alpha)\right)+r_4\left(Sinh(\alpha y^*)+Sinh(\alpha)\right)\right)}{1+y^*}$$

$$A_2=A_1-r_1+r_2-r_3\,Cosh(\alpha)+r_4\,Sinh(\alpha),\ A_3=\frac{r_5\left(1-y^{*2}\right)+r_6\left(1-y^{*3}\right)}{(y^*-1)},\ A_4=-A_3-r_5-r_6$$

$$p_1=-\frac{\left(2A_1^2+r_4^2\alpha^2-r_3^2\alpha^2\right)}{4},\ p_2=-\frac{2A_1 r_1}{3},\ p_3=-\frac{\left(4r_1^2+6A_1 r_2\right)}{12},\ p_4=-\frac{3r_1 r_2}{5},$$

$$p_5=-\frac{3r_2^2}{10},\ p_6=-\frac{\left(r_3^2+r_4^2\right)}{8},\ p_7=-\frac{r_3 r_4}{4},\ p_8=-\frac{\left(2A_1 r_4\alpha^2-8r_1 r_3\alpha+36r_2 r_4\right)}{\alpha^3},$$

$$p_9=-\frac{\left(2A_1 r_3\alpha^2-8r_1 r_4\alpha+36r_2 r_3\right)}{\alpha^3},\ p_{10}=-\frac{\left(4r_1 r_4\alpha-24r_2 r_3\right)}{\alpha^2},\ p_{11}=-\frac{\left(4r_1 r_3\alpha-24r_2 r_4\right)}{\alpha^2},\ p_{12}=-\frac{6r_2 r_4}{\alpha}$$

$$p_{13}=-\frac{6r_2 r_3}{\alpha},\ q_1=-\frac{A_3^2}{2},\ q_2=-\frac{2A_3 r_5}{3},\ q_3=-\frac{\left(2r_5^2+3A_3 r_6\right)}{6},\ q_4=-\frac{3r_5 r_6}{5},\ q_5=-\frac{3r_6^2}{10}$$

$$T_1=-\begin{pmatrix}p_1-p_2+p_3-p_4+p_5+p_6\,Cosh(2\alpha)-p_7\,Sinh(2\alpha)+p_8\,Cosh(\alpha)\\-p_9\,Sinh(\alpha)-p_{10}\,Cosh(\alpha)+p_{11}\,Sinh(\alpha)+p_{12}\,Cosh(\alpha)-p_{13}\,Sinh(\alpha)\end{pmatrix},$$

$$T_2=-(q_1+q_2+q_3+q_4+q_5),$$

$$T_3 = q_1 y^{*2} + q_2 y^{*3} + q_3 y^{*4} + q_4 y^{*5} + q_5 y^{*6} - p_1 y^{*2} - p_2 y^{*3} - p_3 y^{*4} - p_4 y^{*5} - p_5 y^{*6} - p_6\, Cosh(2\alpha y^*) - p_7\, Sinh(2\alpha y^*)$$
$$- p_8\, Cosh(\alpha y^*) - p_9\, Sinh(\alpha y^*) - p_{10}\, y^* Cosh(\alpha y^*) - p_{11} y^* Sinh(\alpha y^*) - p_{12} y^{*2} Cosh(\alpha y^*) - p_{13} y^{*2} Sinh(\alpha y^*)$$

$$T_4 = 2q_1 y^* + 3q_2 y^{*2} + 4q_3 y^{*3} + 5q_4 y^{*4} + 6q_5 y^{*5} - 2p_1 y^* - 3p_2 y^{*2} - 4p_3 y^{*3} - 5p_4 y^{*4} - 6p_5 y^{*5} - 2\alpha p_6\, Sinh(2\alpha y^*)$$
$$- 2\alpha p_7\, Cosh(2\alpha y^*) - p_8 \alpha\, Sinh(\alpha y^*) - \alpha p_9\, Cosh(\alpha y^*) - p_{10}\left(y^* \alpha Sinh(\alpha y^*) + Cosh(\alpha y^*)\right)$$
$$- p_{11}\left(y^* \alpha\, Cosh(\alpha y^*) + Sinh(\alpha y^*)\right) - p_{12}\left(2y^* Cosh(\alpha y^*) + \alpha y^{*2}\, Sinh(\alpha y^*)\right)$$
$$- p_{13}\left(2y^* Sinh(\alpha y^*) + \alpha y^{*2}\, Cosh(\alpha y^*)\right)$$

$$G_1 = -\frac{\left(y^* T_4 + T_1 - T_2 - T_3 - T_4\right)}{2},\ G_2 = \frac{\left(T_1 + T_2 + T_3 + T_4(1 - y^*)\right)}{2},\ G_3 = \frac{\left(-T_1 + T_2 + T_3 - T_4(1 + y^*)\right)}{2},\ G_4 = T_2 - G_3$$

$$R_1 = -\frac{GR_T\, G_2}{2},\ R_2 = -\frac{GR_T\, G_1}{6},\ R_3 = -\frac{GR_T\, p_1}{12},\ R_4 = -\frac{GR_T\, p_2}{20},\ R_5 = -\frac{GR_T\, p_3}{30}, R_6 = -\frac{GR_T\, p_4}{42}, R_7 = -\frac{GR_T\, p_5}{56}$$

$$R_8 = -\frac{GR_T\, p_6}{4\alpha^2},\ R_9 = -\frac{GR_T\, p_7}{4\alpha^2},\ R_{10} = -\frac{\left(p_8\alpha^2 - 2p_{11}\alpha + 6p_{12}\right)GR_T}{\alpha^4}, R_{11} = -\frac{\left(p_9\alpha^2 - 2p_{10}\alpha + 6p_{13}\right)GR_T}{\alpha^4}$$

$$R_{12} = -\frac{\left(p_{10}\alpha - 4p_{13}\right)GR_T}{\alpha^3}, R_{13} = -\frac{\left(p_{11}\alpha - 4p_{12}\right)GR_T}{\alpha^3}, R_{14} = -\frac{GR_T\, p_{12}}{\alpha^2}, R_{15} = -\frac{GR_T\, p_{13}}{\alpha^2}, S_1 = -\frac{GR_T\, G_4}{2}$$

$$S_2 = -\frac{GR_T\, G_3}{6},\ S_3 = -\frac{GR_T\, q_1}{12},\ S_4 = -\frac{GR_T\, q_2}{20},\ S_5 = -\frac{GR_T\, q_3}{30},\ S_6 = -\frac{GR_T\, q_4}{42},\ S_7 = -\frac{GR_T\, q_5}{56}$$

$$T_5 = -\begin{pmatrix} R_1 - R_2 + R_3 - R_4 + R_5 - R_6 + R_7 + R_8\, Cosh(2\alpha) - R_9\, Sinh(2\alpha) + R_{10}\, Cosh(\alpha) \\ -R_{11}\, Sinh(\alpha) - R_{12}\, Cosh(\alpha) + R_{13}\, Sinh(\alpha) + R_{14}\, Cosh(\alpha) - R_{15}\, Sinh(\alpha) \end{pmatrix}$$

$$T_7 = \begin{pmatrix} R_1 y^{*2} + R_2 y^{*3} + R_3 y^{*4} + R_4 y^{*5} + R_5 y^{*6} + R_6 y^{*7} + R_7 y^{*8} + R_8\, Cosh(2\alpha y^*) + R_9\, Sinh(2\alpha y^*) + R_{10}\, Cosh(\alpha y^*) \\ + R_{11}\, Sinh(\alpha y^*) + R_{12}\, y^* Cosh(\alpha y^*) + R_{13} y^* Sinh(\alpha y^*) + R_{14} y^{*2}\, Cosh(\alpha y^*) + R_{15} y^{*2}\, Sinh(\alpha y^*) \end{pmatrix}$$

$$G_5 = \frac{T_7 - T_5}{1 + y^*},\ G_7 = \frac{T_6 - T_8}{1 - y^*},\ G_6 = T_5 + G_5,\ G_8 = T_6 - G_7.$$

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