International Journal of Mathematical Archive-5(3), 2014, 124-128
Available online through www.ijma.info ISSN 2229 – 5046

INTEGRAL INEQUALITIES IN TWO INDEPENDENT VARIABLES FOR RETARDED EQUATION

Jayashree Patil*[1] and D. B. Dhaigude[2]

[1]Vasantrao Naik Mahavidyalaya, Aurangabad - 431003(M.S.), India
[2]Department of mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad (M.S.) India

(Received on: 02-03-14; Revised & Accepted on: 15-03-14)

ABSTRACT

The object of this paper is to establish some new nonlinear retarded integral inequalities in two independent variables which provide explicit bounds on unknown functions. The inequalities given here can be used as handy tools in the theory of partial differential equations with time delays.

Keywords and Phrases: *Retarded integral inequality, Boundedness, explicit bound.*

1.INTRODUCTION

The integral inequalities involving functions of one and more than one independent variables which provide explicit bounds on unknown functions play a fundamental role in the development of the theory of differential equations.

The Gronwall-Bellman inequality states that if u and f are nonnegative continuous functions on an interval $[a, b]$ satisfying

$$u(t) \leq c + \int_a^t f(s)u(s)\, \mathrm{ds} \tag{1.1}$$

for some constant $c \geq 0$, then

$$u(t) \leq c\exp\left(\int_a^t f(s)ds\right),\ \ t\ \in [a, b] \tag{1.2}$$

Inequality (1.2), provides an explicit bound on the unknown function and hence furnishes a handy tool in the study of qualitative and quantitative properties of solution of differential and integral equation.

Due to its importance over the years many generalizations and analogous results of (1.1), have been established $(see, eg., [1-14])$.

In this paper, we establish some new integral inequalities involving functions of two independent variables. Our results extend some results in [7] and [9].

2. MAIN RESULTS

Let R denotes the set of real numbers, $R_+ = [0, \infty)$. Also, $J_1 = [x_0, X)$ and $J_2 = [y_0, Y)$ be the given subset of R; $\Delta = J_1 \times J_2 . D_1 z(x, y), D_2 z(x, y)$ be partial derivative of z with respective x and y respectively

Theorem: 2.1 Let $u, a, c \in C(\Delta, R_+)$, a and c be nondecreasing in each variables, $f_i, h_i, g_i \in C(\Delta, R_+)$, i=1,2,...,n and let $\alpha_i \in C^1(J_1, J_1)$ be nondecreasing with $\alpha_i(t) \leq t$, i=1,2,...,n. and $\beta_i \leq C^1(J_2, J_2)$ be nondecreasing with $\beta_i(t) \leq t$, i=1,2,...,n. Suppose that $q > 0$ is a constant and $\varphi \in C(R_+, R_+)$ is an increasing function with $\varphi(\infty) = \infty$ and $\psi(u)$ is a nondecreasing continuous function for $u \in R$, with $\psi(u) > 0$ for $u > 0$. If

$$\begin{aligned}\varphi(u(x,y)) \quad &\leq a(x,y) + c(x,y)\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(s,t)u^q(s,t) \\ &\left(\psi(u(s,t)) + \int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma\right) + g_i(s,t)u^q(s,t)]dtds\end{aligned} \tag{2.1}$$

for all $(x, y) \in \Delta$, then

Corresponding author: **Jayashree Patil*[1]**
Vasantrao Naik Mahavidyalaya, Aurangabad 431003(M.S), India.
E-mail: jv.patil29@gmail.com

$$u(x,y) \leq \varphi^{-1}\{G^{-1}[\Phi^{-1}(\Phi(k(x_0,y)) + c(x,y)\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)} f_i(s,t)(1+\int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)d\eta d\sigma)dtds)]\} \tag{2.2}$$

for all $(x,y) \in [x_0,x_1]\times[y_0,y_1]$,

where,

$$k(x_0,y) = G(a(x,y)) + c(x,y)\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)} g_i(s,t)dtds$$

$$G(r) = \int_{r_0}^{r}\frac{ds}{[\varphi^{-1}(s)]^q},\ r \geq r_0 > 0 \tag{2.3}$$

$$\Phi(r) = \int_{r_0}^{r}\frac{ds}{\psi(\varphi^{-1}(G^{-1}(s)))},\ r \geq r_0 > 0 \tag{2.4}$$

when G^{-1} and Φ^{-1} denote the inverse functions of G, Φ and $(x_1,y_1)\in\Delta$ is so chosen that

$$\Phi(k(x_0,y)) + c(x,y)\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)} f_i(s,t)(1+\int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)d\eta d\sigma)dtds \in Dom(\Phi^{-1})$$

Proof: Suppose that $a(x,y) > 0$. Fixing numbers $\bar{x}$ and $\bar{y}$ with $x_0 \leq x \leq \bar{x}$ and $y_0 \leq y \leq \bar{y}$, define a positive function $z(x,y)$ as,

$$z(x,y) = a(\bar{x},\bar{y}) + c(\bar{x},\bar{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(s,t)u^q(s,t)\times(\psi(u(s,t)) + \int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma) + g_i(s,t)u^q(s,t)]dtds \tag{2.5}$$

Then $z(x,y) > 0$, $z(x_0,y) = z(x,y_0) = a(\bar{x},\bar{y})$ and (2.1) can be restated

$$u(x,y) \leq \varphi^{-1}(z(x,y)) \tag{2.6}$$

clearly, $z(x,y)$ is continuous nondecreasing function for all $x \in J_1$, $y \in J_2$ and

$$D_1 z(x,y) = c(\bar{x},\bar{y})\sum_{i=1}^{n}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(\alpha_i(x),t)u^q(\alpha_i(x),t)\times(\psi(u(\alpha_i(x),t)) + \int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma) + g_i(\alpha_i(x),t)u^q(\alpha_i(x),t)]\alpha_i'(x)dt$$

Using (2.6) in above inequality,we deduce

$$D_1 z(x,y) \leq c(\bar{x},\bar{y})[\varphi^{-1}(z(x,y))]^q\sum_{i=1}^{n}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(\alpha_i(x),t) \times(\psi(u(\alpha_i(x),t)) + \int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma) + g_i(\alpha_i(x),t)]\alpha_i'(x)dt \tag{2.7}$$

Using the monotonicity of φ^{-1} and z, we deduce

$$[\varphi^{-1}(z(x,y))]^q \geq [\varphi^{-1}(z(x_0,y_0))]^q = [\varphi^{-1}(a(\overline{x},\overline{y}))]^q > 0$$

From the definition of G and (2.7), we have

$$D_1 G\big(z(x,y)\big) = \frac{D_1 z(x,y)}{[\varphi^{-1}(z(x,y))]^q} \leq c(\bar{x},\bar{y})\sum_{i=1}^{n}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(\alpha_i(x),t) \times(\psi(u(\alpha(x),t)) + \int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma) + g_i(\alpha_i(x),t)]\alpha_i'(x)dt \tag{2.8}$$

Keeping y fixed in (2.8) and integrating from x_0 to x, $x \in J_1$ and making change of variable on right-hand side, we have

$$G(z(x,y) \leq G(a(\overline{x},\overline{y}) + c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(s,t)\times(\psi(u(s,t)) + \int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma) + g_i(s,t)]dtds$$

$$G(z(x,y) \le G(a(\overline{x},\overline{y}) + c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)} g_i(s,t)dtds \sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)} g_i(s,t)dtds$$
$$+c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(s,t) \ \times (\psi(u(s,t))$$
$$+\int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma)]dtds \quad (2.9)$$

Now, define a function $k(x,y)$ by

$$k(x,y) = G(a(\overline{x},\overline{y}) + c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(\bar{x})}\int_{\beta_i(y_0)}^{\beta_i(\bar{y})} g_i(s,t)dtds$$
$$+c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(s,t)(\psi(u(s,t)) + \int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma)]dtds$$

Then

$$k(x_0,y) = G(a(\overline{x},\overline{y}) + c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(\bar{x})}\int_{\beta_i(y_0)}^{\beta_i(\bar{y})} g_i(s,t)dtds$$

and (2.9) can be restated as

$$z(x,y) \le G^{-1}[k(x,y)] \quad (2.10)$$

We know $u(x,y) \le \varphi^{-1}(z(x,y)) \le \varphi^{-1}(G^{-1}(k(x,y)))$, also ψ is nondecreasing, we obtained

$$\psi[u(\sigma,\eta)] \le \psi[\varphi^{-1}(z(\sigma,\eta))] \le \psi[\varphi^{-1}(z(\alpha_i(t),t))] \le \psi[\varphi^{-1}(G^{-1}(k(s,t)))]$$

for $\sigma \in [\alpha_i(t_0),\alpha_i(t)]$and $\eta \in [\beta_i(t_0),\beta_i(t)]$.

It is easy to observe that $k(x,y)$ is continuous nondecreasing function for all $x \in J_1, y \in J_2$.

$$D_1(k(x,y)) = c(\overline{x},\overline{y}) \ \sum_{i=1}^{n}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(\alpha_i(x),t) \ \times (\psi(\varphi^{-1}\big(z(\alpha_i(x),t)\big)$$
$$+\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)\psi(u(\sigma,\eta))d\eta d\sigma)dt]\alpha'_i(x)dt$$

$$D_1(k(x,y)) \le c(\overline{x},\overline{y}) \ \psi(\varphi^{-1}(G^{-1}(k(\alpha_i(x),t))))$$
$$+\sum_{i=1}^{n}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(\alpha_i(x),t) \ \times (1+\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)d\eta d\sigma)]\alpha'_i(x)dt$$

From definition of Φ and using the monotonicity of φ^{-1},ψ,G^{-1} and k, we deduce

$$\frac{d}{dx}\Big(\Phi\big(k(x,y)\big)\Big) = \frac{D_1\big(k(x,y)\big)}{\psi\Big(\varphi^{-1}\Big(G^{-1}\big(k(x,y)\big)\Big)\Big)}$$
$$\le c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(\alpha_i(x),t) \times (1+\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)d\eta d\sigma)]\alpha'_i(x)dt \quad (2.11)$$

Integrating the inequality in (2.11) from x_0 to x and making change of variable on right-hand side, we deduce

$$\Phi(k(x,y)) \le \Phi(k(x_0,y)) + c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(s,t) \ \times (1\int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)d\eta d\sigma)]dtds$$

Using (2.6), (2.10) and above inequality, we get

$$u(x,y) \le \varphi^{-1}\{G^{-1}[\Phi^{-1}(\Phi(k(x_0,y)) + c(\overline{x},\overline{y})\sum_{i=1}^{n}\int_{\alpha_i(x_0)}^{\alpha_i(x)}\int_{\beta_i(y_0)}^{\beta_i(y)}[f_i(s,t)(1+\int_{\alpha_i(x_0)}^{s}\int_{\beta_i(y_0)}^{t} h_i(\sigma,\eta)d\eta d\sigma)]dtds)]\} \quad (2.12)$$

Taking $x = \bar{x}, y = \bar{y}$ in the inequality (2.12), since $\bar{x}$ and $\bar{y}$ are arbitrary, we get required inequality. If $a(x,y) = 0$, we carry out the above procedure with $\varepsilon > 0$ instead of $a(x,y)$ and subsequently letting $\varepsilon \to 0$. Hence proof is completed.

Remark:(1) If we put $h_i(\sigma,\eta) = 0$ in above theorem, we get Theorem 2.1 in [9]

3. APPLICATION

In this section we show that our one of the result is useful in proving global existence of the solution of certain non-linear partial integrodifferential equation with time delay. Consider the nonlinear integrodifferential equation involving several retarded arguments,

$$D_2(\varphi(z(x,y))D_1 z(x,y)) = F(x,y,z(x-l_1(x),y-m_1(y)),\cdots,z(x-l_n(x),y-m_n(y)),$$
$$\int_{x_0}^{x}\int_{y_0}^{y} H_i(x,y,s,t,z(s-l_1(s),t-m_1(t)),\cdots, \ \cdots z(s-l_n(s),t-m_n(t)))dtds) \quad (3.1)$$

with the initial boundary conditions

$$\varphi(z(x,y_0)) = e_1(x), \quad \varphi(z(x_0,y)) = e_2(y), \quad e_1(x_0) = e_2(y_0) = 0$$

where $F \in C(\Delta \times R^n \times R, R), H_i \in C(\Delta \times \Delta \times R^n, R), e_1 \in C^1(J_1,R) e_2 \in C^1(J_2,R)$ and $l_i \in C^1(J_1,J_1), m_i \in C^1(J_2,J_2)$ such that $0 < (l_i)^{'}(x) \le 1, 0 < (m_i)^{'}(y) \le 1, l_i(x_0) = x_0, m_i(y_0) = y_0, i = 1,\cdots,n.$

The following theorem deals with a boundedness on the solution of the problem (3.1)

Theorem: 3.1 Assume that $F: \Delta \times R^n \times R \to R$ is a continuous function for which there exists continuous nonnegative functions $f_i(x,y), g_i(x,y); i = 1,\cdots n$ such that

$$|F(x,y,u_1,u_2,\cdots u_n,v)| \le \sum_{i=1}^{n} |u_i|^q \{f_i(x,y)(\psi(|u_i| + |v|)) + g_i(x,y)|u_i|^q\} \tag{3.2}$$

and the function $H_i: \Delta \times \Delta \times R^n \to R$ is a continuous function for which there exist continuous nonnegative function $h_i(x,y), i = 1,\cdots,n$ such that

$$|H_i(x,y,s,t,u_1,u_2 \cdots u_n)| \le \sum_{i=1}^{n} h_i(x,y)\psi(|u_i|) \tag{3.3}$$

and

$$|e_1(x) + e_2(y)| \le a(x,y) \tag{3.4}$$

where $a(x,y) \in C(\Delta, R_+)$ is nondecreasing in each variables, $p > q > 0$ are constants and $\psi(u)$ is a nondecreasing continuous function for $u \in R$ with $\psi(u) > 0$ for $u > 0$. Let

$$M_i = \max_{x \in J_1} \frac{1}{1-l'_i(x)}, \quad N_i = \max_{y \in J_2} \frac{1}{1-m'_i(y)}, \quad i = 1,2,\cdots,n \tag{3.5}$$

if $z(x,y)$ is any solution of problem (3.1) with the initial boundary condition, then

$$|z(x,y)| \le \varphi^{-1}\{G^{-1}[\Phi^{-1}((k(x_0,y)) + \sum_{i=1}^{n=1} \int_{y_0-m_i(y_0)}^{x-l_i(x)} \int_{y_0-m_i(y_0)}^{y-m_i(y)} \overline{f_i}(\sigma,\tau) \left(1 + \int_{x_0-l_i(x_0)}^{s} \int_{y_0-m_i(y_0)}^{t} \overline{h_i}(\phi,\gamma) d\gamma d\phi) d\tau d\sigma)]\}\right)$$

for all$(x,y) \in [x_0,x_1] \times [y_0,y_1]$, where φ, G, Φ are as in Theorem 2.1 and

$$k(x_0,y) = [G|a(x,y)|] + \sum_{i=1}^{n} \int_{x_0-l_i(x_0)}^{x-l_i(x)} \int_{y_0-m_i(y_0)}^{y-m_i(y)} \overline{g_i}(\sigma,\tau) d\tau d\sigma.$$

$$\overline{f_i} = f_i(\sigma + l_i(s), \tau + m_i(t)) M_i N_i$$

$$\overline{h_i} = g_i(\phi + l_i(\xi), \gamma + m_i(\eta)) M_i N_i$$

$$\overline{g_i} = h_i(\sigma + l_i(s), \tau + m_i(t)) M_i N_i$$

$$\sigma, s, \phi, \xi \in J_1, \tau, t, \gamma, \eta \in J_2.$$

Proof: It is easy to see that the solution $z(x,y)$ of problem (3.1) with the initial boundary condition satisfies the equivalent integral equation

$$\varphi(z(x,y)) = e_1(x) + e_2(y) + \int_{x_0}^{x} \int_{y_0}^{y} F(s,t,z(s-l_1(s),t-m_1(t)),\cdots,z(s-l_n(s),t-m_n(t)), \int_{x_0}^{s} \int_{y_0}^{t} H_i(s,t,\xi,\eta,z(\xi-l_1(\xi),\eta-m_1(\eta)),\cdots \cdots,z(\xi-l_n(\xi),\eta-m_n(\eta))) d\eta d\xi) dt ds \tag{3.6}$$

Using (3.2), (3.3), (3.4), (3.5) and making change of variable on right hand side of the above inequality, we have

$$|\varphi(z(x,y))| \le a(x,y) + \sum_{i=1}^{n} \int_{x_0-l_i(x_0)}^{x-l_i(x)} \int_{x_0-m_i(y_0)}^{y-m_i(y)} [|z(\sigma,\tau)|^q \overline{f_i}(\sigma,\tau) \times (\psi(|z(\sigma,\tau)|) + \int_{x_0-l_i(x_0)}^{\sigma} \int_{x_0-m_i(y_0)}^{\tau} \overline{h_i}(\phi,\nu)\psi(|z(\phi,\nu)|) d\nu d\phi) + \overline{g_i}(\sigma,\tau)|z(\sigma,\tau)|^q] \mathrm{d}r \mathrm{d}\sigma \tag{3.7}$$

$$\overline{f_i} = f_i(\sigma + l_i(s), \tau + m_i(t)) M_i N_i,$$

$$\overline{h_i} = h_i(\phi + l_i(\xi), \nu + m_i(\eta)) M_i N_i,$$

$\overline{g_i} = g_i(\sigma + l_i(s), \tau + m_i(t))M_iN_i,$

$\sigma, s, \phi, \xi \in J_1, \tau, t, \nu, \eta \in J_2.$

Now, immediate application of the inequality established in Theorem 2.1 to the inequality (3.7) yields the result.

REFERENCES

[1] R.P.Agarwal,S.Deng,and w.zang,*Generlisation of a retarded Gronwall-like inequalities and its applications,* Appl. Math. Comp.,165,(2005),599-612.

[2] R.Bellman.*Stability Theory of DifferentialEquations,*McGraw-Hill,New York, 1953.

[3] I.Bihari,*A generalisation of a lemma of Bellman and its uniqueness promblems of differential equations,* Acta. Math. Hungarica.,7,(1956),81-94.

[4] W.S.Cheng,*Some new nonlinear inequalities and applications to boundary value promblems,*Nonlinear Analysis:theory,methods and applications,64,(2006),2112-2128.

[5] C.M.Dafermos,*The second law of thermodynamics and stability,*Archive for Rational Mechanics and Analysis, 70, (1979),167-179.

[6] F.M.Dannan,*Integral ineualities of Gronwall-Bellman-Bihari type and asymptotic behavior of certain second order nonlinear differntial equations,*J.of Math.Analy. and Appl.,108, (1985),151-164.

[7] D.B.Dhaigude and J.V.Patil ,*Some nonlinear retarded integral ineualities and applications,* Marath. Math. soc., 14, (2013),56-70.

[8] T.H.Gronwall,*Note on the derivatives with respect to the solutions of a system of differential equations,*The Ann.Math.,20,(1919),292-296.

[9] Y.H.Kim,*Gronwall,Bellman and Pachpatte type integral inequalities,*Nonlinear Anal.71(2009),2641-2656.

[10] D.S,Mitrinovic,J.E.Pecaric and A.M.Fink, *Inequalities Involving Functios and Their Integrals and Derivatives*,Kluwer Academic Publisher,Dodrecht,The Netherlands,1991.

[11] B.G. Pachpatte,*Inequalities For Differential and Integral Equations,* Academic Press, New york, 1998.

[12] B.G. Pachpatte,*Explicit Bounds on certain integral inequalities*,J. Math. anly.and appl.,267,(2002),48-61.

[13] W.S.Wang,*A generalized sum-difference inequality and applications to partial difference equations,*Adva. Diff.Equa.,2008,Article 1D 695495,12 pages,(2008).

[14] H. zang,F. Meng, *On some integral inequalities in two independent variables for retarded equations,* Appl.Math.comp.,203,(2008),608-616.

Source of support: Nil, Conflict of interest: None Declared

