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WEIGHTED SHARING OF A SMALL FUNCTION OF A MEROMORPHIC FUNCTION AND ITS DERIVATIVES

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ABSTRACT

In this paper, we study the relationship between a meromorphic function and its k-th order derivative which share a small function with weight(multiplicities) l(a positive integer)ignoring multiplicity.

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INTRODUCTION AND RESULTS

In this paper we shall use the standard notations of Nevanlinna theory such as $T(r,f), N(r,f), m(r,f)$ and so on (see[3]), where f is a meromorphic function defined on the whole complex plane. The quantity S(r,f) is defined by S(r,f)=0(T(r,f)) as $r \to \infty$ possibly outside a set of finite linear measure. A meromorphic function a(z) is called a small function with respect to f provided T(r,a)= S(r,f) holds.

Suppose that f and g are two non-constant meromorphic functions, a is a small function with respect to f and g and k be a positive integer. Now f and g share 'a' ignoring multiplicities or IM (counting multiplicities or CM) if f - a and g - a have the same zeros ignoring(counting) multiplicities. We denote by $N_{k)}\left(r,\frac{1}{f-a}\right)$, the counting function for zeros of f - a with multiplicity $\le k$ (counting multiplicity), and by $\overline{N}_{k)}\left(r,\frac{1}{f-a}\right)$, the corresponding one for which the multiplicity is not counted. Similarly by $N_{(k}\left(r,\frac{1}{f-a}\right)$, we mean the counting function for zeros of f – a with multiplicity at least k(counting multiplicity) and by $\overline{N}_{(k}\left(r,\frac{1}{f-a}\right)$, we mean the corresponding one for which the multiplicity is not counted.

We denote $N_k\left(r,\frac{1}{f-a}\right)=\overline{N}\left(r,\frac{1}{f-a}\right)+\overline{N}_{(2}\left(r,\frac{1}{f-a}\right)+......+\overline{N}_{(k}\left(r,\frac{1}{f-a}\right)$

where $\overline{N}_{(1}\left(r,\frac{1}{f-a}\right)=\overline{N}\left(r,\frac{1}{f-a}\right)$ and $\delta_p(a,f)=1-\limsup_{r\to\infty}\frac{N_p\left(r,\frac{1}{f-a}\right)}{T(r,f)}$, where p is a positive integer; then clearly $0\le\delta(a,f)\le\delta_k(a,f)\le\Theta(a,f)\le 1$,

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where $\delta(a,f) = 1 - \limsup_{r\to\infty} \dfrac{N\left(r,\frac{1}{f-a}\right)}{T(r,f)}$ *and* $\Theta(a,f) = 1 - \limsup_{r\to\infty} \dfrac{\overline{N}\left(r,\frac{1}{f-a}\right)}{T(r,f)}$

In [6], Q.C. Zhang proved the following theorem about a meromorphic function and its k-th order derivative.

Theorem: A Let f be a non-constant meromorphic function and let k be a positive integer. Suppose that f and $f^{(k)}$ share 1 CM and $2\overline{N}(r,f) + \overline{N}\left(r,\frac{1}{f'}\right) + N\left(r,\frac{1}{f^{(k)}}\right) \le (\lambda + o(1))T(r,f^k)$ *for* $r \in I$, where I is a set of infinite linear measure and λ satisfies $0 < \lambda < 1$ then $\dfrac{f^{(k)}-1}{f-1} \equiv c$ for some constant $c \in C - \{0\}$.

In 2003, Kit-wing [5] discussed the problem of a meromorphic function and its k- th derivative sharing one small function and proved the following result.

Theorem: B Let $k \ge 1$. Let f be a non-constant non-entire meromorphic function, $a \in s(f)$ and $a(\ne 0,\infty)$ and f do not have any common pole. If f,$f^{(k)}$ share a CM and $4\delta(0,f) + 2(8+k)\Theta(\infty,f) > 19 + 2k$, *then* $f = f^{(k)}$.

Two years latter, in 2005, Q.C.Zhang [2] proved the following theorem.

Theorem: C Let f be a non-constant meromorphic function and $k(\ge 1), l(\ge 0)$ be integers. Also let $a \equiv a(z)$ $(not \equiv 0,\infty)$ be a meromorphic function such that T(r,a) = S(r,f). Suppose that f – a and $f^{(k)}$-a share $(0,l)$. If $l \ge 2$ and $(3+k)\Theta(\infty,f) + 2\delta_{2+k}(0,f) > k+4$ or, if l =1 and $(4+k)\Theta(\infty,f) + 3\delta_{2+k}(0,f) > k+6$, or, if l = 0, i.e., f - a and $f^{(k)}$-a share the value 0 IM and $(6+k)\Theta(\infty,f) + 5\delta_{2+k}(0,f) > 2k+10$ *then* $f \equiv f^{(k)}$.

Recently, in 2010, A.Chen, X.Wang and G.Zhang [7] proved the following results.

Theorem: D Let $k(\ge 1), n(\ge 1)$ be integers and f be a non-constant meromorphic function. Also let $a(z)(not \equiv 0,\infty)$ be a small function with respect to f. If f and $[f^n]^{(k)}$ share a(z) IM and

$$4\overline{N}(r,f) + 2\overline{N}\left(r,\frac{1}{\left(\frac{f}{a}\right)'}\right) + 2N_2\left(r,\frac{1}{(f^n)^{(k)}}\right) + \overline{N}\left(r,\frac{1}{(f^n)^{(k)}}\right) \le (\lambda + o(1))T\left(r,(f^n)^{(k)}\right)$$

, or, if f and $(f^n)^{(k)}$ share a(z) CM and $2\overline{N}(r,f) + \overline{N}\left(r,\frac{1}{\left(\frac{f}{a}\right)'}\right) + N_2\left(r,\frac{1}{(f^n)^{(k)}}\right) \le (\lambda + o(1))T\left(r,(f^n)^{(k)}\right)$, for $0 < \lambda < 1$, where $r \in I$ and I is a set of infinite linear measure, then $\dfrac{f-a}{(f^n)^{(k)} - a} = c$, for some constant $c \in C - \{0\}$.

Theorem: E Let $k(\ge 1), n(\ge 1)$ be integers and let f be a non-constant meromorphic function. Also let $a(z)$ $(not \equiv 0,\infty)$ be a small function with respect to f. If f and $(f^n)^{(K)}$ share a(z) IM and $(2k+6)\Theta(\infty,f) + 3\Theta(0,f) + 2\delta_{k+2}(0,f) > 2k+10$, or, if f and $(f^n)^{(K)}$ share a(z) CM and $(k+3)\Theta(\infty,f) + \delta_2(0,f) + \delta_{k+2}(0,f) > k+4$ then $f \equiv (f^n)^{(k)}$.

In this paper, we will prove the following two theorems which will include the behavior of a meromorphic function and its k th derivative sharing a small function with multiplicity not greater than l, a positive integer.

Theorem: 1 Let k, m and n are three positive integers with $m \leq n$ and let f be a non-constant meromorphic function. Also let $a(z)\ (not \equiv 0, \infty)$ be a small function with respect to f. If $\overline{E}_{l)}(a, f^m(z)) = \overline{E}_{l)}(a, f^{n\,(k)})$, where l is a positive integer and

$$\overline{N}(r, f) + 2N_2\left(r, \frac{1}{f}\right) + 2N_2\left(r, \frac{1}{\left(f^n\right)^{(k)}}\right) + \overline{N}\left(r, \frac{1}{\left(f^n\right)^{(k)}}\right) \leq (\lambda + 0(1))T\left(r, \left(f^n\right)^{(k)}\right), \ for\ 0 < \lambda < 1,$$

where $r \in I$ and I is a set of infinite linear measure, then $\frac{\left(f^n\right)^{(k)} - a}{f^m - a} = c$ for some constant $a \in C - \{0\}$ where C is the set of complex numbers.

Theorem: 2 Let f be a non-constant meromorphic function and let k and n be two positive integers. If $\overline{E}_{l)}(a, f) \equiv \overline{E}_{l)}(a, \left(f^n\right)^{(k)})$, where l is a positive integer and $a(z)\ (not \equiv 0, \infty)$ be a small function of f and $(2k+6)\Theta(\infty, f) + \Theta(0, f) + 2\delta_2(0, f) + 2\delta_{k+2}(0, f) > 2k + 10$ then

$f = \left(f^n\right)^{(k)}$.

2. LEMMAS

Here we mention some existing lemmas of the literature which will be frequently used to prove the aforementioned theorems.

Lemma 2.1 (see[7]): Let f be a non-constant meromorphic function and k,p be two positive integers. Then $N_p\left(r, \frac{1}{f^{(k)}}\right) \leq N_{p+k}\left(r, \frac{1}{f}\right) + k\overline{N}(r, f) + S(r, f)$.

Lemma: 2.2 (see[4]) Let f be a non-constant meromorphic function and let n be a positive integer. $P(f) = a_n f^n + a_{n-1} f^{n-1} + \text{........} + a_1 f$

where a_i is a meromorphic function such that $T(r, a_i) = S(r, f)\ (i = 1, 2, \text{........}, n)$. Then

$T(r, P(f)) = nT(r, f) + S(r, f)$.

3. PROOF OF THE THEOREMS

Proof of Theorem: 1 Let $F = \frac{f^m}{a}$ and $G = \frac{\left(f^n\right)^{(k)}}{a}$. Therefore, $F - 1 = \frac{f^m - a}{a}$ and $G - 1 = \frac{\left(f^n\right)^{(k)} - c}{a}$.

Now, $\overline{E}_{l)}(a, f^m) = \overline{E}_{l)}\left(a, \left(f^n\right)^{(k)}\right)$ except the zeros and poles of a(z). Define,

$$H = \left(\frac{F''}{F'} - \frac{2F''}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G''}{G-1}\right).$$

We now consider two cases:

Case: I Suppose $H\ not \equiv 0$. Then m(r,H) = S(r,f). Now if z_0 is a common simple zero of F-1 and G-1 (except the zeros and poles of a(z)), then after simple calculation, we get $H(z_0) = 0$.

So, $\overline{N}_E\left(r, \frac{1}{G-1}\right) \leq N\left(r, \frac{1}{H}\right) + S(r, f) \leq T(r, H) + S(r, f) \leq N(r, H) + S(r, f)$

Again by analysis, we can deduce that,

$$N(r, H) \leq \overline{N}(r, f) + \overline{N}_{(2}\left(r, \frac{1}{F}\right) + \overline{N}_{(2}\left(r, \frac{1}{G}\right) + \overline{N}_L\left(r, \frac{1}{F-1}\right) + \overline{N}_L\left(r, \frac{1}{G-1}\right) + \overline{N}_*\left(r, \frac{1}{F-1}\right)$$
$$+ \overline{N}_*\left(r, \frac{1}{G-1}\right) + \overline{N}_0\left(r, \frac{1}{F'}\right) + \overline{N}_0\left(r, \frac{1}{G'}\right) + S(r, f).$$

Also, $\overline{N}\left(r,\frac{1}{G-1}\right)= N_E^{1)} + \overline{N}_E^{(2}\left(r,\frac{1}{G-1}\right)+\overline{N}_L\left(r,\frac{1}{G-1}\right)+\overline{N}_L\left(r,\frac{1}{F-1}\right)+\overline{N}_*\left(r,\frac{1}{G-1}\right)+S(r,f).$

Therefore,

$$\overline{N}\left(r,\frac{1}{G-1}\right)\le \overline{N}(r,f)+\overline{N}_{(2}\left(r,\frac{1}{F}\right)+\overline{N}_{(2}\left(r,\frac{1}{G}\right)+2\overline{N}_L\left(r,\frac{1}{F-1}\right)+2\overline{N}_L\left(r,\frac{1}{G-1}\right)$$
$$+2\overline{N}_L\left(r,\frac{1}{G-1}\right)+\overline{N}_E^{(2}\left(r,\frac{1}{G-1}\right)+2\overline{N}_*\left(r,\frac{1}{G-1}\right)+\overline{N}_*\left(r,\frac{1}{F-1}\right) \quad (1)$$
$$+\overline{N}_0\left(r,\frac{1}{F'}\right)+\overline{N}_0\left(r,\frac{1}{G'}\right)+S(r,f)$$

Since, $\overline{E}_{l)}(1,F)=\overline{E}_{l)}(1,G)$,

Therefore, $2\overline{N}_L\left(r,\frac{1}{G-1}\right)+2\overline{N}_*\left(r,\frac{1}{G-1}\right)+\overline{N}_E^{(2}\left(r,\frac{1}{G-1}\right)\le 2\,\overline{N}_{(2}\left(r,\frac{1}{G-1}\right).$

From (1), we have,

$$\overline{N}\left(r,\frac{1}{G-1}\right)\le\overline{N}(r,f)+\overline{N}_{(2}\left(r,\frac{1}{F}\right)+\overline{N}_{(2}\left(r,\frac{1}{G}\right)+2\overline{N}_{(2}\left(r,\frac{1}{G-1}\right)$$
$$+2\overline{N}_L\left(r,\frac{1}{F-1}\right)+\overline{N}_*\left(r,\frac{1}{F-1}\right)+\overline{N}_0\left(r,\frac{1}{F'}\right)+\overline{N}_0\left(r,\frac{1}{G'}\right)+S(r,f) \quad (2)$$

We also have,

$$\overline{N}_2\left(r,\frac{1}{F}\right)+2\overline{N}_L\left(r,\frac{1}{F-1}\right)+\overline{N}_*\left(r,\frac{1}{F-1}\right)+\overline{N}_0\left(r,\frac{1}{F'}\right)\le 2\overline{N}\left(r,\frac{1}{F'}\right) \quad (3)$$

Now by the second fundamental theorem we get,

$$T(r,G)\le \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)-N_0\left(r,\frac{1}{G'}\right)+S(r,G) \quad (4)$$

From (4) using (2) and (3) we get,

$$T(r,G)\le 2\overline{N}(r,f)+2\overline{N}\left(r,\frac{1}{F'}\right)+2\overline{N}\left(r,\frac{1}{G'}\right)+\overline{N}\left(r,\frac{1}{G}\right)+S(r,f) \quad (5)$$

By lemma(2.1) we have,

$$T\left(r,\left(f^n\right)^{(k)}\right)\le 6\overline{N}(r,f)+2N_2\left(r,\frac{1}{f}\right)+2N_2\left(r,\frac{1}{\left(f^n\right)^{(k)}}\right)+\overline{N}\left(r,\frac{1}{\left(f^n\right)^{(k)}}\right)+S(r,f)$$

which contradicts the given conditions of the theorem.

Case: II Suppose $H(z)\equiv 0\, i.e., \frac{F''}{F'}-\frac{2F'}{F-1}=\frac{G''}{G'}-\frac{2G''}{G-1}.$ Integrating we get,

$\log F'-2\log(F-1)=\log G'-\log(G-1)+\log A.$ where A is a constant ≠0.

That is, $\log\frac{F'}{(F-1)^2}=\log\frac{AG'}{(G-1)^2}.$

Again integrating we get,

$$\frac{1}{F-1}=\frac{A}{G-1}+B \tag{6}$$

Now if z_0 is a pole of f with multiplicity p which is not the poles and the zeros of a(z), then z_0 is the pole of F with multiplicity mp and the pole of G with multiplicity np+k(≠mp). This contradicts (6). This implies f has no pole, that is f is an entire function.

So, $\overline{N}(r.F)=S(r,f)$ and $\overline{N}(r.G)=S(r,f)$. Now we prove that B = 0.

We first assume that B ≠ 0, then $\frac{1}{F-1}=\frac{B\left(G-1+\frac{A}{B}\right)}{G-1}$.

Therefore, $\overline{N}\left(r,\frac{1}{G-1+\frac{A}{B}}\right)=\overline{N}(r,F)=S(r,f)$

Now we assume $\frac{A}{B}\neq 1$.

By the second fundamental theorem,

$$\begin{aligned}T(r,G)&\leq \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}\left(r,\frac{1}{G-1+\frac{A}{B}}\right)+S(r,G)\\&\leq \overline{N}\left(r,\frac{1}{G}\right)+S(r,f)\\&\leq T(r,G)+S(r,f)\end{aligned}$$

Hence $T(r,G)=\overline{N}\left(r,\frac{1}{G}\right)+S(r,f)$ i.e., $T(r,(f^n)^{(k)})=\overline{N}\left(r,\frac{1}{(f^n)^{(k)}}\right)+S(r,f)$

This contradicts the given condition of the theorem.

Next, we assume $\frac{A}{B}=1$. Then, (AF – A – 1) G = - 1.

So, $\frac{a^2}{f^n(Af^m-Aa-A)}=-\frac{(f^n)^{(k)}}{f^n}$

Now by lemma (2.1) and (2.2), we get,

$$\begin{aligned}(n+m)T(r,f)&=T\left(r,\frac{(f^n)^{(k)}}{f^n}\right)+S(r,f)\\&\leq n\ N\left(r,\frac{1}{f}\right)+k\,\overline{N}(r,f)+S(r,f)\\&\leq nT(r,f)+S(r,f)\end{aligned}$$

i.e., $T(r,f)=S(r,f)$. This is not true.

Hence our assumption is not true and therefore B =0. So, $\frac{G-1}{F-1}=A$

This proves the theorem.

Proof of the theorem: 2

Let $F=\dfrac{f(z)}{a(z)}$ and $G=\dfrac{(f^n)^{(k)}}{a(z)}$. So, $\overline{E}_{l)}(a,f)=\overline{E}_{l)}(a,(f^n)^{(k)})$ implies, $\overline{E}_{l)}(1,F)=\overline{E}_{l)}(1,G)$, except the zeros and poles of a(z).

We define, $H=\left(\dfrac{F''}{F'}-\dfrac{2F'}{F-1}\right)-\left(\dfrac{G''}{G'}-\dfrac{2G'}{G-1}\right)$.

Now we consider two cases:

Case: I Suppose H not ≡ 0.

Then (5) of the proof in theorem1 still holds. Writing (5) for the function F, we get,

$$T(r,F)\le 2\overline{N}(r,f)+2\overline{N}\left(r,\frac{1}{G}\right)+2\overline{N}\left(r,\frac{1}{F'}\right)+\overline{N}\left(r,\frac{1}{G}\right)+S(r,f)$$

i.e. $$T(r,f)\le 2\overline{N}(r,f)+2\overline{N}_2\left(r,\frac{1}{(f^n)^{(k)}}\right)+2\overline{N}(r,f)+\overline{N}_2\left(r,\frac{1}{f}\right)+2\overline{N}(r,f)+\overline{N}\left(r,\frac{1}{f}\right)+S(r,f)$$

$$\le(2k+6)\overline{N}(r,f)+2N_{K+2}\left(r,\frac{1}{f}\right)+2\overline{N}_2\left(r,\frac{1}{f}\right)+\overline{N}\left(r,\frac{1}{f}\right)+S(r,f)$$

i.e., $(2k+6)\Theta(a,f)+\Theta(0,f)+2\delta_2(0,f)+2\delta_{k+2}(0,f)\le 2k+10$

This contradicts the given condition of the theorem.

Case: II Suppose H ≡ 0.

So $\dfrac{1}{F-1}=\dfrac{A}{G-1}+B,$ where A ≠ 0, B are constant. By the same argument of the proof of theorem 1, we get,

$\overline{N}(r,F)=S(r,f)$ and $\overline{N}(r,G)=S(r,f)$.

So, $\Theta(\infty,f)=1.$

Assume that, B ≠ 0, then $\dfrac{B\left(F-1-\frac{1}{B}\right)}{F-1}=-\dfrac{A}{G-1}$

So, $\overline{N}\left(r,\dfrac{1}{F-1+\frac{1}{B}}\right)=\overline{N}(r,G)=S(r,f)$.

If B ≠ -1, then by the second fundamental theorem for F, we have

$$T(r,F)\le\overline{N}(r,F)+\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{F-1+\frac{A}{B}}\right)+S(r,f)$$

$$\le\overline{N}\left(r,\frac{1}{F}\right)+S(r,f)$$

$$\le T(r,F)+S(r,f)$$

So $T(r,F)\le\overline{N}\left(r,\dfrac{1}{F}\right)+S(r,f)$ i.e., $T(r,f)\le\overline{N}\left(r,\dfrac{1}{f}\right)$. Hence, $\Theta(0,f)=0.$

Putting $\Theta(\infty, f)=1; \Theta(0, f)=0$ and also $\delta(0, f)\le\Theta(0, f)=0$ in the given condition of the theorem we have, $\delta_{k+2}(0, f)>2,$ which is not true. Hence B = - 1.

So $\overline{N}\left(r,\frac{1}{F}\right)=S(r, f)$, i.e., $\overline{N}\left(r,\frac{1}{f}\right)=S(r, f)$. Therefore, $\frac{F}{F-1}=\frac{A}{G-1},$

i.e., F(G-1-A) = - A that is $F=\frac{A}{-G+(1+A)}.$

So, $f=\frac{A}{-(f^n)^{(k)}+(1+A)}.$ Therefore, $\overline{N}\left(r,\frac{1}{(f^n)^{(k)}+(1+A)}\right)=\overline{N}(r, f)=S(r, f).$

Hence $T(r, f)=T(r,(f^n)^{(k)})=S(r, f)$. which is also not true. Thus B = 0.

So $\frac{1}{F-1}=\frac{A}{G-1}$, i.e., G – 1 = A (F – 1).

If A ≠ 1 then $G=A\left(F-1+\frac{1}{A}\right).$ So, $N\left(r,\frac{1}{G}\right)=N\left(r,\frac{1}{F-1+\frac{1}{A}}\right).$

By the second fundamental theorem, we have,

$$T(r,F)\le\overline{N}(r,F)+\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{F-1+\frac{1}{A}}\right)+S(r, f).$$

i.e.,

$$\begin{aligned}T(r, f)&\le\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{G}\right)+S(r, f).\\&=\overline{N}\left(r,\frac{1}{f}\right)+\overline{N}\left(r,\frac{1}{(f^n)^{(k)}}\right)+S(r, f).\\&\le\overline{N}\left(r,\frac{1}{f}\right)+N_{k+1}\left(r,\frac{1}{f}\right)+k\,\overline{N}(r, f)+S(r, f)\\&=\overline{N}\left(r,\frac{1}{f}\right)+N_{k+1}\left(r,\frac{1}{f}\right)+S(r, f)\end{aligned}$$

So,

$$\Theta(0, f)+\delta_{k+1}(0, f)\le 1 \tag{7}$$

Now by the given condition of the theorem and by (7) we have, $\Theta(0, f)>2.$ This is not possible.

So, A = 1 and hence F = G i.e., $f=(f^n)^{(k)}$.

This proves the theorem.

REFERENCES

[1] Chen A.,Wang X.,and Zhang G., Unicity of meromorphic function sharing one small function with its derivative, International Journal of Mathematics and Mathematical Sciences,Artical Id 507454,P.11,2010.

[2] Fang M.L.,Hong W., A unicity theorem for entire functions concerning differential polynomials,Indian J. Pure Appl.Math.32(9)(2001), 1343-1348.

[3] Hayman W.K., Meromorphic functions,Oxford Mathematical Monographs Clarendon Press,Oxford,1964.

[4] Lin S., Lin W., Uniqueness of meromorphic function concerning weakly weighted sharing,Kodai mathematical Journal, Vol29,pp.269-280,2006.

[5] Yang C.C., On deficiencies of differential polynomials II,Math.Z.125(1972),107-112.

[6] Yang C.C.,Yi H.X., Uniqueness theory of meromorphic function,Mathematics and its Application, Vol.557, Science Press,Beijing,China;Kluwer Academic,New York,NY,USA,2003.

[7] Yi H.X., Yang C.C., Uniqueness theory of meromorphic functions,Science Press,Beijing,1995.

[8] Yu K.W., On entire and meromorphic functions that share small functions with their derivatives,J.Inequal. Pure and Appl. Math. 4(2003),1-7.

[9] Zhang Q.C., Meromorphic function that shares one small function with its derivative,Journal of Inequalities in pure and Applied Mathematics, Vol.6, no.4, Article 116,P.13,2005.

[10] Zhang Q.C., The uniqueness of meromorphic functions with their derivatives,Kodai Mathematical Journal, Vol21, no.2, pp.179-184,1998.

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