

APPLICATIONS OF FRACTIONAL CALCULUS INVOLVING SERIES REPRESENTATION OF THE H-FUNCTION AND ALEPH FUNCTION

Rakeshwar Purohit<sup>1</sup>, Ashok Singh Shekhawat\*<sup>2</sup> and Jyoti Shaktawat<sup>3</sup>

<sup>1</sup>Department of Mathematics and Statistics, University College of Science,  
 Mohan Lal Sukhadia University, Udaipur, Rajasthan (India)

<sup>2</sup>Department of Mathematics,  
 Arya College of Engineering and Information Technology, Jaipur, Rajasthan (India)

<sup>3</sup>Department of Mathematics,  
 Kautilya Institute of Technology and Engineering, Jaipur, Rajasthan (India)

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ABSTRACT

The object of the present paper is to discuss about some fractional calculus operators involving many families of bilateral expansions with H-function of several complex variable and Aleph function.

The main result of our paper are unified in nature and capable of generate a very large number of new and known result involving simple special function and polynomial of one or more variable as special cases of our main result.

1. INTRODUCTION

The series representation of the H-function of several complex variable [26] given by Olkha and Chaurasia [15] as given:

$$(a) \quad \delta_{g_i}^{(i)}[d_j^{(i)} + G_i'] \neq \delta_j^{(i)}[d_j^{(i)} + G_i], \text{ for } j \neq g_i$$

$$j, g_i = 1, \dots, u^{(i)}; G_i', G_i = 0, 1, 2, \dots \quad (1.1)$$

$$(b) \quad z_i \neq 0$$

$$\nabla_i = \sum_{j=1}^A \theta_j^{(i)} - \sum_{j=1}^C \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \varphi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} < 0 \quad \forall i \in \{1, \dots, r\} \quad (1.2)$$

then

$$H \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \right] = H_{A, C: [B', D']; \dots; (B^{(r)}, D^{(r)})}^{0, \lambda: (u', v'); \dots; (u^{(r)}, v^{(r)})} [z_1, \dots, z_r]$$

$$= \sum_{g_i=1}^{u^{(i)}} \sum_{G_i=0}^{\infty} \frac{\prod_{j=1}^{\lambda} \Gamma \left( 1 - a_j + \sum_{i=1}^r \theta_j^{(i)} \eta_{G_i} \right)}{\prod_{j=\lambda+1}^A \Gamma \left( a_j - \sum_{i=1}^r \theta_j^{(i)} \eta_{G_i} \right) \prod_{j=1}^C \Gamma \left( 1 - c_j + \sum_{i=1}^r \psi_j^{(i)} \eta_{G_i} \right)}$$

$$\frac{\prod_{\substack{j=1 \\ j \neq g_i}}^{u^{(i)}} \Gamma(d_j^{(i)} - \delta_j^{(i)} \eta_{G_i}) \prod_{j=1}^{v^{(i)}} \Gamma(1 - b_j^{(i)} + \varphi_j^{(i)} \eta_{G_i}) \prod_{j=1}^r (z_i) \eta_{G_i} (-1)^{\sum_{i=1}^r (G_i)}}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)} \eta_{G_i}) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma(b_j^{(i)} - \varphi_j^{(i)} \eta_{G_i}) \prod_{j=1}^r (\delta_{g_i}^{(i)} G_i!)}$$

(1.3)

Corresponding author: Ashok Singh Shekhawat\*<sup>2</sup>

<sup>2</sup>Department of Mathematics, Arya College of Engineering and Information Technology,  
 Jaipur, Rajasthan (India), E-mail: [csmaths2004@yahoo.com](mailto:csmaths2004@yahoo.com)

Suppose as usual that the parameter

$$\begin{aligned} a_j, j = 1, \dots, A; b_j^{(i)}, j = 1, \dots, B^{(i)} \\ c_j, j = 1, \dots, C; d_j^{(i)}, j = 1, \dots, D^{(i)} \quad \forall i \in \{1, \dots, r\} \end{aligned} \quad (1.4)$$

are complex number and the associated coefficients

$$\begin{aligned} \theta_j^{(i)}, j = 1, \dots, A; \phi_j^{(i)}, j = 1, \dots, B^{(i)} \\ \varphi_j^{(i)}, j = 1, \dots, C; \delta_j^{(i)}, j = 1, \dots, D^{(i)} \quad \forall i \in \{1, \dots, r\} \end{aligned} \quad (1.5)$$

are positive real number

where

$$\eta_{G_i} = \frac{d_{g_i}^{(i)} + G_i}{\delta_{g_i}^{(i)}}, i = 1, 2, \dots, r \quad (1.6)$$

The integers  $\lambda, A, C, u^{(i)}, B^{(i)}$  and  $D^{(i)}$  are constrained by the inequalities  $0 \leq \lambda \leq A, c \geq 0, L \leq u^{(i)} \leq D^{(i)}$  and  $0 \leq v^{(i)} \leq B^{(i)}$

$\forall i \in \{1, \dots, r\}$  and the equality in (1.2) hold true for suitable restricted values of the complex variable  $z_1, \dots, z_r \sum_{g_i=1}^{u^{(i)}} z_r$  and

$\sum_{G_i=0}^{\infty}$  denote the multiple sums  $\sum_{g_i=1}^m \dots \sum_{g_r=1}^{u^{(r)}}$  and  $\sum_{G_1=0}^{\infty} \dots \sum_{G_r=0}^{\infty}$  respectively  $\forall i \in \{1, \dots, r\}$ .

H.M. Srivastava defined and introduced the generalized polynomials [22, p.185, eq. (7)] in the following manner:

$$S_{n_1, \dots, n_R}^{m_1, \dots, m_R} [z_1, \dots, z_R] = \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_R=0}^{(n_R/m_R)} \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} B(n_1, \alpha_1; \dots; n_R, \alpha_R) Z_1^{\alpha_1} \dots Z_R^{\alpha_R} \quad (1.7)$$

where  $n_i'' = 0, 1, 2, \dots \forall i'' \in (1, \dots, R); m_i'' \neq 0 \forall i'' \in (1, \dots, R)$   $m_i$  is an arbitrary positive integer and the coefficients  $B[n_1, \alpha_1; \dots; n_R, \alpha_R]$  are arbitrary constant, real or complex.

The Aleph ( $\aleph$ )-function, introduced by Südland *et al* [18], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals

$$\begin{aligned} \aleph[z] = \aleph_{P_i, Q_i; \tau_i; r}^{m, n} [z] = \aleph_{P_i, Q_i, Z_i; r}^{m, n} \left[ z \left| \begin{matrix} (a_j, A_j)_{l, n} [\tau_i (a_{ji}, A_{ji})_{n+i, p_i, r}] \\ (b_j, B_j)_{l, m} [\tau_i (b_{ji}, B_{ji})_{m+1, q_i, r}] \end{matrix} \right. \right] \\ = \frac{1}{2\pi\omega} \int_L \Omega_{P_i, Q_i, \tau_i; r}^{m, n}(s) = z^{-s} ds \end{aligned} \quad (1.8)$$

for  $\forall z \neq 0$  where  $\omega = \sqrt{-1}$  and

$$\Omega_{P_i, Q_i, \tau_i; r}^{m, n}(s) = \frac{\prod_{j=1}^m \Gamma(b_j + B_j s) \prod_{j=1}^n \Gamma(1 - a_j - A_j s)}{\sum_{i=1}^r \tau_i \prod_{j=n+1}^{p_i} \Gamma(a_{ji} + A_{ji} s) \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - B_{ji} s)} \quad (1.9)$$

The integration  $L = L_{\gamma-\infty, \gamma+\infty}, \gamma \in \mathbb{R}$  extends from  $\gamma-\infty$  to  $\gamma+\infty$  and is such that the poles assumed to be simple of

$\Gamma(1 - a_j - A_j s), j = 1, \dots, n$  do not coincide with the poles of  $\Gamma(b_j + B_j s), j = 1, \dots, m$  the parameter  $p_i, q_i$  are non-negative integers satisfying  $0 \leq n \leq p_i, 1 \leq m \leq q_i, \tau_i > 0$  for  $i = 1, \dots, r$ . The parameter  $A_j, B_j, A_{ji}, B_{ji} > 0$  and  $a_j, b_j, a_{ji}, b_{ji} \in \mathbb{C}$ . The empty product in (1.2) is interpreted as unity. The existence conditions for the defining integral (1.8) are given below

$$\varphi_\ell > 0 |\arg(z)| < \frac{\pi}{2} \varphi_\ell, \ell = 1, \dots, r \quad (1.10)$$

$$\varphi_\ell \geq 0 |\arg(z)| < \frac{\pi}{2} \varphi_\ell \text{ and } \operatorname{Re}\{\xi_\ell\} + 1 < 0 \quad (1.11)$$

where

$$\varphi_\ell = \sum_{j=1}^n A_j + \sum_{j=1}^m B_j - \tau_\ell \left( \sum_{j=n+1}^{p_\ell} A_{j\ell} + \sum_{j=m+1}^{q_\ell} B_{j\ell} \right) \quad (1.12)$$

$$\xi_\ell = \sum_{j=1}^m b_j - \sum_{j=1}^n a_j + \tau_\ell \left( \sum_{j=m+1}^{q_\ell} b_{j\ell} - \sum_{j=n+1}^{p_\ell} a_{j\ell} \right) + \frac{1}{2}(p_\ell - q_\ell), \ell = 1, \dots, r \quad (1.13)$$

for detailed account of the Aleph ( $\aleph$ )-function see [18] and [19].

The Riemann-Liouville fractional operator  $D_x^\alpha$  defined by [20]

$$D_x^\alpha [F(x)] = \begin{cases} \frac{1}{\Gamma(-\alpha)} \int_0^x (x-t)^{-\alpha-1} F(t) dt, & \operatorname{Re}(\alpha) < 0, \\ \frac{d^m}{dx^m} D_x^{\alpha-m} \{F(x)\}, & m-1 \leq \operatorname{Re}(\alpha) < m, m \in \mathbb{N}, \end{cases} \quad (1.14)$$

where  $m$  is a positive integer and the integral exists.

The Riemann-Liouville fractional operator satisfy generalized Leibnitz rule [20] as

$$D_x^\alpha \{f(x) T(x)\} = \sum_{n=-\infty}^{\infty} k \binom{\sigma}{\mu + kn} D_x^{\sigma-\mu-kn} \{f(x)\} D_x^{\mu+kn} \{T(x)\}, \quad (\sigma, \mu \in \mathbb{C}, 0 \leq k \leq 1) \quad (1.15)$$

We also mention that some what different types of fractional derivatives are used in the study of PDES related to physical models.

## 2. MAIN THEOREMS

The expansion valid under the following conditions:

$$(a) \quad \operatorname{Re}(\gamma) + \sum_{i=1}^r q_i \binom{d_j^{(i)}}{\delta_j^{(i)}} > -1$$

$$(b) \quad \operatorname{Re}(\rho) + \sum_{i=r+1}^{2r} s_i \binom{d_j^{(i)}}{\delta_j^{(i)}} > -1,$$

$$(c) \quad \sigma, \mu \in \mathbb{C}, 0 \leq k \leq 1,$$

$$(d) \quad \nabla_i = \sum_{j=1}^{A+M} \theta_j^{(i)} - \sum_{j=1}^{C+N} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \varphi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} \leq 1; \forall i = 1, \dots, 2r,$$

(e)  $a_i > 0, \beta_{i'} > 0, i=1, \dots, s, i'=1, \dots, R; M_1, \dots, M_s$  and  $m_1, \dots, m_R$  are arbitrary positive integers and  $A(n_1 \alpha_1, \dots, n_r \alpha_R)$  are arbitrary, real or complex.

$$\sum_{n_1, \dots, n_R=0}^{\infty} \sum_{m_1, \dots, m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k \binom{\sigma}{\mu + kn} (-1)^{\sum_{i=1}^r s_i \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^r n_i} \cdot \frac{\phi_1(m_1)}{m_1!} \dots \frac{\phi_r(m_r)}{m_r!} \psi(m_1, \dots, m_r) \frac{\phi_1'(n_1)}{n_1!} \dots \frac{\phi_R'(n_R)}{n_R!} \psi'(n_1, \dots, n_R) \cdot \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} A(n_1, \alpha_1, \dots, n_R, \alpha_R) (x_1^{s_1} \dots x_r^{s_r}) (x_{r+1}^{n_1} \dots x_{2r}^{n_R})$$

$$\begin{aligned}
 & z^{-\sigma+\rho+\nu+\sum_{i=1}^r q_i s_i + \sum_{i=1}^R B_i \alpha_i + \sum_{i=1}^R s_i n_i - 2} \\
 & \cdot \mathfrak{N}_{p_i+1, q_i+1, \tau_i; r}^{\sigma, m, n+1} \left[ z \left| \begin{array}{l} (a_j, A_j)_{1, n}, (1-\rho-\gamma+\sum_{i=1}^r q_i s_i - \sum_{i=1}^R \beta_i \alpha_i - \sum_{i=1}^R s_i n_i, 1), [\tau_i (a_{ji}, A_{ji})_{n+1}, p_i; r \\ (b_j, B_j)_{1, m}, (1-\rho-\nu-\sum_{i=1}^r q_i s_i - \sum_{i=1}^R \beta_i \alpha_i - \sum_{i=1}^R s_i n_i + \sigma, 1), [\tau_i (b_{ji}, B_{ji})_{m+1}, q_i; r \end{array} \right. \right] \\
 = & \sum_{n_1, \dots, n_R=0}^{\infty} \sum_{m_1, \dots, m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k \left( \begin{array}{c} \sigma \\ \mu + kn \end{array} \right) (-1)^{\sum_{i=1}^r s_i + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R n_i} \\
 & \cdot \frac{\phi_1(m_1)}{m_1!} \dots \frac{\phi_r(m_r)}{m_r!} \psi(m_1, \dots, m_r) \frac{\phi'_1(n_1)}{n_1!} \dots \frac{\phi'_R(n_R)}{n_R!} \psi'(n_1, \dots, n_R) \\
 & \cdot \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} A(n_1, \alpha_1, \dots, n_R, \alpha_R) (x_1^{s_1} \dots x_r^{s_r}) (x_{r+1}^{n_1} \dots x_{2r}^{n_R}) \\
 & z^{-\sigma+\rho+\nu+\sum_{i=1}^r q_i s_i + \sum_{i=1}^R B_i \alpha_i + \sum_{i=1}^R s_i n_i - 2} \Gamma \left( \rho + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i \right)_i \\
 & \cdot \frac{\Gamma \left( \rho + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i - \mu + kn \right)}{\Gamma \left( \rho + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i \right)} \\
 & \cdot \mathfrak{N}_{p_i+1, q_i+1, \tau_i; r}^{\sigma, m, n+1} \left[ z \left| \begin{array}{l} (a_j, A_j)_{1, n}, (1-\nu-\sum_{i=1}^r q_i s_i, 1), [\tau_i (a_{ji}, A_{ji})_{n+1}, p_i; r \\ (b_j, B_j)_{1, m}, (1-\nu-\sum_{i=1}^r q_i s_i - \sigma - \mu - kn; 1), [\tau_i (b_{ji}, B_{ji})_{m+1}, q_i; r \end{array} \right. \right] \tag{2.1}
 \end{aligned}$$

**Proof:** We take

$$F(z) = z^{\nu-1} \chi_{p_i, q_i; \tau_i; r}^{m, n} [z] H_{A, C; [B; D'+1]; \dots; [B^{(r)}, D^{(r)+1}] }^{0, 1; (1, \nu'), \dots; (1, \nu^{(r)})} [X_1 z^{q_1}, \dots, X_r z^{q_r}]$$

and

$$T(x) = z^{\rho-1} S_{n_1, \dots, n_R}^{m_1, \dots, m_R} [-z^{\beta_1}, \dots, -z^{\beta_R}] \cdot H_{M, N; (B^{(r+1)}, D^{(r+1)+1}); \dots; (B^{(r)}, D^{(r)+1})}^{0, 1; (1, \nu^{(r+1)}), \dots; (1, \nu^{(2r)})} [x_{r+1} z^{s_{r+1}}, \dots, x_{2r} z^{s_{2r}}]$$

Using result (1.15), we get

$$\begin{aligned}
 \text{LHS } D_z^\sigma & \left\{ z^{\nu-1} \mathfrak{N}_{p_i, q_i; \tau_i; r}^{\sigma, m, n} [z] H_{A, C; (B; D'+1), \dots, (B^{(r)}, D^{(r)+1})}^{0, 1; (1, \nu'), \dots, (1, \nu^{(r)})} [X_1 z^{q_1}, \dots, X_r z^{q_r}] \right. \\
 & \cdot z^{\rho-1} \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_R=0}^{(n_R/m_R)} \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} \\
 & \cdot A(n_1, \alpha_1, \dots, n_R, \alpha_R) (-1)^{\sum_{i=1}^R \beta_i \alpha_i} (z)^{\sum_{i=1}^R \beta_i \alpha_i} \\
 & \cdot H_{M, N; (B^{(r+1)}, D^{(r+1)+1}), \dots, (B^{(2r)}, D^{(2r)+1})}^{0, 1; (1, \nu^{(r+1)}), \dots; (1, \nu^{(2r)})} [x_{r+1} z^{s_{r+1}}, \dots, x_{2r} z^{s_{2r}}] \tag{2.2}
 \end{aligned}$$

In (2.2) using the result (1.14) and the series representation of the multivariable H-function.

### 3. PARTICULAR CASES

I. If we take  $\rho = 1$  and  $x_{r+1} = \dots = x_{2r} = 0$  in (2.1), we get the following theorem

**Theorem: (a)** Under the hypothesis preceding the assertion of the main theorem (2.1)

$$\begin{aligned}
 & \sum_{n_1, \dots, n_R=0}^{\infty} \sum_{m_1, \dots, m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k \left( \begin{matrix} \sigma \\ m + kn \end{matrix} \right) (-1)^{\sum_{i=1}^r s_i + \sum_{i=1}^r n_i + \sum_{i=1}^R \beta_i \alpha_i} \\
 & \cdot \frac{\phi_1(m_1)}{m_1!} \dots \frac{\phi_r(m_r)}{m_r!} \psi(m_1, \dots, m_r) \frac{\phi'_1(n_1)}{n_1!} \dots \frac{\phi'_R(n_R)}{n_R!} \psi'(n_1, \dots, n_R) \\
 & \cdot \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} A(n_1, \alpha_1, \dots, n_R, \alpha_R) (x_1^{s_1} \dots x_r^{s_r}) \\
 & \cdot Z^{-\sigma + \nu + \sum_{i=1}^r q_i s_i + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i - 1} \\
 & \cdot \mathcal{N}_{P_i+1, Q_i+1, \tau_i; r}^{m, n+1} \left[ Z \left| \begin{matrix} (a_j, A_j)_{l, n} \cdot (-\nu + \sum_{i=1}^r q_i s_i - \sum_{i=1}^R \beta_i \alpha_i - \sum_{i=1}^R s_i n_i, 1), [\tau_i (a_{ji}, A_{ji})_{n+1}, P_i; r \\ (b_j, \beta_j)_{l, m} \cdot (-\nu - \sum_{i=1}^r q_i s_i - \sum_{i=1}^R \beta_i \alpha_i - \sum_{i=1}^R s_i n_i + \sigma, 1), [\tau_i (b_{ji}, B_{ji})_{m+1}, Q_i; r \end{matrix} \right. \right] \\
 & = \sum_{n_1, \dots, n_R=0}^{\infty} \sum_{m_1, \dots, m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k \left( \begin{matrix} \sigma \\ \mu + kn \end{matrix} \right) (-1)^{\sum_{i=1}^r s_i + \sum_{i=1}^r n_i + \sum_{i=1}^R \beta_i \alpha_i} \\
 & \cdot \frac{\phi_1(m_1)}{m_1!} \dots \frac{\phi_r(m_r)}{m_r!} \psi(m_1, \dots, m_r) \frac{\phi'_1(n_1)}{n_1!} \dots \frac{\phi'_R(n_R)}{n_R!} \psi'(n_1, \dots, n_R) \\
 & \cdot \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} A(n_1, \alpha_1, \dots, n_R, \alpha_R) (x_1^{s_1} \dots x_r^{s_r}) \\
 & \cdot Z^{-\sigma + \nu + \sum_{i=1}^r q_i s_i + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i - 1} \\
 & \cdot \frac{\Gamma(1 + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i)}{\Gamma(1 + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i - \mu - kn)} \\
 & \cdot \mathcal{N}_{P_i+1, Q_i+1, \tau_i; r}^{m, n+1} \left[ Z \left| \begin{matrix} (a_j, A_j)_{l, n} \cdot (1 - \nu - \sum_{i=1}^r q_i s_i, 1), [\tau_i (a_{ji}, A_{ji})_{n+1}, P_i; r \\ (b_j, \beta_j)_{l, m} \cdot (1 - \nu - \sum_{i=1}^r q_i s_i - \sigma - \mu - kn, 1), [\tau_i (b_{ji}, B_{ji})_{m+1}, Q_i; r \end{matrix} \right. \right] \tag{3.1}
 \end{aligned}$$

If we take  $\nu = 1$  and  $x_1, \dots, x_r = 0$  in (2.1), we find the following theorem

**Theorem: (b)** Under the hypothesis preceding the assertion of the main theorem (2.1)

$$\begin{aligned}
 & \sum_{n_1, \dots, n_R=0}^{\infty} \sum_{m_1, \dots, m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k \left( \begin{matrix} \sigma \\ \mu + kn \end{matrix} \right) (-1)^{\sum_{i=1}^r s_i + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^r n_i} \\
 & \cdot \frac{\phi_1(m_1)}{m_1!} \dots \frac{\phi_r(m_r)}{m_r!} \psi(m_1, \dots, m_r) \frac{\phi'_1(n_1)}{n_1!} \dots \frac{\phi'_R(n_R)}{n_R!} \psi'(n_1, \dots, n_R) \\
 & \cdot \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} A(n_1, \alpha_1, \dots, n_R, \alpha_R) (x_{r+1}^{n_1} \dots x_{2r}^{n_R}) \\
 & \cdot Z^{-\sigma + \nu + \sum_{i=1}^r q_i s_i + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i - 1} \\
 & \cdot \mathcal{N}_{P_i+1, Q_i+1, \tau_i; r}^{m, n+1} \left[ Z \left| \begin{matrix} (a_j, A_j)_{l, n} \cdot (-\rho - \sum_{i=1}^r q_i s_i - \sum_{i=1}^R \beta_i \alpha_i - \sum_{i=1}^R s_i n_i, 1), [\tau_i (a_{ji}, A_{ji})_{n+1}, P_i; r \\ (b_j, \beta_j)_{l, m} \cdot (-\rho - \sum_{i=1}^r q_i s_i - \sum_{i=1}^R \beta_i \alpha_i - \sum_{i=1}^R s_i n_i + \sigma, 1), [\tau_i (b_{ji}, B_{ji})_{m+1}, Q_i; r \end{matrix} \right. \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \sum_{n_1, \dots, n_R=0}^{\infty} \sum_{m_1, \dots, m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \dots \sum_{\alpha_r=0}^{(n_r/m_r)} \sum_{n=-\infty}^{\infty} k \left( \begin{matrix} \sigma \\ \mu + kn \end{matrix} \right) (-1)^{\sum_{i=1}^r s_i + \sum_{i=1}^r n_i + \sum_{i=1}^R \beta_i \alpha_i} \\
 &\cdot \frac{\phi_1(m_1)}{m_1!} \dots \frac{\phi_r(m_r)}{m_r!} \psi(m_1, \dots, m_r) \frac{\phi_1'(n_1)}{n_1!} \dots \frac{\phi_R'(n_R)}{n_R!} \psi'(n_1, \dots, n_R) \\
 &\cdot \frac{(-n_1)_{m_1 \alpha_1}}{\alpha_1!} \dots \frac{(-n_R)_{m_R \alpha_R}}{\alpha_R!} A(n_1, \alpha_1, \dots, n_R, \alpha_R) (x_{r+1}^{n_1} \dots x_{2r}^{n_R}) \\
 &\cdot \frac{z^{-\sigma + \rho + \sum_{i=1}^r q_i s_i + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i - 1}}{\Gamma \rho + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i} \\
 &\cdot \frac{1}{\Gamma \rho + \sum_{i=1}^R \beta_i \alpha_i + \sum_{i=1}^R s_i n_i - \mu + kn} \\
 &\cdot \mathcal{N}_{p_i+1, q_i+1, \tau_i, r}^{m, n+1} \left[ z \left| \begin{matrix} (a_j, A_j)_{l, n}, (-\sum_{i=1}^r q_i s_i, 1), [\tau_i (a_{ji}, B_{ji})_{n+1, p_i, r}] \\ (b_j, B_j)_{l, m}, (-\sum_{i=1}^r q_i s_i - \sigma - \mu - kn; 1), [\tau_i (b_{ji}, B_{ji})_{m+1, q_i, r}] \end{matrix} \right. \right]. \tag{3.2}
 \end{aligned}$$

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