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ON P-SASAKIAN MANIFOLD SATISFYING CERTAIN CONDITIONS ON THE CONCIRCULAR CURVATURE TENSOR OF A QUARTER-SYMMETRIC METRIC CONNECTION

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ABSTRACT

The object of present paper is to study the quarter-symmetric metric connection on a P-Sasakian manifold and we find the necessary and sufficient condition with respect to the quarter-symmetric metric connection for P- Sasakian manifold satisfying the conditions like $\tilde{Z}(\xi,X).\tilde{Z}=0, \tilde{Z}(\xi,X).\tilde{R}=0, \tilde{R}(\xi,X).\tilde{Z},\ \tilde{Z}(\xi,X).\tilde{S}=0$ *and* $\tilde{Z}(\xi,X).\tilde{C}=0$.

Key words: *Concircular curvature tensor, Weyl conformal curvature tensor, P- Sasakian manifold and Quarter-symmetric connection.*

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1. INTRODUCTION

In 1975,Golab[7] defined and studied quarter-symmetric connection in a Riemannian manifold with affine connection. This was further developed by S.C. Rastogi [12], [13] R.S. Mishra and S.N. Pandey[8], K. Yano and Imai[15], Mukhopadhyay, Roy and Barua [9] U.C.De and Biswas[4] and many other geometers.

A linear connection $\tilde{\nabla}$ on an n-dinmensional Riemannian manifold is said to be a quarter-symmetric connection[7] if its torsion tensor T of the connection $\tilde{\nabla}$

$$T(X,Y)=\tilde{\nabla}_X Y-\tilde{\nabla}_Y X-[X,Y]$$

Satisfies

(1.1) $T(X,Y)=\eta(Y)\phi X-\eta(X)\phi Y,$

where η is $1-$ from and ϕ is a (1,1) tensor field.

In particular, if $\phi X=X$, then the quarter-symmetric connection reduces to the semi-symmetric connection [6]. Thus the notion of quarter-symmetric connection generalizes the idea of the semi-symmetric connection. If moreover, a quarter-symmetric connection $\tilde{\nabla}$ satisfies the condition

(1.2) $(\tilde{\nabla}_X g)(Y,Z)=0$ for all $X,Y\in TM$.

Then$\tilde{\nabla}$ is said to be a quarter-symmetric metric connection. The paper is organized as follows. In section 2, we give a brief account of P- Sasakian manifold. In section 3, we establish the relation between the Riemannian connection and the quarter-symmetric metric connection. In section 4, we study curvature tensor$\tilde{R}$, Ricci tensor$\tilde{S}$, scalar curvature $\tilde{r}$, concircular curvature tensor $\tilde{Z}$ and Weyl conformal curvature tensor $\tilde{C}$ with respect to the quarter-symmetric metric connection. In section 5, we find necessary and sufficient condition with respect to the quarter-symmetric metric connection for P- Sasakian manifold satisfying the conditions like

$\tilde{Z}(\xi,X).\tilde{Z}=0, \tilde{Z}(\xi,X).\tilde{R}=0, \tilde{R}(\xi,X).\tilde{Z}, \tilde{Z}(\xi,X).\tilde{S}=0$ and $\tilde{Z}(\xi,X).\tilde{C}=0$

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2. P-SASAKIAN MANIFOLD

Let M be an n-dimensional differentiable manifold on which there exists a (1,1) tensor field ϕ, a vector field ξ and $1-$from η satisfying

(2.1) $\phi^2 = I - \eta \otimes \xi$

(2.2) $\eta(\xi) = 1$

(2.3) $\eta o \phi = 0$

(2.4) $\phi \xi = 0$

is called an almost para contact manifold and the structure (ϕ, ξ, η) is called an almost para contact structure.

The first and one of the remaining last three above relations imply the other two relations. Let g be a compatible Riemannian metric with(ϕ, ξ, η) -structure such that

(2.5) $g(\phi X, \phi Y) = g\ (X, Y) - \eta(X)\eta(Y)$ or, equivalently,

(2.6) $g(\phi X, Y) = g\ (X, \phi Y)$ and $g(X, \xi) = \eta(X)$ for all $X, Y \in TM$.

Then M is called an almost para contact Riemannian manifold or an almost para contact metric manifold with an almost para contact Riemannian structure-(ϕ, ξ, η, g).

Definition: An almost para contact Riemannian manifold is called P-Sasakain manifold if

(2.7) $(\nabla_X \phi)(Y) = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi$ for all $X, Y \in TM$.

where ∇ denotes the operator of co-variant differentiation with respect to Riemannian metric g. On P-Sasakian manifold, we have

(2.8) $(\nabla_X \eta)(Y) = g(\phi X, Y) = (\nabla_Y \eta)(X)$

(2.9) $(\nabla_X \eta)(Y) = \Phi(X, Y)$ where $\Phi(X, Y) \stackrel{\text{def}}{=} g(\phi X, Y)$

(2.10) $(\nabla_X \xi) = \phi X$

Also in an P- Sasakian manifold M, the curvature tensor R, the Ricci tensor S, and the Ricci orperator Q satisfy

(2.11) $R(X, Y)\xi = \eta(X)Y - \eta(Y)X$

(2.12) $R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi$

(2.13) $R(\xi, X)\xi = X - \eta(X)\xi$

(2.14) $S(X, \xi) = -(n-1)\eta(X)$

(2.15) $Q\xi = -(n-1)\xi$

(2.16) $\eta(R(X, Y)U) = g(X, U)\eta(Y) - g(Y, U)\eta(X)$

(2.17) $\eta(R(X, Y)\xi) = 0$

(2.18) $\eta(R(\xi, X)Y) = \eta(X)\eta(Y) - g(X, Y)$

(2.19) $S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y)$

(2.20) $S(X, \phi Y) = S(\phi X, Y)$

Definition: An almost paracontact Riemannian manifold is said to be η-Einstein [2] if the Ricci tensor S satisfy

(2.21) $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$

where a and b are smooth functions on the manifold .In particular, if $b = 0$, then M is an Einstein manifold.

3. RELATION BETWEEN THE RIEMANNIAN CONNECTION AND THE QUARTER-SYMMETRIC CONNECTION

Let $\tilde{\nabla}$ be a linear connection and ∇ be a Riemannian connection of an almost paracontact metric manifold M such that

(3.1) $\tilde{\nabla}_X Y = \nabla_X Y + \mathcal{U}(X,Y)$

where $\mathcal{U}$ is tensor of type $(1,1)$. For $\tilde{\nabla}$ to be the quarter-symmetric connection in M, then we have [7].

(3.2) $\mathcal{U}(X,Y) = \dfrac{1}{2}[T(X,Y) + T'(X,Y) + T'(Y,X)]$

Where

(3.3) $g(T'(X,Y),Z) = g(T(Z,X),Y)$

From (1.1) an(3.3) , we get

(3.4) $T'(X,Y) = \eta(X)\phi Y - g(\phi X, Y)\xi$

Using (1.1) and (3.4) in (3.2), we have

(3.5) $\mathcal{U}(X,Y) = \eta(Y)\phi X - g(\phi X, Y)\xi$

Thus a quarter-symmetric metric connection $\tilde{\nabla}$ in a para-Sasakian manifold is given by

(3.6) $\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g(\phi X, Y)\xi$

Conversely, we show that a linear connection $\tilde{\nabla}$ in a para-Sasakian manifold defined by

(3.7) $\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g(\phi X, Y)\xi$ denotes a quarter-symmetric metric connection.

Using (3.7) the torsion tensor of the connection $\tilde{\nabla}$ is given by

$$T(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y]$$

(3.8) $T(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y$

The above equation shows that the connection $\tilde{\nabla}$ is quarter-symmetric connection [7]. Also we have

$$(\tilde{\nabla}_X g)(Y,Z) = Xg(Y,Z) - g(\tilde{\nabla}_X Y, Z) - g(Y, \tilde{\nabla}_X Z)$$

(3.9) $(\tilde{\nabla}_X g)(Y,Z) = 0$

From (3.8) and (3.9) we conclude that $\tilde{\nabla}$ is quarter-symmetric metric connection.

Therefore equation (3.6) is the relation between the Riemannian connection and the quarter- symmetric metric connection on a para-Sasakian manifold.

4. CURVATURE TENSOR OF A P-SASAKIAN MANIFOLD WITH RESPECT TO THE QUARTER-SYMMETRIC METRIC CONNECTION

Let $\tilde{R}$ and R be the curvature tensor with respect to the connection $\tilde{\nabla}$ and ∇ respectively. Then we have from [11]

(4.1) $\tilde{R}(X,Y)U = R(X,Y)U + 3g(\phi X, U)\phi Y - 3g(\phi Y, U)\phi X + \eta(U)[\eta(X)Y - \eta(Y)X]$
$\qquad - [\eta(X)g(Y,U) - \eta(Y)g(X,U)]\xi$

Where $\tilde{R}(X,Y)U = \tilde{\nabla}_X\tilde{\nabla}_Y U - \tilde{\nabla}_Y\tilde{\nabla}_X U - \tilde{\nabla}_{[X,Y]}U$

From(4.1) , it follows that

(4.2) $\tilde{S}(Y,U) = S(Y,U) + 2g(Y,U) - (n+1)\eta(Y)\eta(U)$

where $\tilde{S}$ and S are the Ricci tensors of the connections $\tilde{\nabla}$ and ∇ respectively.

Again contracting (4.2), we have

(4.3) $\tilde{r} = r + n - 1$

where $\tilde{r}$ and r is the scalar curvature of the connections $\tilde{\nabla}$ and ∇ respectively.

Let (M, g) be a Riemannian manifold. Then the concircular curvature tensor Z and the Weyl conformal curvature tensor C are defined by [16]

(4.4) $$Z(X,Y)U = R(X,Y)U - \frac{r}{n(n-1)}\{g(Y,U)X - g(X,U)Y\}$$

(4.5) $$C(X,Y)U = R(X,Y)U - \frac{1}{n-2}\{S(Y,U)X - S(X,U)Y + g(Y,U)QX - g(X,U)QY\} + \frac{r}{(n-1)(n-2)}\{g(Y,U)X - g(X,U)Y\}$$

for all $X, Y, U \in TM$,where r is scalar curvature.

Let $\tilde{Z}$ and $\tilde{C}$ be the concircular curvature tensor and the Weyl conformal curvature tensor with respect to quarter-symmetric metric connection $\tilde{\nabla}$,then by using (3.6) and (4.4), we have

(4.6) $$\tilde{Z}(X,Y)U = Z(X,Y)U + 3g(\phi X,U)\phi Y - 3g(\phi Y,U)\phi X + \eta(U)[\eta(X)Y - \eta(Y)X] - [\eta(X)g(Y,U) - \eta(Y)g(X,U)]\xi + \frac{1}{n}[g(X,U)Y - g(Y,U)X]$$

Where

(4.7) $\tilde{Z}(X,Y)U = \tilde{R}(X,Y)U - \frac{\tilde{r}}{n(n-1)}\{g(Y,U)X - g(X,U)Y\}$ and using (3.6) and (4.5), we have

(4.8) $$\tilde{C}(X,Y)U = C(X,Y)U + 3g(\phi X,U)\phi Y - 3g(\phi Y,U)\phi X - \tfrac{3}{n-2}\{g(Y,U)X - g(X,U)Y + \eta(U)(\eta(X)Y - \eta(Y)X) + (\eta(Y)g(X,U) - \eta(X)g(Y,U)\xi)\}$$

Where

(4.9) $$\tilde{C}(X,Y)U = \tilde{R}(X,Y)U - \frac{1}{n-2}\{\tilde{S}(Y,U)X - \tilde{S}(X,U)Y + g(Y,U)\tilde{Q}X - g(X,U)\tilde{Q}Y\} + \tfrac{\tilde{r}}{(n-1)(n-2)}\{g(Y,U)X - g(X,U)Y\}$$

5. MAIN RESULTS

In this section, we obtain necessary and sufficient conditions for P- Sasakian manifolds satisfying the derivation conditions $\tilde{Z}(\xi,X).\tilde{Z} = 0, \tilde{Z}(\xi,X).\tilde{R} = 0, \tilde{R}(\xi,X).\tilde{Z}, \ \tilde{Z}(\xi,X).\tilde{S} = 0$ and $\tilde{Z}(\xi,X).\tilde{C} = 0$.

Theorem (5.1): The concircular curvature tensor with respect to quarter-symmetric metric connection in a P- Sasakian manifold satisfies $\tilde{Z}(\xi,X).\tilde{Z} = 0$ if and only if either the scalar curvature r of M is $r = (2n+1)(1-n)$ or M is η-Einstein.

Proof: In a P- Sasakian manifold from (2.11), (2.12), (4.4) and(4.6),we have

(5.1) $$\tilde{Z}(\xi,X)Y = \left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)(\eta(Y)X - g(X,Y)\xi)$$

and

(5.2) $$\tilde{Z}(X,Y)\xi = \left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)(\eta(X)Y - \eta(Y)X)$$

From the conditions $\tilde{Z}(\xi,U).\tilde{Z} = 0$, we get

$$[\tilde{Z}(\xi,U),\tilde{Z}(X,Y)]\xi - \tilde{Z}\big(\tilde{Z}(\xi,U)X,Y\big)\xi - \tilde{Z}\big(X,\tilde{Z}(\xi,U)Y\big)\xi = 0$$

In view of (5.1), we have

$$\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)\{\eta\big(\tilde{Z}(X,Y)\xi\big)U - g\big(U,\tilde{Z}(X,Y)\xi\big)\xi - \eta(X)\tilde{Z}(U,Y)\xi + g(U,X)\tilde{Z}(\xi,Y)\xi - \eta(Y)\tilde{Z}(X,U)\xi + g(U,Y)\tilde{Z}(X,\xi)\xi - \tilde{Z}(X,Y)U + \eta(U)\tilde{Z}(X,Y)\xi\} = 0$$

Equation (5.2)then gives

$$\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)\left(\tilde{Z}(X,Y)U-\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)(g(X,U)Y-g(Y,U)X)\right)=0$$

Therefore either the scalar curvature $r=(2n+1)(1-n)$, or

$$\tilde{Z}(X,Y)U-\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)(g(X,U)Y-g(Y,U)X)=0$$

Using (4.1) and (4.7), we have

$$R(X,Y)U=-3g(\phi X,U)\phi Y+3g(\phi Y,U)\phi X-\eta(U)[\eta(X)Y-\eta(Y)X]+[\eta(X)g(Y,U)-\eta(Y)g(X,U)]\xi \\ +2(g(X,U)Y-g(Y,U)X)$$

Contracting, we get

$$S(Y,U)=-2ng(Y,U)+(n+1)\eta(Y)\eta(U)$$

Therefore M is η- Einstein manifold. The converse is trivial.

Using the fact that $\tilde{Z}(\xi,X).\tilde{R}$ denotes $\tilde{Z}(\xi,X)$ acting on $\tilde{R}$ as a derivation, we have the following theorem as a corollary of theorem (5.1).

Theorem (5.2): The concircular curvature tensor with respect to quarter-symmetric metric connection in a P- Sasakian manifold satisfies $\tilde{Z}(\xi,X).\tilde{R}=0$ if and only if either M is η-Einstein or M has the scalar curvature

$$r=(2n+1)(1-n).$$

Theorem (5.3): If the concircular curvature tensor with respect to quarter-symmetric metric connection in a P- Sasakian manifold satisfies $\tilde{R}(\xi,X).\tilde{Z}=0$, then the manifold M is η-Einstein.

Proof: The condition $\tilde{R}(\xi,X).\tilde{Z}=0$ implies that

(5.3) $[\tilde{R}(\xi,U),\tilde{Z}(X,Y)]\xi-\tilde{Z}(\tilde{R}(\xi,U)X,Y)\xi-\tilde{Z}(X,\tilde{R}(\xi,U)Y)\xi=0$

In P-Sasakian manifold, we have

(5.4) $\tilde{R}(X,Y)\xi=2(\eta(X)Y-\eta(Y)X)$

and

(5.5) $\tilde{R}(\xi,X)U=2(\eta(U)X-g(X,U)\xi)$

Using (5.5) in (5.3), we have

(5.6) $\tilde{Z}(X,Y)U-\eta(U)\tilde{Z}(X,Y)\xi+\eta(Y)\tilde{Z}(X,U)\xi-g(U,Y)\tilde{Z}(X,\xi)\xi+\eta(X)\tilde{Z}(U,Y)\xi-g(U,X)\tilde{Z}(\xi,Y)\xi$
$-\eta(\tilde{Z}(X,Y)\xi)U=0$

From (5.2), (5.4)and (5.6),we have

(5.7) $\tilde{Z}(X,Y)U+\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)(g(Y,U)X-g(X,U)Y)=0$

From (4.4) (4.6) and (5.7), we have

$$R(X,Y)U=2(g(X,U)Y-g(Y,U)X)-3g(\phi X,U)\phi Y+3g(\phi Y,U)\phi X-\eta(U)[\eta(X)Y-\eta(Y)X] \\ +[\eta(X)g(Y,U)-\eta(Y)g(X,U)]\xi$$

Contracting, we get

$$S(Y,U)=-2ng(Y,U)+(n+1)\eta(Y)\eta(U)$$

Therefore M is η- Einstein manifold.

Theorem(5.4): The concircular curvature tensor with respect to quarter-symmetric metric connection in a P- Sasakian manifold satisfies$\tilde{Z}(\xi,X).\tilde{S}=0$if and only if either M has the scalar curvature $r=(2n+1)(1-n)$ or M is η-Einstein.

Proof: The condition $\tilde{Z}(\xi,X).\tilde{S}=0$ implies that

$$\tilde{S}\big(\tilde{Z}(\xi,X)Y,\xi\big)+\tilde{S}\left(Y,\tilde{Z}(\xi,X)\right)\xi=0$$

In view of (5.1) gives

$$\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)\left(\eta(Y)\tilde{S}(X,\xi)-g(X,Y)\tilde{S}(\xi,\xi)+\tilde{S}(Y,X)-\eta(X)\tilde{S}(Y,\xi)\right)=0$$

Using (2.14) and (4.2),we have

$$\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)\big(S(X,Y)+2ng(X,Y)-(n+1)\eta(X)\eta(Y)\big)=0$$

Therefore either the scalar curvature $r=(2n+1)(1-n)$, or

$$S(X,Y)=-2ng(X,Y)+(n+1)\eta(X)\eta(Y)$$

Thus M is η- Einstein manifold. The converse part is trivial.

Theorem(5.5): The Weyl conformal curvature tensor with respect to quarter-symmetric metric connection in a P-Sasakian manifold satisfies $\tilde{Z}(\xi,X).\tilde{C}=0$ if and only if either M has the scalar curvature $r=(2n+1)(1-n)$ or M is conformally flat in which case M is a SP- Sasakian manifold.

Proof: $\tilde{Z}(\xi,U).\tilde{C}=0$ implies that

$$\big[\tilde{Z}(\xi,U),\tilde{C}(X,Y)\big]W-\tilde{C}\big(\tilde{Z}(\xi,U)X,Y\big)W-\tilde{C}(X,\tilde{Z}(\xi,U)Y)W=0$$

In view of (5.1),we have

$$\left(\frac{(2n+1)(n-1)+r}{n(n-1)}\right)\big[\eta\big(\tilde{C}(X,Y)W\big)U-\tilde{C}(X,Y,W,U)\xi-\eta(X)\tilde{C}(U,Y)W+g(U,X)\tilde{C}(\xi,Y)W-\eta(Y)\tilde{C}(X,U)W$$
$$+g(U,Y)\tilde{C}(X,\xi)W-\eta(W)\tilde{C}(X,Y)U+g(U,W)\tilde{C}(X,Y)\xi\big]=0$$

So either the scalar curvature $r=(2n+1)(1-n)$, or the equation

$$\eta\big(\tilde{C}(X,Y)W\big)U-\tilde{C}(X,Y,W,U)\xi-\eta(X)\tilde{C}(U,Y)W+g(U,X)\tilde{C}(\xi,Y)W-\eta(Y)\tilde{C}(X,U)W+g(U,Y)\tilde{C}(X,\xi)W$$
$$-\eta(W)\tilde{C}(X,Y)U+g(U,W)\tilde{C}(X,Y)\xi=0$$

holds on M.Taking the inner product of the last equation with ξ,we get

(5.8) $$\eta\big(\tilde{C}(X,Y)W\big)\eta(U)-\tilde{C}(X,Y,W,U)-\eta(X)\eta(\tilde{C}(U,Y)W)+g(U,X)\eta(\tilde{C}(\xi,Y)W)-\eta(Y)\eta(\tilde{C}(X,U)W)$$
$$+g(U,Y)\eta(\tilde{C}(X,\xi)W)-\eta(W)\eta(\tilde{C}(X,Y)U)+g(U,W)\eta(\tilde{C}(X,Y)\xi)=0$$

From (4.8), in P-Sasakian manifold, we have

(5.9) $$\eta\big(\tilde{C}(X,Y)U\big)=\eta(C(X,Y)U)$$

(5.10) $$\tilde{C}(X,Y,W,U)=C(X,Y,W,U)$$

Using (5.9) and(5.10) in (5.8), we get

(5.12) $$\eta(C(X,Y)W)\eta(U)-C(X,Y,W,U)-\eta(X)\eta(C(U,Y)W)+g(U,X)\eta(C(\xi,Y)W)-\eta(Y)\eta(C(X,U)W)$$
$$+g(U,Y)\eta(C(X,\xi)W)-\eta(W)\eta(C(X,Y)U)+g(U,W)\eta(C(X,Y)\xi)=0$$

Now using (2.14), (2.16) and (4.5) in (5.12), we get

$$(5.13)\quad g(U,Y)g(X,W)-g(U,X)g(Y,W) + \frac{1-n}{n-2}\{-g(Y,W)g(X,U)+g(X,W)g(U,Y)+g(X,U)\eta(Y)\eta(W)-g(U,Y)\eta(X)\eta(W)\} + \frac{1}{n-2}\{S(Y,U)\eta(X)\eta(W)-S(X,U)\eta(Y)\eta(W)+g(Y,W)S(X,U)-g(X,W)S(Y,U)\} - R(X,Y,W,U)=0$$

Contracting (5.13), we have

$$(5.14)\quad S(Y,U)=\left(1+\frac{r}{n-1}\right)g(Y,U)+\left(-n+\frac{r}{n-1}\right)\eta(Y)\eta(U)$$

This implies that M is an η-Einstein manifold.So using (5.14) in (5.12) we obtain $C=0$ on M.Thus using the fact from [1] that a conformally flat P- Sasakian manifold is an SP-Sasakian, M becomes an SP-Sasakian manifold. The converse statement is trivial.

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