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AN ITERATIVE ALGORITHM FOR EFFICIENT ESTIMATION OF SQUARE OF THE NORMAL POPULATION MEAN FOR SMALL SAMPLES

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ABSTRACT

Artikayala et. al. (2011) used a monte-carlo simulation study to compare the relative bias the relative efficiencies of some well-known estimators of the square of the normal population mean, with respect to their mean-squared-errors. They concluded that the estimator of Singh & Zaidi (2000) is the best in terms of the relative efficiency, while the 'relative biases' of all the estimators are negligible. The present paper aims at developing an iterative algorithm for making this estimator increasingly more efficient, as supported by a similar monte-carlo simulation study with a more convincingly large number of replication of '51,000'. The simulation study is implemented using MATLAB 2010b.

Key Words: *Mean square error; Relative efficiency; Monte-Carlo simulation & Replication.*

Mathematics Subject Classification: *62F10.*

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1. INTRODUCTION:

In estimating the square of population coefficient of variation, we do need to have an estimate of population variance and of the square of population mean. The square of population coefficient of variation is quite useful in derivations ([2], [3], [4], [5], and [6]) of the estimates of variance of the population.

Recently, Artikayala et. al. (2011) ([1]) used a monte-carlo simulation study to compare the relative bias the relative efficiencies of some well-known estimators of the square of the normal population mean, with respect to their mean-squared-errors. They concluded that the estimator of Singh & Zaidi (2000) is the best in terms of the relative efficiency, while the 'relative biases' of all the estimators are negligible. The present paper aims at developing an iterative algorithm for making this estimator increasingly more efficient, as supported by a similar monte-carlo simulation study with a more convincingly large number of replication of '51,000'. The simulation study is implemented using *MATLAB 2010b*.

2. AN ITERATIVE ALGORITHM FOR MORE EFFICIENT ESTIMATION OF POPULATION MEAN:

In this section we detail an iterative algorithm for progressively more efficient estimation of the square of the population mean. Based on a random sample ($y_1, y_2, y_3, \ldots, y_n$) of size 'n' from the population with mean 'μ', and variance 'σ^2', we have:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{i=n} y_i \quad \text{And} \quad s^2 = \frac{1}{(n-1)}\sum_{i=1}^{i=n}(y_i - \bar{y})^2$$

as an unbiased estimate, respectively, of 'μ'& 'σ^2', i.e. of the population mean & of the population variance.

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We have to estimate the square of the population mean, i.e. 'μ^2'. A common-sensical estimator would be its sample counterpart, namely the:

'Square of Sample Mean' '$\bar{y}^2$' (2.1)

Artikayala et. al. (2011) considered all the prominently known estimators of square of the population mean, i.e. 'μ^2', in the recent literature, and compared their relative efficiencies (in terms of their mean-squared-errors) through a 'Monte-Carlo Simulation' study. They determined that the best estimator is that of Singh & Zaidi (2000) ([7]):

Say $T_6 = \bar{y}^2 * F$; (2.2)

Wherein, the factor F is:

$$F = \frac{1}{\{1 + \frac{u}{(1+3.5*u)}\}} \quad \text{with } u = s^2/(n*\bar{y}^2) \tag{2.3}$$

Implicitly, the factor 'F' defined in (2.3) makes the estimator of Singh & Zaidi (2000) ([7]) in (2.2) better than the estimator 'Square of Sample Mean' '$\bar{y}^2$'. This fact that the factor 'F' is the 'NICE' factor making the sample-counterpart estimator 'Square of Sample Mean' '$\bar{y}^2$'more efficient is seminal to the proposition and the make-up of the 'Iterative-Algorithm'. Let us rename the estimator in (2.2) as the estimator at the 'FIRST' iteration of the proposed algorithm. Therefore;

Say $T_6[1] = \bar{y}^2 * F$; (2.3)

Subsequently, that at the 'SECOND 'iteration of the algorithm is;

Say $T_6[2] = \bar{y}^2 * F^2$; (2.4)

That at the 'THIRD' iteration of the algorithm is;

Say $T_6[3] = \bar{y}^2 * F^3$; (2.5)

So on and so-forth, that at the 'Ith.' iteration of the algorithm is;

Say $T_6[I] = \bar{y}^2 * F^I$; (2.6)

I = 1 (1) N; N being any arbitrary positive integer!

The aforesaid details do define our proposed "Iterative Algorithm' for progressively more efficient estimation of the 'Square of the population mean' "μ^2".

3. EMPIRICAL MONTE-CARLO SIMULATION STUDY:

The Monte-carlo simulation method is adopted since the analytical comparisons of the Singh & Zaidi (2000)'s estimator with our proposed 'iteratively progressively efficient' estimators in (2.6) for various plausible values of the 'Iteration-Indicator' index I (= 2 (1) N) are not possible because of the resultant 'mean-squared-errors' being inextricably complicated and because of their involving 'unknown' population parameters.

We have used a ***'MATLAB 2010b'*** program to generate 51,000 (Replications) of random samples from a Normal population N (3, 2^2). For the empirical study we have confined to samples of illustrative-sizes: 2, 3, 4, 5, 6, 7, 8, 9, 10, 12 and 14. Also we have confined our study to I = 2 (1) 11. For each respective sample we have generated the squared-error of the estimator '●', i.e. $(● - 9)^2$, and thence determined the MSE (●) by taking the average over all the resultant "51,000" squared-errors. The 'Relative Efficiencies' of the estimators (relative to 'Square of Sample Mean' '$\bar{y}^2$') are simply calculated by the following formula:

$$\text{PREff}(●) = [\text{MSE}(\bar{y}^2)*100/\text{MSE}(●)]\ \% \tag{3.1}$$

The resultant "Percentage Relative Efficiencies" are tabulated in the 'Table # 1' to follow. The results are heartening and supportive of our claim that the proposed "Iterative-Algorithm" is seminal to progressively more efficient estimators of the 'Square of the population Mean'! The relative efficiencies of the last most efficient estimator for a particular 'n' {T_6[9] for n =2, T_6[6] for n = 3, T_6[5] for n = 4, T_6[4] for n = 5 & 6, T_6[3] for n = 7, 8, & 9, T_6[2] for n = 10, 12, and 14.} are highlighted. For the sample size as large as '15' the betterments are not-so-significant!

Table - 1

Relative Efficiencies of the Estimators of Square of Normal Population Mean [Relative to Sample Mean squared (%)]

Estimator↓\n→	2	3	4	5	6	7	8	9	10	12	14
Squared Sample Mean	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
T_6[1]	109.1	107.9	106.5	105.4	104.5	103.7	103.0	102.7	102.4	101.7	101.5
T_6[2]	116.6	114.3	111.3	109.0	107.4	105.8	104.7	104.0	103.5	102.3	101.8
T_6[3]	122.4	118.9	114.4	111.1	108.8	106.4	104.9	104.1	103.3	101.7	101.2
T_6[4]	126.8	121.9	115.9	111.7	108.9	105.9	103.9	102.9	102.1	100.2	99.7
T_6[5]	129.9	123.7	116.3	111.2	107.8	104.3	102.0	100.9	100.1	97.9	97.4
T_6[6]	131.9	124.2	115.7	109.8	105.9	101.9	99.5	98.3	97.4	95.0	94.6
T_6[7]	133.1	123.9	114.3	107.7	103.4	99.0	96.4	95.1	94.3	91.8	91.5
T_6[8]	133.7	123.1	112.4	105.2	100.6	95.9	93.1	91.8	90.9	88.3	88.1
T_6[9]	133.9	121.8	110.2	102.4	97.5	92.6	89.7	88.2	87.4	84.8	84.6
T_6[10]	133.7	120.3	107.8	99.5	94.3	89.3	86.2	84.7	83.9	81.2	81.0
T_6[11]	133.3	118.5	105.3	96.6	91.2	85.9	82.8	81.3	80.4	77.7	77.5

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