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# PERISTALTIC TRANSPORT OF A JOHNSON - SEGALMAN FLUID THROUGH A POROUS MEDIUM IN A CHANNEL

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*ABSTRACT*

*In this paper, we investigated the peristaltic motion of a Johnson-Segalman fluid through a porous medium in a two - dimensional channel under the assumptions of long-wavelength and low-Reynolds number. The flow is investigated in a wave frame of reference moving with velocity of the wave. A Perturbation solution for small Weissenberg number is obtained for the axial velocity, axial pressure gradient and pressure rise per one wavelength. The effects o various emerging parameters on the pressure gradient, pumping characteristics and friction force are discussed through graphs in detail.*

**Key words:** *Darcy number; Johnson-Segalman fluid; peristaltic flow; Weissenberg number.*

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## 1. INTRODUCTION:

Peristalsis is a well known mechanism of pumping biological and industrial fluids. Even though it is observed in living systems for many centuries; the mathematical modeling of peristaltic transport began with trend setting works by Shapiro et al. [1] using wave frame of reference and Fung and Yih [2] using laboratory frame of reference. Many of the contributors to the area of peristaltic transport have either followed Shapiro or Fung.

Even though, there are many models to describe non-Newtonian behavior of the fluids but in recent years, the Johnson-Segalman fluid has acquired a special status, as it includes as special cases the classical Newtonian fluid and Maxwell fluid. The Johnson-Segalman model is a viscoelastic fluid model which was developed to allow for non-affine deformations Johnson and Segalman [3]. The following researchers [4,5,6] used this model to explain the phenomenon of “spurt”. The term “spurt” is used to describe the large increase in the volume through put for a small increase in the driving pressure gradient (Vinogardov et al. [7]; Denn [8]) at a critical value of the pressure gradient that is observed in the flow of many non-linear fluids. Some experiments relevant to this issue have also been carried out by (Kraynik and Schowalter [9], Lim and Schowalter [10], Malkus et al. [5], Migler et al. [11], Migler et al. [12], Ramamurthy [13]). Experimentalists usually associate “spurt” with slip at the wall. Rao and Rajagopal [14] also studied three distinct flows of Johnson-Segalman fluid. Unlike most other fluid models, the Johnson-Segalman (JS) fluid allows for a non-monotonic relationship between the shear stress and the rate of share in a shear flow for certain values of the material parameter. While the JS model offers a very interesting means for explaining “spurt”, it seems more likely that the phenomenon is because of the “stick slip” that takes place at the boundary Rao and Rajagopal [14]. Hayat et al. [15] investigated the peristaltic motion of a Johnson-Segalman fluid in a planner channel. The MHD peristaltic motion of Johnson-Segalman fluid in a planar channel was studied by Elshahed and Haroun [16].

In all the above mentioned studies, no porous medium has been taken into account. But it is well known that flow through a

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porous medium has practical applications especially in geophysical fluid dynamics. Examples of natural porous media are beach sand, sandstone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. In the arterial system of human or animal, it is quite common to find localized narrowings, commonly caused by intravascular plaques. This stenosis disturbs the normal pattern of blood flow through the artery. Acknowledge of flow characteristics in the vicinity of stenosis may help to further the understanding of some major complications which can arise such as an in growth of tissue in the artery, the development of a coronary thrombosis, the weakening and bulging of the artery downstream from stenosis, etc. The investigations of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis. In some pathological situations, the distribution of fatty cholesterol and artery clogging blood clots in the lumen of coronary artery can be considered as equivalent to porous medium. El Shehawey and Husseny [17] and El Shehawey et al. [18] studied the peristaltic mechanism of a Newtonian fluid through a porous medium. Hall effects on peristaltic flow of a Maxwell fluid through a porous medium in a channel was studied by Hayat et al. [19]

In view of these, we studied the peristaltic flow of a Johnson-Segalman fluid through a porous medium in a two - dimensional channel. The flow is investigated in a wave frame of reference moving with velocity of the wave under the assumptions of long-wavelength and low-Reynolds number. A Perturbation solution for small Weissenberg number is obtained for the axial velocity, axial pressure gradient and pressure rise per one wavelength. The effects o various emerging parameters on the pressure gradient, pumping characteristics and friction force are discussed in detail.

## 2. THE MATHEMATICAL MODEL:

We consider an incompressible, Johnson-Segalman fluid through a porous medium in a two dimensional infinite symmetric channel of width $2a$. We employ a rectangular coordinate system with $X$ parallel to and $Y$ normal to the channel walls. Moreover, we consider an infinite wave train traveling with velocity $c$ along the channel walls. Fig. 1 shows the physical model of the problem. The symmetric channel walls are defined as

$$\pm H(X,t) = \pm a \pm b \sin\left[\frac{2\pi}{\lambda}(X - ct)\right] \tag{2.1}$$

where $b$ is the amplitude of the wave, $t$ is the time and $\lambda$ is the wavelength.

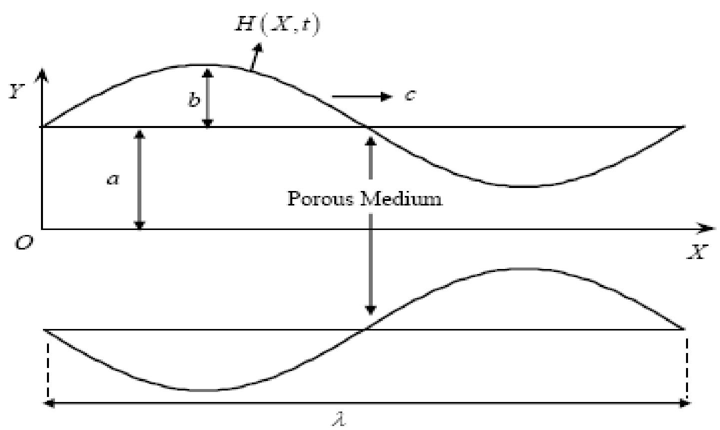


**Fig. 1.** The Physical model

The equations governing the flow of an incompressible fluid are

$$\operatorname{div} V = 0 \qquad \operatorname{div}\sigma + \rho f = \rho \frac{dV}{dt} \tag{2.2}$$

where $V$ is the velocity field, $f$ - the body force per unit mass, $\rho$ - the fluid density, $\frac{d}{dt}$ - the material derivative and $\sigma$ - the Cauchy stress tensor given by [3]:

$$\sigma = -pI + T \tag{2.3}$$

$$T = 2\mu D + S \tag{2.4}$$

$$S + m\left[\frac{dS}{dt} + S\left(W - eD\right) + \left(W - eD\right)^T S\right] = 2\eta D \tag{2.5}$$

$$D = \frac{1}{2}\left[L + L^T\right], \qquad W = \frac{1}{2}\left[L - L^T\right], \qquad L = \text{grad}V \tag{2.6}$$

The equations above include the scalar pressure $p$, the identity tensor $I$, the dynamic viscosities $\mu$ and $\eta$, the relaxation time $m$, the slip parameter $e$ and the respective symmetric and skew symmetric part of the velocity gradient $D$ and $W$. Note that, our model reduces to the Maxwell fluid model for $e = 1$ and $\mu = 0$, and for $m = 0 = \mu$, it reduces to the classical Navier-Stokes fluid model.

The velocity for unsteady two-dimensional flows is defined as

$$V = \left[U\left(X, Y, t\right), V\left(X, Y, t\right), 0\right] \tag{2.7}$$

From Eqs. (2.2) - (2.7) we obtain, when body forces are absent,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2.8}$$

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial p}{\partial X} + \mu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} - \frac{\mu}{k}u \tag{2.9}$$

$$\rho\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial p}{\partial X} + \mu\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} - \frac{\mu}{k}v \tag{2.10}$$

$$2\eta\frac{\partial U}{\partial X} = S_{XX} + m\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)S_{XX} - 2emS_{XX}\frac{\partial U}{\partial X} + m\left[\left(1-e\right)\frac{\partial V}{\partial X} - \left(1+e\right)\frac{\partial U}{\partial Y}\right]S_{XY} \tag{2.11}$$

$$\begin{aligned}\eta\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right) = S_{XY} + m\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)S_{XY} + \frac{m}{2}\left[\left(1-e\right)\frac{\partial U}{\partial Y} - \left(1+e\right)\frac{\partial V}{\partial X}\right]S_{XX} \\ + \frac{m}{2}\left[\left(1-e\right)\frac{\partial V}{\partial X} - \left(1+e\right)\frac{\partial U}{\partial Y}\right]S_{YY}\end{aligned} \tag{2.12}$$

$$2\eta\frac{\partial V}{\partial Y} = S_{YY} + m\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)S_{YY} - 2emS_{YY}\frac{\partial V}{\partial Y} + m\left[\left(1-e\right)\frac{\partial U}{\partial Y} - \left(1+e\right)\frac{\partial V}{\partial X}\right]S_{XY} \tag{2.13}$$

In the fixed frame $\left(X, Y\right)$ the motion is unsteady, while it becomes steady in the wave frame $\left(x, y\right)$. The transformation from the fixed frame of reference $\left(X,\ Y\right)$ to the wave frame of reference $\left(x,\ y\right)$ is given by

$$x = X - ct,\ y = Y,\ u = U - c,\ v = V,\ p\left(x\right) = P\left(X, t\right) \tag{2.14}$$

Here *u, v* and *U, V* are the velocity components in the wave frame and in the fixed frame, respectively. We put Eq. (2.14) into the Eqs. (2.8) - (2.13) and using the following non - dimensional variables:

$$\bar{x}=\frac{x}{\lambda},\quad \bar{y}=\frac{y}{a},\quad \bar{u}=\frac{u}{c},\quad \bar{v}=\frac{v}{c},\ h=\frac{H}{a},\ \bar{S}=\frac{a}{\mu c}S,\ \bar{p}=\frac{2\pi a^2}{\lambda(\mu+\eta)c}p,\ \delta=\frac{2\pi a}{\lambda},\quad \mathrm{Re}=\frac{\rho c a}{\mu},$$
$$Wi=\frac{mc}{a},\phi=\frac{b}{a}\ , \tag{2.15}$$

we have (after dropping the bars)

$$\delta\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.16}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\left(\frac{\mu+\eta}{\mu}\right)\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\frac{1}{Da}(u+1) \tag{2.17}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\left(\frac{\mu+\eta}{\mu}\right)\frac{\partial p}{\partial y}+\delta\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\delta^2\frac{\partial S_{xy}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y}-\frac{\delta}{Da}v \tag{2.18}$$

$$\left(\frac{2\eta\delta}{\mu}\right)\frac{\partial u}{\partial x}=S_{xx}+Wi\delta\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)S_{xx}-2eWi\delta S_{xx}\frac{\partial U}{\partial x}+Wi\left(\delta^2(1-e)\frac{\partial v}{\partial x}-(1+e)\frac{\partial u}{\partial y}\right)S_{xy} \tag{2.19}$$

$$\frac{\eta}{\mu}\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)=S_{xy}+Wi\delta\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)S_{xy}+\frac{Wi}{2}\left[(1-e)\frac{\partial u}{\partial y}-(1+e)\delta^2\frac{\partial v}{\partial x}\right]S_{xx}$$
$$+\frac{Wi}{2}\left[(1-e)\delta^2\frac{\partial v}{\partial x}-(1+e)\frac{\partial u}{\partial y}\right]S_{yy} \tag{2.20}$$

$$\left(\frac{2\eta\delta}{\mu}\right)\frac{\partial v}{\partial y}=S_{yy}+Wi\delta\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)S_{yy}-2eWi\delta S_{yy}\frac{\partial v}{\partial y}+Wi\left((1-e)\frac{\partial u}{\partial y}-\delta^2(1+e)\frac{\partial v}{\partial x}\right)S_{xy} \tag{2.21}$$

Under lubrication approach (i.e., neglecting the terms of order $\delta$ and Re), from Eqs. (2.17) and (2.18), we get

$$0=-\left(\frac{\mu+\eta}{\mu}\right)\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial S_{xy}}{\partial y}-\frac{1}{Da}(u+1) \tag{2.22}$$

$$\frac{\partial p}{\partial y}=0 \tag{2.23}$$

where

$$S_{xx}=Wi(1+e)\frac{\partial u}{\partial y}S_{xy} \tag{2.24}$$

$$\frac{\eta}{\mu}\frac{\partial u}{\partial y}=S_{xy}+\frac{Wi}{2}(1-e)\frac{\partial u}{\partial y}S_{xx}-\frac{Wi}{2}(1+e)\frac{\partial u}{\partial y}S_{yy} \tag{2.25}$$

$$S_{yy} = -Wi(1-e)\frac{\partial u}{\partial y}S_{xy} \tag{2.26}$$

From Eqs. (2.24) – (2.26), we write

$$S_{xy} + Wi^2\left(1-e^2\right)\left(\frac{\partial u}{\partial y}\right)^2 S_{xy} = \frac{\eta}{\mu}\frac{\partial u}{\partial y} \tag{2.27}$$

Using Eqs. (2.23) and (2.27), the Eq. (2.22) can be rewritten as

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial x^2} + Wi^2\alpha_1\frac{\partial}{\partial y}\left[\left(\frac{\partial u}{\partial y}\right)^3\right] - \alpha^2(u+1) \tag{2.28}$$

where $\alpha_1 = \dfrac{\left(e^2-1\right)\eta}{(\mu+\eta)}$ and $\alpha^2 = \dfrac{\eta}{(\mu+\mu)Da}$.

The corresponding non-dimensional boundary conditions are

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{2.29}$$

$$u_0 = -1 \quad \text{at} \quad y = h = 1+\phi\sin\pi x \tag{2.30}$$

The volume flow rate in a wave frame is given by

$$q = \int_0^h u\,dy \tag{2.31}$$

The flux at any axial station in the laboratory frame is

$$Q(x,t) = \int_0^h (u+1)dy = q + h \tag{2.32}$$

The average volume flow rate over one wave period T (= $\lambda / c$ ) of the peristaltic wave is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T Q\,dt = q + 1 \tag{2.33}$$

**3. SOLUTION:**

The Eq. (2.28) is non-linear and its closed form solution is not possible. thus, we linearize this equation in terms of $Wi^2$, since $Wi$ is small for the type of flow under consideration. So we expand $u$, $p$ and $q$ as

$$u = u_0 + Wi^2 u_1 + o\left(Wi^4\right)$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + Wi^2\frac{dp_1}{dx} + o\left(Wi^4\right)$$

 

$$q = q_0 + Wi^2 q_1 + o\left(Wi^4\right) \tag{3.1}$$

**3.1 Equations of order $Wi^0$:**

$$\frac{dp_0}{dx} = \frac{\partial^2 u_0}{\partial y^2} - \alpha^2 \left(u_0 + 1\right) \tag{3.2}$$

The corresponding boundary conditions are

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{3.3}$$

$$u_0 = -1 \quad \text{at} \quad y = h \tag{3.4}$$

**3.2 Equations of order $Wi^2$:**

$$\frac{dp_1}{dx} = \frac{\partial^2 u_1}{\partial y^2} + \alpha_1 \frac{\partial}{\partial y}\left[\left(\frac{\partial u_1}{\partial y}\right)^3\right] - \alpha^2 u_1 \tag{3.5}$$

The corresponding boundary conditions are

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{3.6}$$

$$u_1 = 0 \quad \text{at} \quad y = h \tag{3.7}$$

**3.3 Solution of order $Wi^0$:**

Solving the Equation (3.2) by using the boundary conditions (3.3) and (3.5), we obtain

$$u_0 = \frac{1}{\alpha^2}\frac{dp_0}{dx}\left[\frac{\cosh \alpha y}{\cosh \alpha h} - 1\right] - 1 \tag{3.8}$$

and the volume flow rate $q_0$ is given by

$$q_0 = \int_0^h u_0 dy = \frac{1}{\alpha^3}\frac{dp_0}{dx}\left[\frac{\left(\sinh \alpha h - \alpha h \cosh \alpha h\right)}{\cosh \alpha h}\right] - h \tag{3.9}$$

From Equation (3.7), we obtain

$$\frac{dp_0}{dx} = \frac{\alpha^3\left(q_0 + h\right)\cosh \alpha h}{\sinh \alpha h - \alpha h \cosh \alpha h} \tag{3.10}$$

**3.4 Solution of order $Wi^2$:**

Solving the Eq. (3.5) by using Eq. (3.8) and the boundary conditions (3.6) and (3.7), we obtain

 

$$u_1=\frac{1}{\alpha^2}\frac{dp_1}{dx}\left(\frac{\cosh\alpha y}{\cosh\alpha h}-1\right)+\frac{3\alpha_1}{2\alpha^2}\left(\frac{dp_0}{dx}\right)^3\left(\frac{\cos h\alpha y}{\cosh^4\alpha h}\right)\left[\frac{\cosh 3\alpha h}{16\alpha^2}-\frac{h\sinh\alpha h}{4\alpha}\right]$$
$$-\frac{3\alpha_1}{2\alpha^2}\left(\frac{dp_0}{dx}\right)^3\left(\frac{1}{\cosh 3\alpha h}\right)\left[\frac{\cosh 3\alpha y}{16\alpha^2}-\frac{y\sinh\alpha y}{4\alpha}\right] \tag{3.11}$$

and the volume flow rate $q_1$ is given by

$$q_1=\frac{1}{\alpha^3}\frac{dp_1}{dx}\left(\frac{\sinh\alpha h-\alpha h\cosh\alpha h}{\cosh\alpha h}\right)+\frac{3\alpha_1}{2\alpha^4}\left(\frac{B}{\cosh^3\alpha h}\right)\left[\frac{\alpha^3\left(q_0+h\right)\cosh\alpha h}{\left(\sinh\alpha h-\alpha h\cosh\alpha h\right)}\right]^3 \tag{3.12}$$

where

$$B=\frac{\tanh\alpha h\cosh 3\alpha h}{16\alpha}-\frac{h\tanh\alpha h\sinh\alpha h}{4}-\frac{\sinh 3\alpha h}{48\alpha}+\frac{h\cosh\alpha h}{4}-\frac{\sinh\alpha h}{4\alpha}$$

From Equation (3.10), we obtain

$$\frac{dp_1}{dx}=\frac{\alpha^3\cosh\alpha h q_1}{\sinh\alpha h-\alpha h\cosh\alpha h}-\frac{3\alpha_1\alpha^8 B}{2}\left[\frac{\left(q_0+h\right)^3\cosh\alpha h}{\left(\sinh\alpha h-\alpha h\cosh\alpha h\right)^4}\right] \tag{3.13}$$

Substituting Equations (3.10) and (3.13) into the second equation of (3.1) and neglecting terms greater than $O\left(W_i^4\right)$, we get

$$\frac{dp}{dx}=\frac{\alpha^3\left(q+h\right)\cosh\alpha h}{\sinh\alpha h-\alpha h\cosh\alpha h}-\frac{3\alpha_1 B\alpha^8}{2}Wi^2\left[\frac{\left(q+h\right)^3\cosh\alpha h}{\left(\sinh\alpha h-\alpha h\cosh\alpha h\right)^4}\right] \tag{3.14}$$

The dimensionless pressure rise and frictional force per one wavelength in the wave frame are defined, respectively as

$$\Delta p=\int_0^{2\pi}\frac{dp}{dx}dx \tag{3.15}$$

and $$F=\int_0^{2\pi}h\left(-\frac{dp}{dx}\right)dx. \tag{3.16}$$

## 4. DISCUSSION OF THE RESULTS:

Fig. 2 shows the effect of Darcy number $Da$ on the variation of pressure gradient $\frac{dp}{dx}$ with $x$ for $Wi=0.01$, $e=0.8,\eta=1,\mu=1$ and $\phi=0.6$. It is noted that, the magnitude of $\frac{dp}{dx}$ decreases with an increase in $Da$. The effect of Weissenberg number $Wi$ on the variation of pressure gradient $\frac{dp}{dx}$ with $x$ for $Da=0.25$, $e=0.8,\eta=1,\mu=1$ and $\phi=0.6$ is shown in Fig. 3. It is observed that, the magnitude of the $\frac{dp}{dx}$ increases with increasing $Wi$.

Fig. 4 represents the effect of amplitude ratio $\phi$ on the variation of pressure gradient $\frac{dp}{dx}$ with $x$ for $Da = 0.5$, $e = 0.8,\ \eta = 1,\ \mu = 1$ and $Wi = 0.01$. It is noted that, the magnitude of the pressure gradient $\frac{dp}{dx}$ increases with an increase in $\phi$ .

Fig. 5 shows the effect of Darcy number $Da$ on the variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for $Wi = 0.01$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$ .It is observed that, in the pumping region $(\Delta p > 0)$, the $\overline{Q}$ decreases with increasing $Da$ while it increases with increasing $Da$ , the both pumping and co-pumping regions.

In order to study the effect of Weissenberg number $Wi$ on the variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for $Da = 0.25$ , $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$ is shown in Fig. 6. It is observed that, the pumping increases with increasing $Wi$ . Further it is observed that, the pumping is more for Johnson-Segalman fluid than that of Newtonian fluid. Moreover for large $Da$ the pumping curves co-insides for all $Wi$ is shown in Fig. 7.

Fig. 8 depicts the effect of amplitude ratio $\phi$ on the variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for $Da = 0.5$, $e = 0.8, \eta = 1, \mu = 1$ and $Wi = 0.01$. It is noted that, in the pumping region, the $\overline{Q}$ increases with increasing amplitude ratio $\phi$ while in co-pumping region (for an appropriately chosen $(\Delta p < 0)$ it decreases with increasing $\overline{Q}$ .

The effect of Darcy number $Da$ on the variation of friction force $F$ with time averaged flux $\overline{Q}$ for $Wi = 0.01$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$ is presented in Fig. 9. It is observed that, the friction force $F$ initially increases and then decreases with increasing $Da$ .

Fig. 10 shows the effect of Weissenberg number $Wi$ on the variation of friction force $F$ with time averaged flux $\overline{Q}$ for $Da = 0.25$ , $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$ . It is noted that, the friction force $F$ decreases with increasing $Wi$ , for large $Da$ the friction force $F$ co-insides for all $Wi$ is shown in Fig. 11.

The effect of amplitude ratio $\phi$ on the variation of friction force $F$ with time averaged flux $\overline{Q}$ for $Da = 0.5$, $e = 0.8, \eta = 1, \mu = 1$ and $Wi = 0.01$ is depicted in Fig. 12. It is observed that, the friction force $F$ first decreases and then increases with increasing $\phi$ .

## 5. CONCLUSIONS:

In this paper, we investigated the peristaltic transport of a Johnson-Segalman fluid through a porous medium in a two - dimensional channel under the assumptions of long-wavelength and low-Reynolds number. A Perturbation solution for small Weissenberg number is obtained for the axial velocity, axial pressure gradient, friction force and pressure rise per one wavelength. It is found that both the pressure gradient $\frac{dp}{dx}$ and the time averaged flux $\overline{Q}$ increases with increasing Weissenberg number $Wi$ as well as amplitude ratio $\phi$ , whereas they decreases with increasing Darcy number $Da$ . The friction force $F$ initially increases and then decreases with increasing $Da$ . The friction force $F$ decreases with increasing $Wi$ . The friction force $F$ first decreases and then increases with increasing $\phi$ . Further it is observed that the pumping is less for Newtonian fluid than that of Johnson-Segalman fluid.

**REFERENCES:**

[1] A.H. Shapiro and M.Y. Jaffrin and S.L. Weinberg, Peristaltic pumping with long wavelengths at low Reynolds number, J. Fluid Mech. 37(1969), 799-825.

[2] Y.C. Fung and C.S. Yih, Peristaltic transport, Trans. ASME J. Appl. Mech. 35(1968), 669-675.

[3] M. W. Johnson and D. Segalman, A model for viscoelastic fluid behavior which allows nonaffine deformation, J. Non-Newtonian Fluid Mech. 2 (1977) 255–270.

[4] R. W. Kolkka, D. S. Malkus, M. G. Hansen, G. R. Ieriy and A. R. Worthing, Spurt phenomenon of the Johnson–Segalman fluid and related models, J. Non-Newtonian Fluid Mech. 29 (1988) 303–335.

[5] D. S. Malkus, J. A. Nohel and B. J. Plohr, Dynamics of shear flows of non-Newtonian fluids, J. Comput. Phys. 87 (1990), 464–497.

[6] T. C. B. Mcleish and R.C. Ball, A molecular approach to the spurt effect in polymer melt flow, J. Polym. Sci. (B) 24 (1986) 1735–1745.

[7] G. V. Vinogradov, A. Ya., Yu. G. Malkin, E. K. Yanovskii, Borisenkiva, B. V. Yarlykov, and G. V. Berezhnaya,''Viscoelastic properties and flow of narrow distribution polybutadienes and polyisoprenes,'' J. Polym. Sci., Part A-2 10(1972), 1061–1084.

[8] M.M. Denn**,** Issues in viscoelastic fluid mechanics**,** Annu**.** Rev**.** Fluid Mech. 22 (1990), 13–34.

[9] A. M. Kraynik, and W. R.Schowalter, Slip at the wall and extrudate roughness with aqueous solutions of polyvinyl alcohol and sodium borate, J. Rheol. 25 (1981), no. 1, 95–114.

[10] F. J. Lim and W. R. Schowalter, Wall slip of narrow molecular weight distribution polybutadienes, J. Rheol. 33 (1989), no. 8, 1359–1382.

[11] K. B. Migler, H. Hervert, and L. Leger, Slip transition of a polymer melt under shear stress, Phys. Rev. Lett. 70 (1990), 287–290.

[12] K. B. Migler, G. Massey, H. Hervert and L. Leger, The slip transition at the polymer-solid interface, J. Phys. Condens.Matter 6 (1994), 301–A304.

[13] A. V. Ramamurthy, Wall slip in viscous fluids and influence of materials of construction, J. Rheol. 30 (1986), 337–357.

[14] I. J. Rao, and K. R. Rajagopal, Some simple flows of a Johnson-Segalman fluid, Acta Mech. 132 (1999), 209–219.

[15] T. Hayat, Y. Wang, A. M. Siddiqui and K. Hutter, Peristaltic motion of a Johnson-Segalman fluid in a planar channel, Mathematical Problems in Engineering, 2003 (2003), no. 1, 1-23.

[16] M. El Shahed and M. H. Haroun, Peristaltic transport of Johnson-Segalman fluid under effect of a magnetic field, Math. Probl. Eng. 6(2005) 663 – 667.

[17] E. F. El Shehawey and S. Z. A. Husseny, Effects of porous boundaries on peristaltic transport through a porous medium, Acta Mechanica, 143 (2000), 165-177.

[18] E.F. El Shehawey, Kh. S. Mekheimer, S. F. Kaldas, and N. A. S. Afifi, Peristaltic transport through a porous medium, J. Biomath. 14 (1999).

[19] T. Hayat, N. Ali and S. asghar, Hall effects on aperistaltic motion of a Maxwell fluid in a porous medium, Physics Letters A, 363 (2007). 397- 403.

 

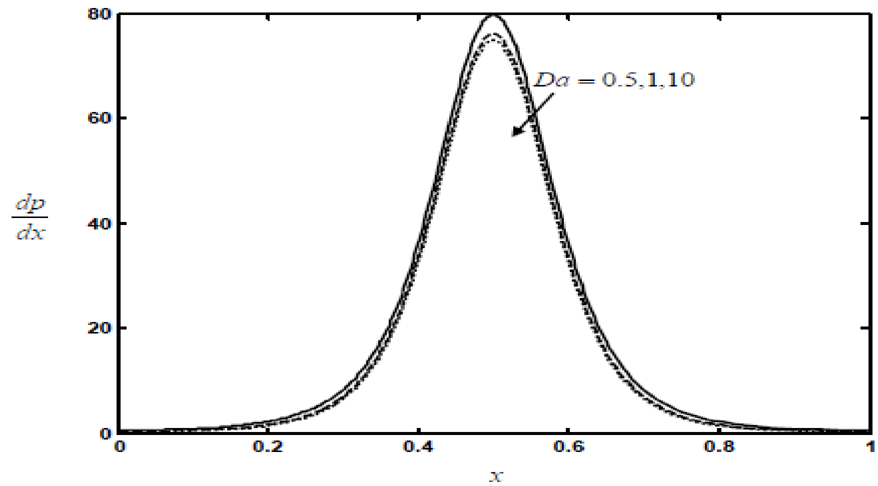


**Fig. 2.** Effect of Darcy number $Da$ on the variation of pressure gradient $\frac{dp}{dx}$ with $x$ for $Wi = 0.01$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

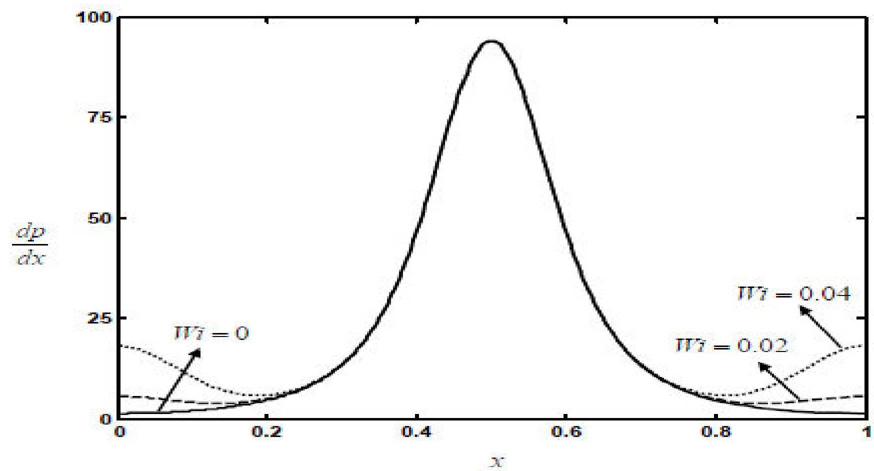


**Fig. 3.** Effect of Weissenberg number $Wi$ on the variation of pressure gradient $\frac{dp}{dx}$ with $x$ for $Da = 0.25$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

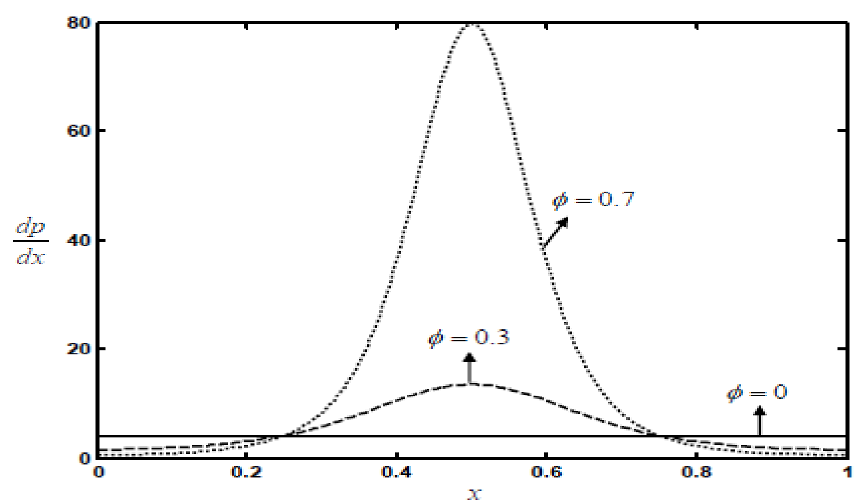


**Fig. 4.** Effect of amplitude ratio $\phi$ on the variation of pressure gradient $\frac{dp}{dx}$ with $x$ for $Da = 0.5$, $e = 0.8, \eta = 1, \mu = 1$ and $Wi = 0.01$.

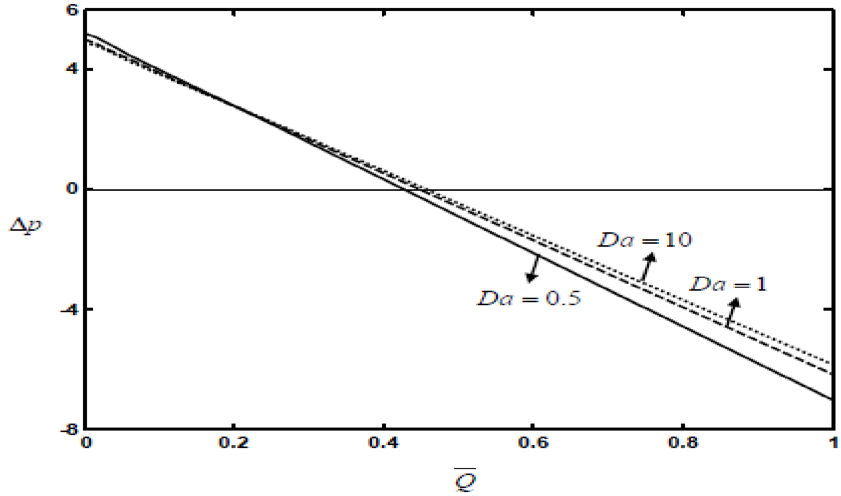


**Fig. 5.** Effect of Darcy number $Da$ on the variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for $Wi = 0.01$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

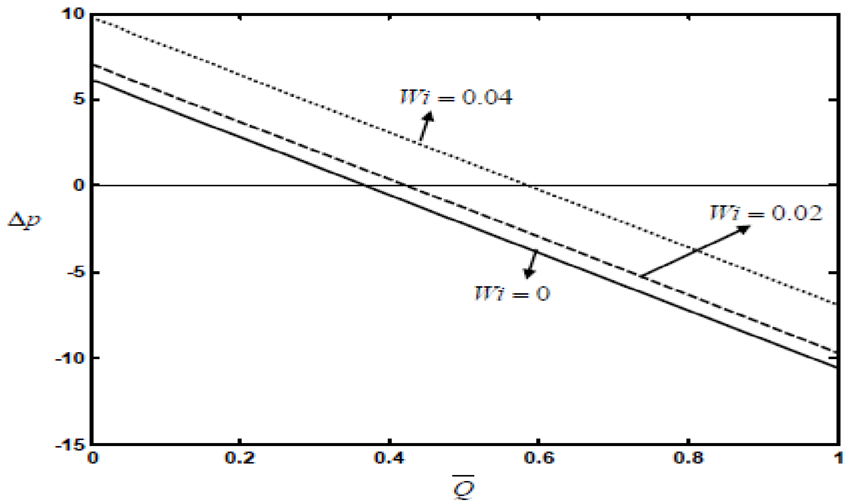


**Fig. 6.** Effect of Weissenberg number $Wi$ on the variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for $Da = 0.25$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

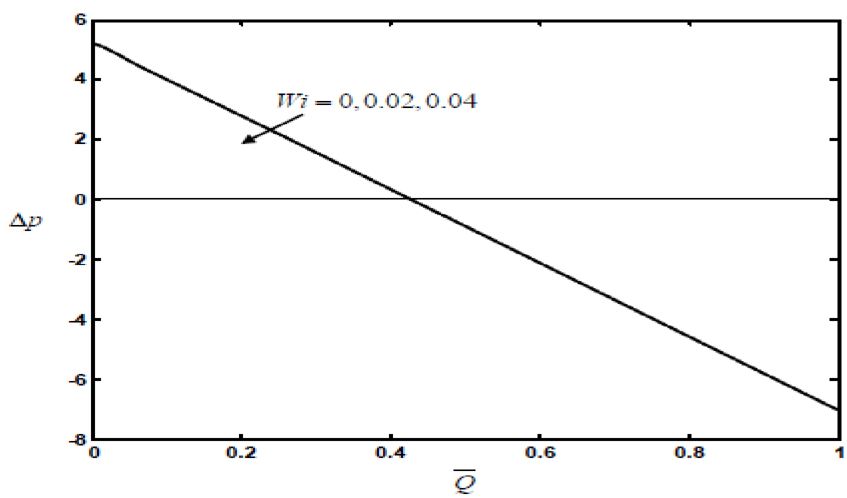


**Fig. 7.** Effect of Weissenberg number $Wi$ on the variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for $Da = 0.5$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

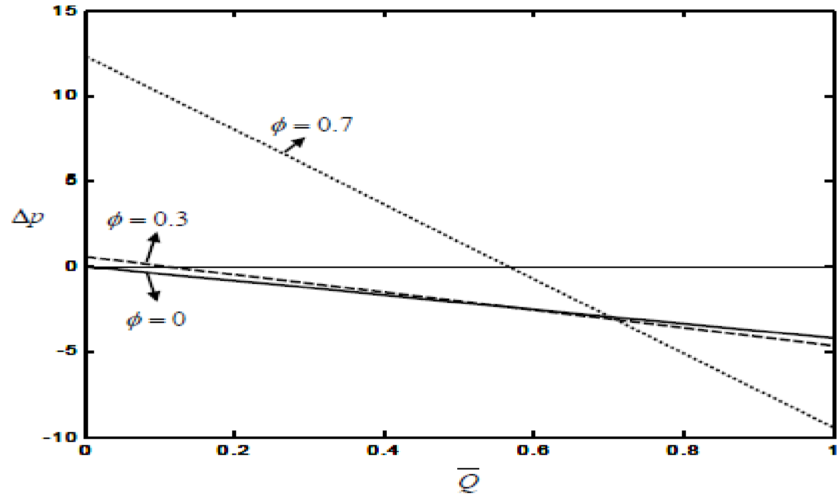


**Fig. 8.** Effect of amplitude ratio $\phi$ on the variation of pressure rise $\Delta p$ with time averaged flux $\overline{Q}$ for $Da = 0.5$, $e = 0.8, \eta = 1, \mu = 1$ and $Wi = 0.01$.

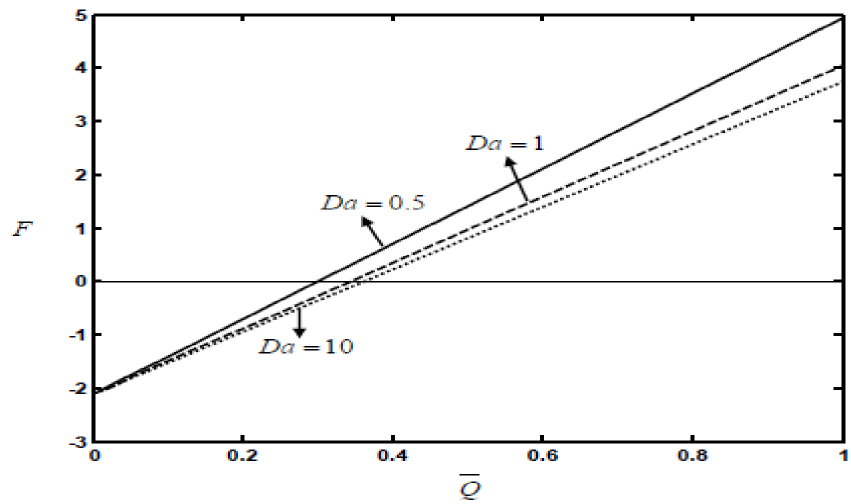


**Fig. 9.** Effect of Darcy number $Da$ on the variation of friction force $F$ with time averaged flux $\overline{Q}$ for $Wi = 0.01$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

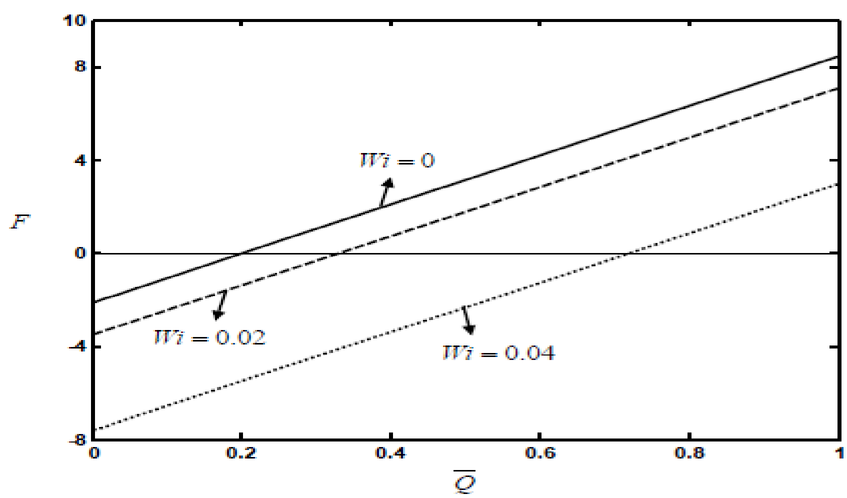


**Fig. 10.** Effect of Weissenberg number $Wi$ on the variation of friction force $F$ with time averaged flux $\overline{Q}$ for $Da = 0.25$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

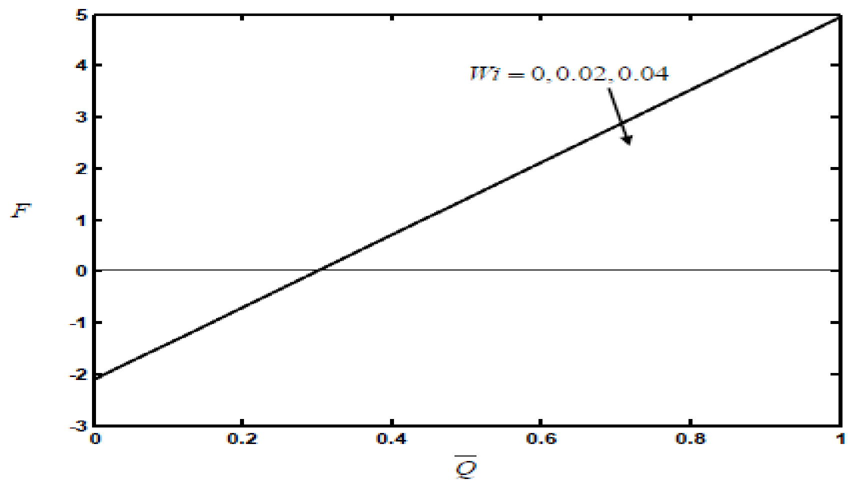


**Fig. 11.** Effect of Weissenberg number $Wi$ on the variation of friction force $F$ with time averaged flux $\overline{Q}$ for $Da = 0.5$, $e = 0.8, \eta = 1, \mu = 1$ and $\phi = 0.6$.

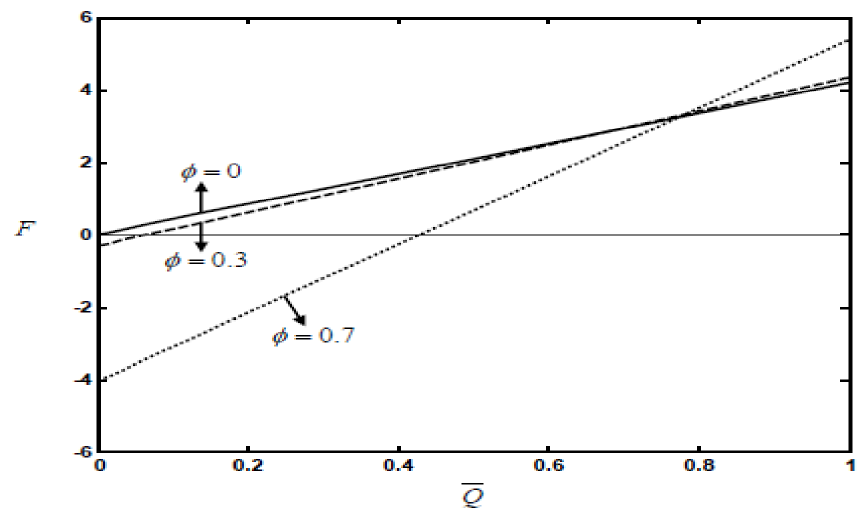


**Fig. 12.** Effect of amplitude ratio $\phi$ on the variation of friction force $F$ with time averaged flux $\overline{Q}$ for $Da = 0.5$, $e = 0.8, \eta = 1, \mu = 1$ and $Wi = 0.01$.

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