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# SPHERICAL SYMMETRIC COSMOLOGICAL MODEL WITH COSMIC STRINGS COUPLED WITH PERFECT FLUID IN BIMETRIC RELATIVITY

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*ABSTRACT*

*Spherical symmetric Kantowski –Sachs space-time is studied in Rosen's bimetric relativity, considering the source of gravitation as cosmic strings coupled with perfect fluid distribution.It is shown that a macro cosmological model represented by cosmic string coupled with perfect distribution does not exist and only a vacuum model can be constructed.*

***Keywords:*** *Spherical symmetric, Kantowski-Sachs model, cosmic string.*

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## INTRODUCTION:

The spherical symmetry has its own importance in general relativity theory by virtue of its comparative simplicity. Many noteworthy space-time, such as the Schwarzschild solutions (exterior and interior) ,the Robertson-Walker model of expanding universe etc. are all spherically symmetric. Rosen N. [6, 7, 8, 9, 10] introduced a new theory known as bimetric theory of relativity. It is based on two metric tensors $g_{ij}$ and $\gamma_{ij}$ . The first metric tensor described the curved space-time and thereby the gravitational field. The second metric tensor refers to the flat space-time and described the inertial force associated with acceleration of the frame of reference. Israelit [1, 2] studied several aspects of bimetric theory of gravitation. Recently Mohanty et-al. [4, 5] constructed some physical viable models in this theory.

In this paper we have shown the spherical symmetric cosmological model with cosmic

string does not exist. Mahurpawa and Ronghe[3] have shown that cosmic strings dose not exist in this modelin the context of bimetric relativity.

## 1. FIELD EQUATIONS:

Accordingly, at each space-time point one has two line elements

$$ds^2 = g_{ij}\, dx^i dx^j \tag{1.1}$$

and

$$d\sigma^2 = \gamma_{ij}\, dx^i dx^j \tag{1.2}$$

This theory is based on a simple form of Lagrangian and has a simpler mathematical structure than that of the general relativity.

The field equations of the bimetric theory of gravitation formulated by Rosen N. are

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$$K_i^{\ j} = N_i^{\ j} - \frac{1}{2} N g_i^{\ j} = -8\pi\kappa T_i^j \tag{1.3}$$

where $$N_i^{\ j} = \frac{1}{2}\gamma^{\alpha\beta}(g^{hi} g_{hj} \mid \alpha) \mid \beta \tag{1.4}$$

$\kappa = (\frac{g}{\gamma})^{\frac{1}{2}}$ with g-the determinant of $g_{ij}$ and $\gamma$-determinant of $\gamma_{ij}$.

The vertical bar ( | )stands for $\gamma$-differentiation and $T_i^j$ is the energy-momentum tensor. We considered here the spherically symmetric Kantowaski-Sachs space-time in the form

$$ds^2 = dt^2 - \lambda dr^2 - k^2(d\theta^2 - Sin^2\theta d\phi^2) \tag{1.5}$$

where $\lambda$ and k are functions of "t" only

the background metric corresponding to the metric (1.5) is

$$d\sigma^2 = dt^2 - dr^2 - d\theta^2 - Sin^2\theta d\phi^2 \tag{1.6}$$

In this case we have taken the source of gravitation cosmic strings coupled with perfect fluid distribution. The energy-momentum tensor for cosmic string coupled with perfect fluid distribution is given

$$T_i^j = T_{i\ string}^{\ j} + (\varepsilon + p) v_i v^j + p g_i^{\ j} \tag{1.7}$$

where $$T_{i\ string}^{\ j} = \rho v_i v^j - \lambda x_i x^j \tag{1.8}$$

Here $\rho$ is energy density for a cloud of cosmic strings with particle attached to them, $\lambda$ the string tensor density, $v^i$ are four velocity vector of cosmic string distribution, $x^i$ is an anisotropic direction or say direction of strings, and $p$ are $\varepsilon$ proper pressure and matter density. The particle density associated with configuration is given by

$$\rho = \rho_p + \lambda \tag{1.9}$$

where $\rho_p$ is the particle density in the string cloud

$$-x_i x^j = v_i v^j = 1, v_i x^j - 0 \text{ if } i \neq j$$

We considered the anisotropic direction along x direction

$$x_1 x^1 = -1, v_4 v^4 = 1$$

So $$T_1^1 = p + \lambda, T_2^2 = p = T_3^3, T_4^4 = \varepsilon + 2p + \rho \tag{1.10}$$

Using equations (1.1) to (1.10) the field equations are

$$\left(\frac{\lambda_4}{\lambda}\right)_4 - 2\left(\frac{k_4}{k}\right)_4 = 16\pi\kappa(p + \lambda) \tag{1.11}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = 16\pi\kappa(\rho) \tag{1.12}$$

 

$$\left(\frac{\lambda_4}{\lambda}\right)_4 + 2\left(\frac{k_4}{k}\right)_4 = 16\pi\kappa\left(\varepsilon + 2p + \rho\right) \tag{1.13}$$

Here suffix "4" following an unknown function denotes an ordinary differentiation with respect to time "t".Equation (1.11) and (1.13) with help of (1.12) gives

$$p + \varepsilon + \lambda + \rho = 0 \tag{1.14}$$

Since $p \geq 0, \lambda \geq 0, \rho \geq 0, \rho \geq 0$

So $p = \lambda = \varepsilon = \rho = 0$ (1.15)

Using equation (1.14) in the field equations (1.11) to (1.13), we have

$$\left(\frac{\lambda_4}{\lambda}\right)_4 - 2\left(\frac{k_4}{k}\right)_4 = 0 \tag{1.16}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = 0 \tag{1.17}$$

$$\left(\frac{\lambda_4}{\lambda}\right)_4 + 2\left(\frac{k_4}{k}\right)_4 = 0 \tag{1.18}$$

Equations (1.17) and (1.18), gives

$$\left(\frac{\lambda_4}{\lambda}\right)_4 = \left(\frac{k_4}{k}\right)_4 = 0 \tag{1.19}$$

We have

$$\lambda = e^{c_{1t}} \tag{1.20}$$

And $k = e^{c_{2t}}$ (1.21)

Using equations (1.20) and (1.21) the line element (1.5) becomes

$$ds^2 = dt^2 - e^{2c_{1t}} dr^2 - e^{2c_{2t}} \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{1.22}$$

By proper choice of coordinates this metric can be transform

$$ds^2 = d\tau^2 - e^{2\tau} \left(dr^2 + d\theta^2 + \sin^2\theta d\phi^2\right) \tag{1.23}$$

Which is free from singularity at $\tau = 0$

## 2. CONCLUSION:

We have studied spherically symmetric Kantowaski-Sachs cosmological model with cosmic string coupled with perfect fluid as energy-momentum tensor and observed that the cosmic string coupled with perfect fluid does not accommodate in this model only a vacuum model can be constructed. There is no singularity at $\tau = 0$.

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