

ON SEMI COMPACT SPACES

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ABSTRACT

In a recent paper [1] some properties of semi compact spaces have been presented. The present paper continuous further study of the properties of semi compact spaces. For example, semi continuous image of a semi compact space is compact.

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Key words: Semi open, Semi continuous, semi compact spaces.

1. INTRODUCTION

Some important results on the topic are contained in [2] and [3]. Let (X, τ) be a topological space. Let A be a subset of (X, τ) . Then A is said to be semi open if $A \subset \text{cl}(\text{int}(A))$. A is semi closed if $A \subset \text{int}(\text{cl}(A))$.

Note that every open set is semi open. If every open cover of X has a finite sub cover then X is called a compact space. If every semi open cover has a finite sub cover, then X is a semi compact space. (X, τ) is said to have semi Hausdorff space if $x \neq y$ in X implies existence of semi open neighbourhoods U and V of x and y such that $U \cap V = \phi$.

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a semi continuous function if $f^{-1}(G)$ is a semi open set in X for each open set G in Y . (X, τ) is called an s -normal space if given two disjoint closed sets A and B in X , there exist disjoint semi open neighbourhoods U and V of A and B respectively.

2. MAIN RESULTS

Theorem: 2.1 Every semi closed subsets of a semi compact space is semi compact.

Proof: Suppose that A is a semi closed subset of a semi compact space (X, τ) .

We shall show that $(A, \tau/A)$ is semi compact.

Let $\mathcal{C}_A = \{G_\alpha \cap A : \alpha \in \Lambda\}$ be any relatively semi open cover of $(A, \tau/A)$.

Then $\mathcal{C} = \{G_\alpha : \alpha \in \Lambda\}$ is a semi open cover of (X, τ) .

But (X, τ) is semi compact.

Hence \mathcal{C} contains a finite sub over $\{G_{\alpha_1}, G_{\alpha_2}, \dots, G_{\alpha_n}\}$ of (X, τ) .

Consequently, $\{G_{\alpha_1} \cap A, \dots, \dots, \dots, G_{\alpha_n} \cap A\}$ is a finite sub cover of A .

Hence $(A, \tau/A)$ is semi compact.

This completes the proof.

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Theorem: 2 Semi continuous image of a semi compact space is compact.

Proof: Suppose that (X, τ) is semi compact.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be semi continuous surjection.

We shall show that (Y, σ) is compact.

Let $\mathcal{C} = \{G_\alpha : \alpha \in \Lambda\}$ be any open cover of (Y, σ) .

Then $f^{-1}(G_\alpha)$ is a semi open set in (X, τ) and $\mathcal{D} = \{f^{-1}(G_\alpha) : \alpha \in \Lambda\}$ a semi open cover of X .

But (X, τ) is semi compact.

Accordingly, \mathcal{D} contains a finite sub cover $\{f^{-1}(G_{\alpha_1}), \dots, f^{-1}(G_{\alpha_n})\}$.

But then $\{G_{\alpha_1}, G_{\alpha_2}, \dots, G_{\alpha_n}\}$ is a finite open subcover of Y .

Hence (Y, σ) is a compact space.

This finishes the proof.

Theorem: 3 Suppose that A is a semi compact subset of a semi Hausdorff space X . Let $x \in X - A$. Then there exist disjoint semi open neighborhoods U and v of A and x respectively.

Proof: By hypothesis, X is semi Hausdorff.

Let $a \in A$ arbitrarily.

Then there exist disjoint semi open neighborhoods U_a and V_x of a and x respectively.

The collection $\mathcal{C} = \{U_a : a \in A\}$ is a semi open cover of A .

But A is semi compact.

Accordingly, this collection \mathcal{C} has a finite sub cover $\{U_{a_1}, U_{a_2}, \dots, U_{a_n}\}$.

Let $U = U_{a_1} \cup \dots \cup U_{a_n}$.

Put $V = V_{a_1} \cap \dots \cap V_{a_n}$.

Then $A \subset U$ and $x \in V$.

Also U and V are semi open.

Since $U_{a_i} \cap V_{a_i} = \phi$ for $1 \leq i \leq n$.

We obtain that $U \cap V = \phi$.

We have proved the result.

Theorem: 4 Every semi compact subset of a semi Hausdorff space is semi closed.

Proof: Let X be a semi Hausdorff space.

Suppose that A is a semi compact subset of X .

Let $x \in X - A$.

By theorem 3, there exist disjoint semi open sets U_x and V_x containing x and A respectively.

Therefore, $x \in U_x \subset X - V_x \subset X - A$.

It follows that $X - A$ is semi open.

Consequently, A is semi closed.

This proves the theorem.

Theorem: 5 Every semi compact, semi Hausdorff space is s - normal.

Proof: Suppose that X is a semi compact, semi Hausdorff space.

Let A and B be two disjoint closed subsets of X .

But then A is semi compact.

It follows, by theorem 3, that for each $x \in B$ there exist disjoint semi open sets U_x and V_x such that $x \in U_x$ and $B \subset V_x$.

The collection $\mathcal{C} = \{U_x : x \in B\}$ is a semi open cover of B .

But B is semi compact.

Hence the collection \mathcal{C} has a sub cover $\{U_{x_1}, U_{x_2}, \dots, U_{x_n}\}$.

Let $U = U_{x_1} \cup \dots \cup U_{x_n}$.

Put $V = V_{x_1} \cap \dots \cap V_{x_n}$.

Then U and V are semi open with $A \subset U$ and $B \subset V$.

Since $U_{x_i} \cap V_{x_i} = \phi$ for $1 \leq i \leq n$.

It follows that $U \cap V = \phi$.

Hence theorem holds.

Theorem: 6 Let X and Y be non empty topological spaces. The product space $X \times Y$ is semi compact if both X and Y are semi compact.

Proof: Suppose that X and Y are semi compact.

Let \mathcal{C} be a semi open cover of $X \times Y$, consisting of basic semi open sets of the form $U \times V$, where U is a semi open set in X and V is a semi open set in Y .

Let $x \in X$.

Then for each $y \in Y$ there exists a set $(U_y \times V_y)$ in \mathcal{C} containing (x, y) .

The collection $\{V_y : y \in Y\}$ is a semi open cover of Y .

But Y is semi compact.

Consequently, this collection has a finite sub cover $\{V_{y_1}, V_{y_2}, \dots, V_{y_n}\}$.

Consider, the corresponding sets $U_{y_1}, U_{y_2}, \dots, U_{y_n}$.

Put $U_x = U_{y_1} \cap \dots \cap U_{y_n}$.

Then $\{x\} \times Y = U_x \times Y$
 $= U_x \times \{V_{y_1} \cup \dots \cup V_{y_n}\}$
 $= (U_{y_1} \times V_{y_1}) \cup \dots \cup (U_{y_n} \times V_{y_n}).$

Thus for each x in X there is a set U_x such that $\{x\} \times Y \subset U_x \times Y$ and that $U_x \times Y$ is contained in a finite number of sets in \mathcal{C} .

But the collection $\{U_x : x \in X\}$ covers X .

Since X is semi compact, this collection has a finite sub cover $\{U_{x_1}, U_{x_2}, \dots, U_{x_m}\}$.

$$\begin{aligned} \text{Then } X \times Y &= (U_{x_1} \cup \dots \cup U_{x_m}) \times Y \\ &= (U_{x_1} \times Y) \cup \dots \cup (U_{x_m} \times Y) \end{aligned}$$

But $(U_{x_i} \times Y) \subset$ union of a finite number of sets in \mathcal{C} for each i with $1 \leq i \leq m$.

It follows that $X \times Y =$ union of a finite number of sets in \mathcal{C} .

Hence \mathcal{C} has a finite sub cover.

Therefore, $X \times Y$ is semi compact

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