International Journal of Mathematical Archive-5(1), 2014, 288-292
Available online through www.ijma.info ISSN 2229 – 5046

ON g*-$\mathcal{G}$-CLOSED SETS

#K. Indirani, *P. Sathishmohan and *V. Rajendran

#Department of Mathematics, Nirmala College for Women, Coimbatore, (T.N.), India.
*Department of Mathematics, KSG College of Arts and Science, Coimbatore, (T.N.), India.

(Received on: 23-11-13; Revised & Accepted on: 09-01-14)

ABSTRACT

In this paper, we introduced and study the notions of g^*-$\mathcal{G}$-closed sets and study some of their properties.

1. INTRODUCTION

In 1970, Levine [8] first introduced the concept of generalized closed (briefly, g-closed) sets were defined and investigated. The idea of grill on a topological space was first introduced by Choquet [4] in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many topological concept quite effectively. In [15], Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. The aim of this paper is to introduce g^*-$\mathcal{G}$-closed sets and investigate the relations of g^*-$\mathcal{G}$-closed sets between such sets.

2. PRELIMINARIES

Throughout this paper, (X, τ) (or X) represent a topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A, respectively. The power set of X will be denoted by $\wp(X)$. A collection $\mathcal{G}$ of a nonempty subsets of a space X is called a grill [1] on X if

(1) $A \in \mathcal{G}$ and $A \subseteq B \Longrightarrow B \in \mathcal{G}$.
(2) $A, B \subseteq X$ and $A \cup B \in \mathcal{G} \Longrightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$.

For any point x of a topological space (X, τ), $\tau(x)$ denote the collection of all open neighbourhoods of x. We recall the following results which are useful in the sequel.

Definition: 2.1 [15] Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. The mapping $\phi: \wp(X) \to \wp(X)$, denoted by $\phi_G(A, \tau)$ for $A \in \wp(X)$ or simply $\phi(A)$ called the operator associated with the grill $\mathcal{G}$ and the topology τ and is defined by $\phi_G(A)=\{x \in X\ A \cap U \in \mathcal{G}, \forall U \in \tau(x)\}$. Let $\mathcal{G}$ be a grill on a space X. Then a map $\Psi: \wp(X) \to \wp(X)$ is defined by $\Psi(A)= A \cup \phi(A)$, for all $A \in \wp(X)$. The map Ψ satisfies Kuratowski closure axioms. Corresponding to a grill $\mathcal{G}$ on a topological space (X, τ), there exits a unique topology τ_G on X given by $\tau_G = \{U \subset X / \Psi(X\text{-}U) = X\text{-}U\}$, where for any $A \subset X$, $\Psi(A) = A \cup \phi(A) = \tau_G - cl(A)$. For any grill $\mathcal{G}$ on a topological space by $(X, \tau, \mathcal{G})$.

Definition: 2.2 A subset A of a topological space (X, τ) is called
1) a pre-open set [12] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.
2) a semi-open set [7] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
3) a semi-preopen set [1] if $A \subseteq cl(int(cl(A)))$ and a semi-pre-closed set [2] if $(int(cl(A))) \subseteq A$.

Definition: 2.3 A subset A of a topological space (X, τ) is called
1) a generalized closed set (briefly g-closed) [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2) a generalized semi-closed set (briefly gs-closed) [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
3) an α-generalized closed set (briefly αg-closed) [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (x, τ).
4) a generalized semi-preclosed set (briefly gsp-closed) [5] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
5) a generalized preclosed set (briefly gp-closed) [11] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
6) a generalized grill closed set (briefly g-$\mathcal{G}$-closed) [6] if $\Psi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
7) a generalized closed set (briefly g^*-closed) [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

Corresponding author: *P. Sathishmohan
#Department of Mathematics, Nirmala College for Women, Coimbatore, (T.N.), India.
E-mail: iiscsathish@yahoo.co.in

Theorem: 2.4 [15]
1) If $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on a space X with $\mathcal{G}_1 \subset \mathcal{G}_2$, then $\tau_{\mathcal{G}1} \subset \tau_{\mathcal{G}2.}$
2) If $\mathcal{G}$ is a grill on a space X and B $\notin \mathcal{G}$, then B is closed in (X, τ, $\mathcal{G}$).
3) For any subset A of a space X and any grill $\mathcal{G}$ on X, ϕ(A) is $\tau_{\mathcal{G}}$-closed.

Theorem: 2.5 [15] Let (X, τ) be a topological space and $\mathcal{G}$ be any grill on X. Then
1) $A \subseteq B\ (\subseteq X) \Longrightarrow \phi(A) \subseteq \phi(B)$;
2) $A \subseteq X$ and $A \notin \mathcal{G} \Longrightarrow \phi(A) = \phi$;
3) $\phi(\phi(A)) \subseteq \phi(A) = cl(\phi(A)) \subseteq cl(A)$, for any $A \subseteq X$;
4) $\phi(A \cup B) = \phi(A) \cup \phi(B)$ for any $A, B \subseteq X$;
5) $A \subseteq \phi(A) \Longrightarrow cl(A) = \tau_{\mathcal{G}}\text{- } cl(A) = cl(\phi(A)) = \phi(A)$;
6) $U \in \tau$ and $\tau \setminus \{ \phi \} \subseteq \mathcal{G} \Longrightarrow U \subseteq \phi(U)$;
7) If $U \in \tau$ then $U \cap \phi(A) = U \cap \phi(U \cap A)$, for any $A \subseteq X$.

Theorem: 2.6 Let (X, τ) be a topological space and $\mathcal{G}$ be any grill on X. Then, for any A, B ⊆ X.
1) $A \subseteq \Psi(A)$ [15];
2) $\Psi(\phi) = \phi$ [15];
3) $\Psi(A \cup B) = \Psi(A) \cup \Psi(B)$ [15];
4) $\Psi(\Psi(A)) = \Psi(A)$ [15];
5) $int(A) \subset int(\Psi(A))$;
6) $int(\Psi(A \cap B)) \subset int(\Psi(A))$;
7) $int(\Psi(A \cap B)) \subset int(\Psi(B))$;
8) $int(\Psi(A)) \subset \Psi(A)$;
9) $A \subseteq B \Longrightarrow \Psi(A) \subseteq \Psi(B)$.

Theorem: 2.7 [16] Let (X, τ) be a topological space and $\mathcal{G}$ be any grill on X. Then, for any A, B ⊆ X,
(1) $\phi(A) \subseteq \Psi(A) = \tau_{\mathcal{G}}\text{- } cl(A) \subseteq cl(A)$;
(2) $A \cup \Psi(int(A)) \subseteq cl(A)$;
(3) $A \subseteq \phi(A)$ and $B \subseteq \phi(B) \Rightarrow \Psi(A \cap B) \subseteq \Psi(A) \cap \Psi(B)$.

3. g^*-$\mathcal{G}$-CLOSED SETS

Definition: 3.1 A subset A of (X, τ) is called a g^*-$\mathcal{G}$-closed set if $\Psi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

Theorem: 3.2 Every closed set is a g^*-$\mathcal{G}$-closed. But not conversely.

Proof: Let A be a closed set. Then cl(A)=A. Let U be any g-open set such that $A \subseteq U$. Then $cl(A) \subseteq U$. We know that $\Psi(A) = A \cup \phi(A) = \tau_{\mathcal{G}}\text{- } cl(A) \subseteq cl(A)$. Therefore $\Psi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open.

Example: 3.3 Let X = {a, b, c}, τ = {{ϕ}, {b}, {X}} and $\mathcal{G}$ = {{a}, {a, b}, {X}}. Let A = {c}. A is a g^*-$\mathcal{G}$-closed set but not closed set.

Theorem: 3.4 Every g-closed set is a g^*-$\mathcal{G}$-closed. But not conversely.

Proof: Let A be a g-closed set. Then $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open. We know that $\Psi(A) = A \cup \Psi(A) = \tau_{\mathcal{G}}\text{- } cl(A) \subseteq cl(A)$. Therefore $\Psi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open.

Example: 3.5 Let X = {a, b, c}, τ = {{ϕ},{a},{b},{a, b},{X}} and $\mathcal{G}$ = {{a},{a, b},{X}}. Let A = {c}. A is a g^*-$\mathcal{G}$-closed set but not g-closed set of (X, τ).

Theorem: 3.6 Every g^*-$\mathcal{G}$-closed set is a gsp closed. But not conversely.

Proof: Let A be a g^*-$\mathcal{G}$-closed set in (X, τ). Then $\Psi(A) \subseteq U$. Whenever $A \subseteq U$ and U is g-open. From the above theorem $cl(A) \subseteq U$. But every closed set is a semi-pre-closed set, we have $spcl(A) \subseteq U$ and also every open set is g-open. Therefore $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Example: 3.7 Let X = {a, b, c}, τ = {{ϕ}, {b}, {X}} and $\mathcal{G}$ = {{a}, {a, b}, {X}}. Let A = {b}. A is a g^*-$\mathcal{G}$-closed set but not gsp-closed set of (X, τ).

Theorem: 3.8 Every g^*-$\mathcal{G}$-closed set is a gp-closed. But not conversely.

Proof: Let A be a g^*-$\mathscr{g}$-closed set in (X, τ). Then $\Psi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open. From the above $cl(A) \subseteq U$. But every closed set is a pre-closed set, we have $pcl(A) \subseteq U$ and also every open set is g-open. Therefore $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Example: 3.9 Let $X = \{a, b, c\}$, $\tau = \{\{\phi\},\{b\},\{X\}\}$ and $\mathscr{g} = \{\{a\},\{a, b\},\{X\}\}$. Let $A = \{b\}$. Then A is gp-closed set but not g^*-$\mathscr{g}$-closed set of (X, τ).

Remark: 3.10 If A and B are g^*-$\mathscr{g}$-closed set, then $A \cup B$ is also a g^*-$\mathscr{g}$-closed set.

Theorem: 3.11 Every g^*-$\mathscr{g}$-closed set is a αg-closed set. But not conversely.

Proof: Let A be a g^*-$\mathscr{g}$-closed set in (X, τ). Then $\Psi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open. We know $cl(A) \subseteq U$. From the above and every closed set is α-closed set. We have $\alpha cl(A) \subseteq U$ and also every open set is g-open. Therefore $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Example: 3.12 Let $X = \{a, b, c\}$, $\tau = \{\{\phi\}, \{b\}, \{X\}\}$ and $\mathscr{g} = \{\{a\}, \{a, b\}, \{X\}\}$. Let $A = \{b\}$. Then A is a αg-closed set but not g^*-$\mathscr{g}$-closed set of (X, τ).

Theorem: 3.13 Every g^*-$\mathscr{g}$-closed set is a gs-closed set. But not conversely.

Proof: Let A be g^*-$\mathscr{g}$-closed set in (X, τ). Then $\Psi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open. We know $cl(A) \subseteq U$ from the above and every closed set is semi-closed set. We have $scl(A) \subseteq U$ and also every open set is g-open. Therefore $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Example: 3.14 Let $X = \{a, b, c\}$, $\tau = \{\{\phi\}, \{b\}, \{X\}\}$ and $\mathscr{g} = \{\{a\},\{a, b\},\{X\}\}$. Let $A = \{b, c\}$. Then A is a gs-closed set but not a g^*-$\mathscr{g}$-closed set of (X, τ).

Theorem: 3.15 A subset A of (X, τ) if a g^*-$\mathscr{g}$-closed set if and only if $\Psi(A) - A$ does not contain any non-empty g-$\mathscr{g}$-closed set.

Proof: Necessity: Let F be a g-$\mathscr{g}$-closed set of (X, τ) such that $F \subseteq \Psi(A) - A$. Then $A \subseteq X$-F.

Since A is g^*-$\mathscr{g}$-closed and X-F is g-$\mathscr{g}$-open, $\Psi(A) \subseteq X - F$. This implies $F \subseteq X - \Psi(A)$.

So $F \subseteq (X - \Psi(A)) \cap (\Psi(A) - A) \subseteq (X - \Psi(A)) \cap \Psi(A) = \phi$. Therefore $F = \phi$.

Sufficiency: Suppose A is a subset of (X, τ) such that $\Psi(A) - A$ does not contain any non-empty g-$\mathscr{g}$-closed set. Let U be a g-$\mathscr{g}$-closed set of (X, τ) such that $A \subseteq U$. If $\Psi(A) \nsubseteq U$, then $\Psi(A) \cap \Psi(U) \neq \phi$. Since $\Psi(A)$ is a closed set. Then we have $\phi \neq \Psi(A) \cap \Psi(U)$ is a g- $\mathscr{g}$-closed set of (X, τ). Then $\phi \neq \Psi(A) \cap \Psi(U) \subseteq \Psi(A) - A$. So $\Psi(A) - A$ contains a non-empty g-$\mathscr{g}$-closed set. This contradicts the hypothesis. Therefore A is a g^*-$\mathscr{g}$-closed set.

Theorem: 3.16 If A is a g^*-$\mathscr{g}$-closed set of (X, τ) such that $A \subseteq B \subseteq \Psi(A)$, then B is also a g^*-$\mathscr{g}$-closed set of (X, τ).

Proof: Let U be a g-$\mathscr{g}$-closed set of (X, τ) such that $B \subseteq U$. Then $A \subseteq U$. Since A is g^*-$\mathscr{g}$-closed, $\Psi(A) \subseteq U$. Now $\Psi(B) \subseteq \Psi(\Psi(A)) = \Psi(A) \subseteq U$. Therefore B is also a g^*-$\mathscr{g}$-closed set of (X, τ).

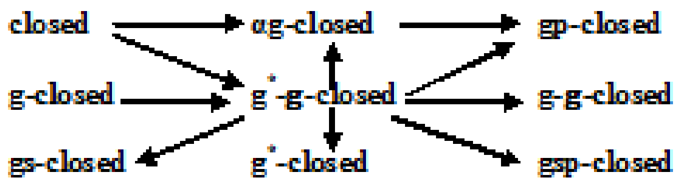
Theorem: 3.17 Every g^*-$\mathscr{g}$-closed set is a g^*-closed. But not conversely.

Example: 3.18 Let $X = \{a, b, c\}$, $\tau = \{\{\phi\}, \{a\}, \{c\}, \{a, c\}, \{X\}\}$ and $\mathscr{g} = \{\{a\}, \{a, b\}, \{X\}\}$. Let $A = \{a, c\}$. Then A is a g^*-closed but not g^*-$\mathscr{g}$-closed set of (X, τ).

Theorem: 3.19 Every g^*-$\mathscr{g}$-closed set is a g-$\mathscr{g}$-closed set. But not conversely.

Example: 3.20 Let $X = \{a, b, c\}$, $\tau = \{ \{\phi\},\{a\},\{c\},\{a, c\},\{X\} \}$ and $\mathscr{g} = \{\{a\},\{a, b\},\{X\}\}$. Let $A =\{c\}$. Then A is a g-$\mathscr{g}$-closed set but not g^*- $\mathscr{g}$-closed set of (X, τ).

Remark: 3.21



4. g^*-$\mathcal{G}$-CONTINUOUS

Definition: 4.1 A function f : $(X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is called g^*-$\mathcal{G}$-continuous if $f^{-1}(V)$ is a g^*-$\mathcal{G}$-closed set of $(X, \tau, \mathcal{G})$ for every closed set V of (Y, σ).

Theorem: 4.2 Every continuous map is g^*-$\mathcal{G}$-continuous. But not conversely.

Proof: Let f: $(X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a continuous map. Let V be a closed set in Y. Then $f^{-1}(V)$ is closed. Since every closed set is g^*-$\mathcal{G}$-closed set. We have $f^{-1}(V)$ is a g^*-$\mathcal{G}$-closed set. Therefore f is g^*-$\mathcal{G}$-continuous.

Example: 4.3 Let X = Y = {a, b, c}, τ = {{ϕ},{b},{X}} and σ = {{ϕ}, {a},{a, b},{Y}} and $\mathcal{G}$ = {{a},{a, b},{X}}. Define f: $(X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ by f(a) = a, f(b) = b and f(c) = c. f is not continuous since {c} is a closed set of (Y, σ). But $f^{-1}(c)$ = {c} is not closed set of $(X, \tau, \mathcal{G})$. However f is g^*-$\mathcal{G}$-continuous.

Theorem: 4.4 Every g-continuous map is g^*-$\mathcal{G}$-continuous map. But not conversely.

Proof: Let f: $(X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a g-continuous map. Let V be a g-closed set in Y. Then $f^{-1}(V)$ is g-closed. Since every g-closed is g^*-$\mathcal{G}$-closed set. We have $f^{-1}(V)$ is a g^*-$\mathcal{G}$-closed set. Therefore f is g^*-$\mathcal{G}$-continuous.

Example: 4.5 Let X = Y = {a, b, c}, τ = σ = {{ϕ}, {a},{a, b},{x} } and $\mathcal{G}$ = {{a},{a, c},{x}}. Define f: $(X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ by f(a) = a, f(b) = b and f(c) = c. f is not g-continuous. Since {c} is a closed set in (Y, σ). But $f^{-1}(c)$ = {c} is not a g-closed set of $(X, \tau, \mathcal{G})$. However f is g^*-$\mathcal{G}$-continuous.

Theorem: 4.6 Every g^*-$\mathcal{G}$-continuous map is gsp continuous and hence an αg-continuous, gp-continuous and gs-continuous. But not conversely.

Proof: Let f: $(X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a g^*-$\mathcal{G}$-continuous map. Let V be a closed set of (Y, σ). Since f is g^*-$\mathcal{G}$-continuous, $f^{-1}(V)$ is a g^*- $\mathcal{G}$-closed set of $(X, \tau, \mathcal{G})$. By the theorems 3.6, 3.8, 3.11 and 3.13, $f^{-1}(V)$ is gsp-closed, gp-closed, αg-closed and gs-closed set of (X, τ).

Example: 4.7 Let X = Y = {a, b, c}, τ = {{ϕ}, {a}, {a, b},{x}}, σ = {{ϕ},{c},{x}} and $\mathcal{G}$ = {{a},{a, c},{x}}. Define g : $(X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ by g(a) = a, g(b) = b and g(c) = c. f is not is g^*-continuous. Since {a, b} is a closed set in (Y, σ), but $f^{-1}(a, b)$ = {a, b} is not g^*- $\mathcal{G}$-closed. However f is gsp continuous, gp-continuous, αg-continuous and gs-continuous.

REFERENCES

[1] D. Andrijevic, Semi-preopen sets, Mat. Vesnik, 38(1) (1986), 24-32.

[2] S.P. Arya and T. Nour, Characterizations of s-normal spaces, Indian J. Pure. Appl. Math., 21(8) (1990), 717-719.

[3] P. Bhattacharya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29(3)(1987), 375-382.

[4] G. Choquet, Sur less notions de filter et grille, comptes Rendus Acad. Sci. Paris, 224(1947), 171-173.

[5] J. Dontchev, On generalizing semi-preopen sets, Mem.Fac.Sci.Kochi Univ. Ser. A, Math., 16(1995), 35-48.

[6] K. Indirani, P. Sathishmohan and V. Rajendran, On Generalized Grill Closed Sets, International Journal of Applied Mathematics, vol 3, Issue 2(Oct 2012-Mar '13)

[7] N. Levine, Semi-open sets and semi-continuity in topological Spaces, Amer. Math. Monthly, 70(1963), 36-41.

[8] N. Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2) (1970), 89-96.

[9] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 15(1994), 51-63.

[10] H. Maki, R. Devi and K. Balachandran, Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed. Part III, 42(1993), 13-21.

[11] H. Maki, J. Uniehara and T. Noiri, Every topological Spaces in pre-$T_{1/2}$, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 17(1996). 33-42.

[12] A.S. Mashhour, M.E. Abd EI-Monsef and S.N. EI-Deeb, On Pre-Continuous and weak Pre-continuous mappings, Proc. Math. And Phys. Soc. Egypt, 53(1982), 47-53.

[13] A.S. Mashhour, I.A. Hasanein and S.N. EI-Deep, α-continuous and α-open mappings, Acta. Math.Hung., 41(3-4) (1983), 213-218.

[14] O.Njastad, On some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.

[15] B. Roy and M.N. Mukherjee, On a typical topology induced by a grill, Soochow J. Math., 33(4)(2007), 771-786.

[16] O .Ravi and S. Ganesan On g-α-open sets and g-α-continuous functions submitted.

[17] M.K.R.S Veerakumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 21(2000), 1-19.

Source of support: Nil, Conflict of interest: None Declared

