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# SYNCHRONIZATION & ANTI-SYNCHRONIZATION OF A SATELLITE'S MOTION UNDER AIR DRAG VIA ACTIVE CONTROL

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***ABSTRACT***

*In this paper, we have investigated the synchronization and anti-synchronization behaviour of two identical planar oscillation of a satellite in circular orbit under air drag evolving from different initial conditions using the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The designed controllers, with our own choice of the coefficient matrix of the error dynamics are found to be effective to achieve synchronization and anti-synchronization between the states variables of two nonlinear dynamical systems under consideration. The results are validated by numerical simulations using mathematica.*

***Keywords:*** *Synchronization; Active Control; Lyapunov stability; Satellite.*

## 1. INTRODUCTION

In the last two decades, considerable research has been done on chaotic systems. One of the most important aspects of chaotic systems is synchronization that represents the entrainment of frequency of oscillators due to weak interactions [1–5]. It is basically the design of control law for full chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with the slave system. Hence, the slave chaotic system completely traces the dynamics of the master in the course of time. On the other hand, a similar phenomenon, anti-synchronization (AS), which is the vanishing of the sum of the relevant state variables of synchronized systems, has been investigated both experimentally and theoretically in many physical systems [6–14]. Various techniques have been proposed and implemented successfully for achieving stable synchronization between identical and non-identical systems. Notable among these methods, the active control scheme proposed by Bai and Lonngren [15] has received and successfully implemented in almost all the field of nonlinear sciences for synchronization as well as AS phenomenon [16–28].

Motivated by the aforementioned we aim to study the synchronization as well as AS for the two identical planar oscillation of a chaotic satellite in circular orbit under air drag using active control technique based on the lyapunov stability theory & Routh–Hurwitz criteria. The designed controller of our own choice of the coefficient matrix of the error dynamics that satisfy the lyapunov stability & the Routh–Hurwitz criteria, are found to be effective to achieve the synchronization & AS between the two identical systems chosen for our study. Numerical simulations are shown to verify the analytical results using *mathematica*.

## 2. DESCRIPTION OF THE MODEL

The determination and prediction of the orbit of a satellite in the near-earth environment is complicated by the fact that the satellite is influenced by the dissipative effects of the earth's atmosphere. For many artificial satellites, this fluctuation in the drag is one of the fundamental source of error in the orbital predictions.

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Let the attitude motion of a satellite in a circular orbit under the influence of central body of mass *M* and its moon of mass *m*, whose orbit is assumed to be circular and coplanar with the orbit of the satellite. The body is assumed to be tri-axial body with principal moments of inertia $A < B < C$ at its centre of mass, *C* is the moment of inertia about the spin axis which is perpendicular to the orbital plane and $\vec{r}$ be the instantaneous radius vector of the centre of mass of the satellite, $\theta$ be the angle that the long axis of the satellite makes with a fixed line lying in the orbital plane and $\frac{\delta}{2}$ the angle between the radius vector $\vec{r}$ and the long axis. Euler's equation of motion of the considered satellite's dynamics [29] about *z*-axis given below:

$$\frac{d^2\eta}{d\nu^2} + \omega^2 \sin\eta - \varepsilon\left(\nu^2 - b\nu - d\right)\sin\nu = 0 \tag{2.1}$$

where $\omega^2 = \frac{3(\mathrm{B}-\mathrm{A})}{C}$, $\varepsilon = \frac{\rho S C_d a^2}{C\Omega^4}$, $b = \frac{\Omega\left(2V_1 - l\omega\right)}{a}$, and $d = \frac{\Omega^2 V_1\left(V_1 - l\omega\right)}{a^2}$ are all constants and $\nu$ is true anomaly taken as independent variable.

## 3. SYNCHRONIZATION VIA ACTIVE CONTROL

In order to formulate the active controllers, we write the system (2.1) in two first order differential equations as shown below:

Let $\frac{d\eta}{d\nu} = x_1'$ and $\frac{d^2\eta}{d\nu^2} = x_2'$, then we have

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= -\omega^2 \sin x_1 + \varepsilon\left(\nu^2 - b\nu - d\right)\sin\nu \end{aligned} \tag{3.1}$$

Let us define another identical system

$$\begin{aligned} y_1' &= y_2 + u_1(\nu), \\ y_2' &= -\omega^2 \sin y_1 + \varepsilon\left(\nu^2 - b\nu - d\right)\sin\nu + u_2\left(\nu\right) \end{aligned} \tag{3.2}$$

where (3.1) and (3.2) are called the master and the slave system respectively and in slave system, $u_1(\nu)$ and $u_2(\nu)$ are control functions to be determined. Let $e_1(\nu) = y_1(\nu) - x_1(\nu)$ and $e_2(\nu) = y_2(\nu) - x_2(\nu)$ be the synchronization errors such that $\lim_{\nu\to\infty} e_i(\nu) \to 0$ for $i = 1, 2$. From (3.1) and (3.2), we have

$$\begin{aligned} e_1'\left(\nu\right) &= e_2\left(\nu\right) + u_1\left(\nu\right) \\ e_2'(\nu) &= -\omega^2\left(\sin y_1 - \sin x_1\right) + u_2(\nu) \end{aligned} \tag{3.3}$$

In order to express (3.3) as only linear terms in $e_1(\nu)$ and $e_2(\nu)$, we redefine the control functions as follows:

$$\begin{aligned} u_1(\nu) &= v_1(\nu), \\ u_2(\nu) &= \omega^2\left(\sin y_1 - \sin x_1\right) + v_2(\nu) \end{aligned} \tag{3.4}$$

From (3.3) and (3.4), we have

$$\begin{aligned} e_1'(\nu) &= e_2(\nu) + v_1(\nu), \\ e_2'(\nu) &= v_2(\nu) \end{aligned} \tag{3.5}$$

Equation (3.5) is the error dynamics, which can be interpreted as a control problem where the system, to be controlled is a linear system with control inputs $v_1(\nu) = v_1\left(e_1(\nu), e_2(\nu)\right)$ and $v_2(\nu) = v_2\left(e_1(\nu), e_2(\nu)\right)$. As long as these feedbacks stabilize the system, $\lim_{\nu\to\infty} e_i(\nu) \to 0$ for $i = 1, 2$. This simply implies that the two systems (3.1) and (3.2) evolving from different initial conditions are synchronized. As functions of $e_1(\nu)$ and $e_2(\nu)$, we choose $v_1(\nu)$ and $v_2(\nu)$ as follows:

$$\begin{pmatrix} v_1(\nu) \\ v_2(\nu) \end{pmatrix} = D \begin{pmatrix} e_1(\nu) \\ e_2(\nu) \end{pmatrix} \tag{3.6}$$

where $D = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, is a $2\times 2$ constant feedback matrix to be determined. Hence the error system (3.5) can be written as:

$$\begin{pmatrix} e_1'(\nu) \\ e_2'(\nu) \end{pmatrix} = C \begin{pmatrix} e_1(\nu) \\ e_2(\nu) \end{pmatrix} \tag{3.7}$$

where $C = \begin{pmatrix} a_1 & 1+b_1 \\ c_1 & d_1 \end{pmatrix}$, is the coefficient matrix.

According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$\begin{aligned} & a_1 + d_1 < 0, \\ & c_1(1+b_1) - a_1 d_1 < 0 \end{aligned} \tag{3.8}$$

then the eigen values of the coefficient matrix of error system (3.5) must be real or complex with negative real parts and, hence, stable synchronized dynamics between systems (3.1) and (3.2) is guaranteed. Let

$$a_1 + d_1 = c_1(1+b_1) - a_1 d_1 = -E\ , \tag{3.9}$$

where $E > 0$ is a real number which is usually set equal to 1. There are several ways of choosing the constant elements $a_1$, $b_1$, $c_1$, $d_1$ of matrix *D* in order to satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria (3.8).

**4. NUMERICAL SIMULATION FOR SYNCHRONIZATION**

For the constant elements of feedback matrix, choosing $a_1 = d_1 = -0.5$ and for the parameters involved in system under investigation, $b = 0.2$, $d = 0.3$, $\varepsilon = 0.1$ and $\omega_0 = 0.3$ with the initial conditions $\left[x_1(0), x_2(0)\right] = \left[0,0\right]$ and $\left[y_1(0), y_2(0)\right] = \left[20,30\right]$, we have simulated the identical systems under consideration using *mathematica*. The results obtained show that the system under consideration achieved synchronization. Time series of states variables $\left(x \,\&\, y\right)$ have been plotted in figures 1 & 2. Where as in figure 3, the time series of the errors $\left(e_1 \,\&\, e_2\right)$ is the witness of achieving synchronization between master and slave system. Further, it also has been confirmed by the convergence of the synchronization quality defined by

$$e(\nu) = \sqrt{e_1^2(\nu) + e_2^2(\nu)} \tag{4.1}$$

Figure (7) confirms the convergence of the synchronization quality defined by (4.1).

In order to discuss the stability, Let us consider the lyapunov function (positive definite also):

$$V(\nu) = \frac{1}{2} e_1^{\,2}(\nu) + \frac{1}{2} e_2^{\,2}(\nu) \tag{4.2}$$

In figure 8, the plot of the $V'(\nu)$ confirm that the synchronization is globally stable for the two identical systems under consideration.

 

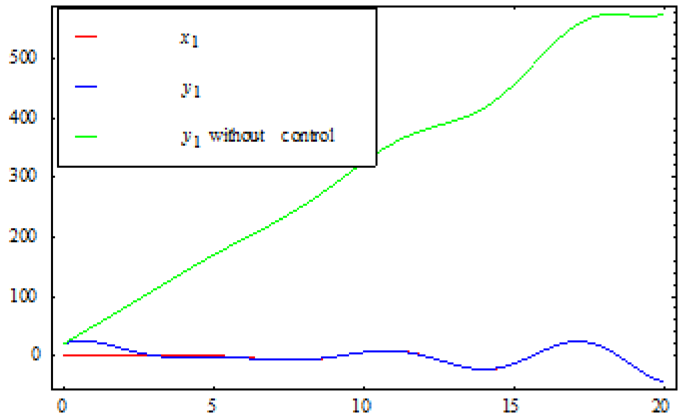

Fig 1: Time Series of $x_1$ & $y_1$ during Synchronization

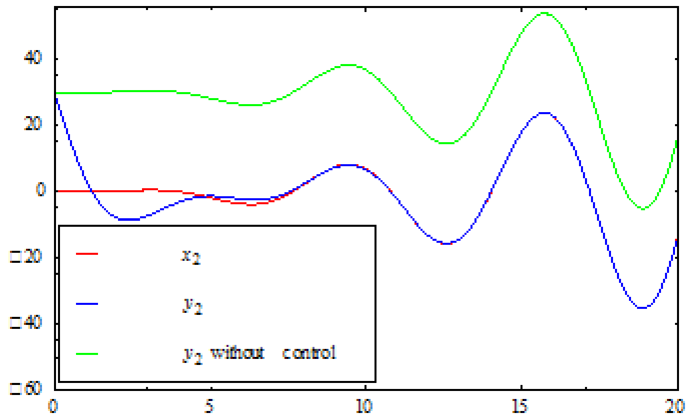

Fig 2: Time Series of $x_2$ & $y_2$ during Synchronization

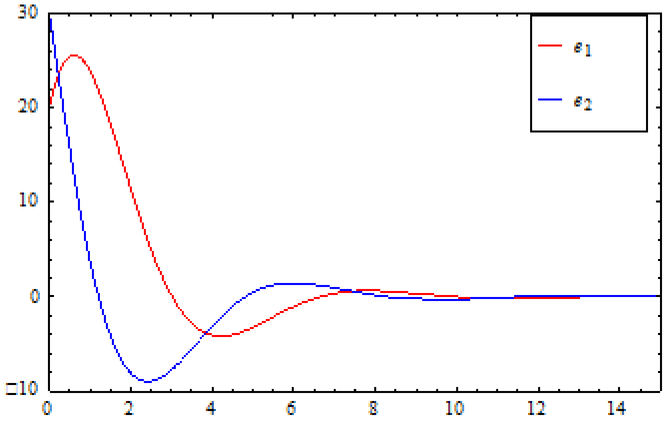

Fig 3: Time Series of errors during Synchronization

## 5. ANTI-SYNCHRONIZATION VIA ACTIVE CONTROL

Anti-synchronization of two coupled systems $\dot{x} = f(x, y)$ (master system) and $\dot{y} = g(x, y)$ (slave system) means $\lim_{t\to\infty} |x(t) + y(t)| \to 0$. This phenomenon has been investigated both experimentally and theoretically in many physical systems [6-9, 11-14, 30-35].

In order to formulate the active controllers for Anti-synchronization, we need to redefine the error functions as $e_1(\nu) = y_1(\nu) + x_1(\nu)$ and $e_2(\nu) = y_2(\nu) + x_2(\nu)$, where $e_1(\nu)$ and $e_2(\nu)$ are called the anti-synchronization errors such that $\lim_{\nu\to\infty} e_i(\nu) \to 0$ for $i = 1, 2$. From (2.1) and (2.2), error dynamics can be written as:

$$e_1'(\nu) = e_2(\nu) + u_1(\nu),$$

$$e_2'(\nu) = -\omega^2\left(\sin y_1 + \sin x_1\right) + 2\varepsilon\left(\nu^2 - b\nu - d\right)\sin\nu + u_2\left(\nu\right) \tag{5.1}$$

In order to express (5.1) as only linear terms in $e_1(\nu)$ and $e_2(\nu)$, we redefine the control functions as follows:

$$u_1(\nu) = v_1(\nu),$$
$$u_2(\nu) = \omega^2\left(\sin y_1 + \sin x_1\right) - 2\varepsilon\left(\nu^2 - b\nu - d\right)\sin\nu + v_2\left(\nu\right) \tag{5.2}$$

From (5.1) and (5.2), we have

$$e_1'(\nu) = e_2(\nu) + v_1(\nu),$$
$$e_2'(\nu) = v_2(\nu) \tag{5.3}$$

Equation (5.3) is the error dynamics, which can be interpreted as a control problem where the system, to be controlled is a linear system with control inputs $v_1(\nu) = v_1\left(e_1(\nu), e_2(\nu)\right)$ and $v_2(\nu) = v_2\left(e_1(\nu), e_2(\nu)\right)$. As long as these feedbacks stabilize the system, $\lim_{\nu\to\infty} e_i(\nu) \to 0$ for $i = 1, 2$. This simply implies that the two systems (2.1) and (2.2) evolving from different initial conditions are synchronized. As functions of $e_1(\nu)$ and $e_2(\nu)$, we choose $v_1(\nu)$ and $v_2(\nu)$ as follows:

$$\begin{pmatrix} v_1(\nu) \\ v_2(\nu) \end{pmatrix} = D\begin{pmatrix} e_1(\nu) \\ e_2(\nu) \end{pmatrix} \tag{5.4}$$

where $D = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, is a $2\times 2$ constant feedback matrix to be determined. Hence the error system (4.3) can be written as:

$$\begin{pmatrix} e_1'(\nu) \\ e_2'(\nu) \end{pmatrix} = C\begin{pmatrix} e_1(\nu) \\ e_2(\nu) \end{pmatrix} \tag{5.4}$$

where $C = \begin{pmatrix} a_2 & 1+b_2 \\ c_2 & d_2 \end{pmatrix}$, is the coefficient matrix.

According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$a_2 + d_2 < 0,$$
$$c_2(1+b_2) - a_2 d_2 < 0 \tag{5.5}$$

then the eigen values of the coefficient matrix of error system (5.3) must be real or complex with negative real parts and, hence, stable synchronized dynamics between systems (3.1) and (3.2) is guaranteed. Let

$$a_2 + d_2 = c_2(1+b_2) - a_2 d_2 = -E\,, \tag{5.6}$$

Where $E > 0$ is a real number which is usually set equal to 1. There are several ways of choosing the constant elements $a_2, b_2, c_2, d_2$ of matrix *D* in order to satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria (5.3).

 

## 6. NUMERICAL SIMULATION FOR ANTI-SYNCHRONIZATION

For the constant elements of feedback matrix, choosing $a_2 = -0.75$ , $d_2 = -0.25$ and for the parameters involved in system under investigation, $b = 0.2$, $d = 0.3$, $\varepsilon = 0.1$ and $\omega_0 = 0.3$ with the initial conditions $[x_1(0), x_2(0)] = [0,0]$ and $[y_1(0), y_2(0)] = [20,30]$, we have simulated the system under consideration using *mathematica*. The results obtained show that the system under consideration achieved AS. Time series analysis of state variables and errors are the witness of achieving AS between master and slave system (figure 4-6).

Furthermore, the convergence of the errors towards zero is shown in figure 7 for anti-synchronization for the error function in ( 4.1). Figure 8 is the witness of global stability of the AS through a Lyapunov function defined by (4.2).

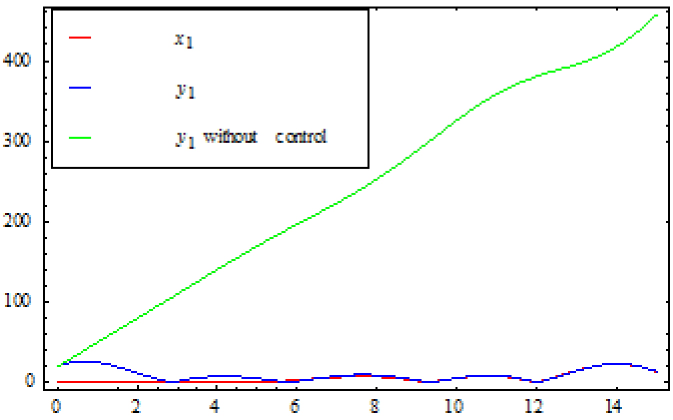


Fig 4: Time Series of $x_1$ & $y_1$ during AS

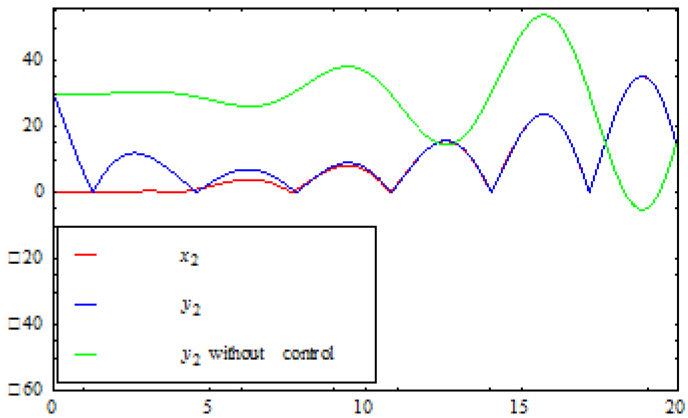


Fig 5: Time Series of $x_2$ & $y_2$ during AS

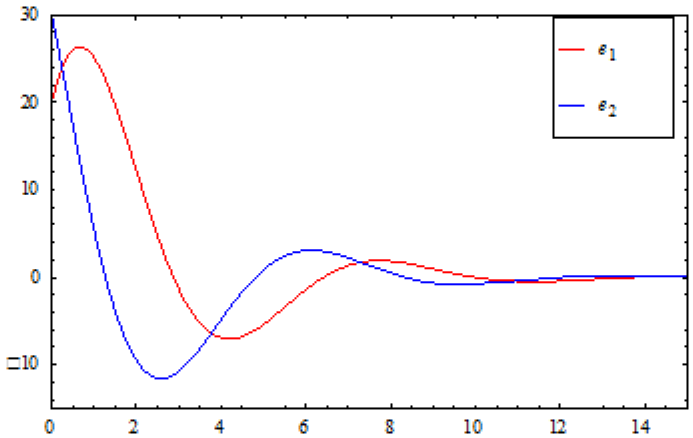


Fig 6: Time Series of errors during AS

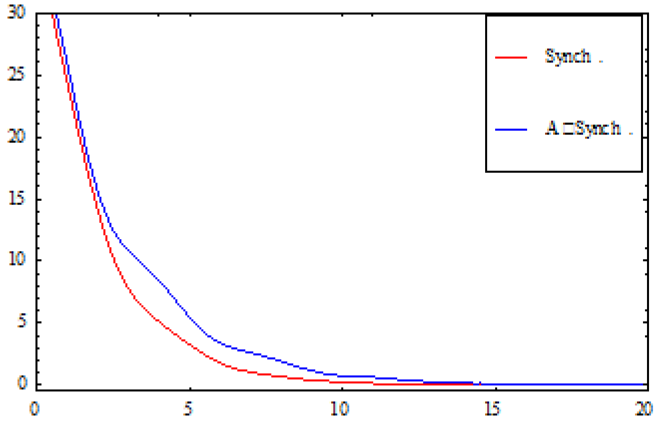


Fig 7: Convergence of errors

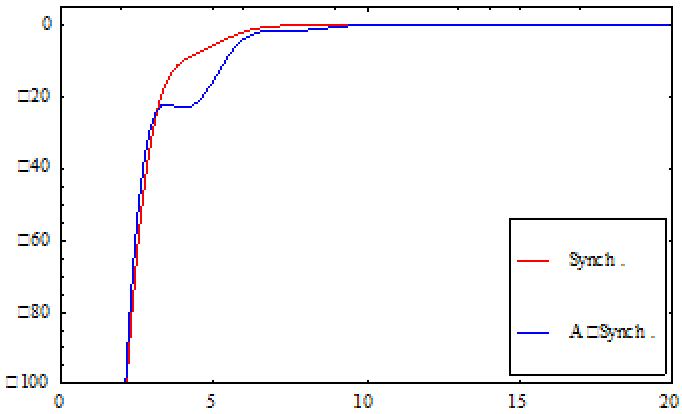


Fig 8: Derivatives of Lyapunove Function

## 7. CONCLUSION

In this paper, we have investigated the synchronization and anti-synchronization behaviour of the two identical planar oscillation of a satellite in circular orbit under air drag evolving from different initial conditions via the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The results were validated by numerical simulations using *mathematica*. For the errors in synchronization and anti-synchronization behavior of the system under study, we have observed that both synchronization and AS can be achieved fastly and are found globally stable. Our computational results validate all this.

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