International Journal of Mathematical Archive-4(12), 2013, 274-276
Available online through www.ijma.info ISSN 2229 – 5046

HOMOMORPHISM AND ANTI-HOMOMORPHISM OF BIPOLAR-VALUED FUZZY SUBGROUPS OF A GROUP

M. S. Anitha
Department of Mathematics, H. H. The Rajahs College, Pudukkottai–622001, Tamilnadu, India.

K. L. Muruganantha Prasad
Department of Mathematics, H. H. The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

K. Arjunan*
Department of Mathematics, H. H. The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

(Received on: 18-11-13; Revised & Accepted on: 19-12-13)

ABSTRACT

In this paper, we made an attempt to study the algebraic nature of bipolar-valued fuzzy subgroups under homomorphism and anti-homomorphism and prove some results on these.

Key Words: *Bipolar-valued fuzzy set, bipolar-valued fuzzy subgroup, bipolar-valued fuzzy normal subgroup.*

INTRODUCTION

In 1965, Zadeh [10] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [7]. Lee [6] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [6, 7]. We introduce the concept of bipolar-valued fuzzy subgroup under homomorphism, antihomomorphism and established some results.

1. PRELIMINARIES

1.1 Definition: A bipolar-valued fuzzy set (BVFS) A in X is defined as an object of the form A = {< x, $A^+(x)$, $A^-(x)$ >/ x∈X}, where $A^+ : X \to [0, 1]$ and $A^- : X \to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued fuzzy set A. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.1 Example: A = {< a, 0.5, –0.3 >, < b, 0.1, –0.7 >, < c, 0.5, –0.4 >} is a bipolar-valued fuzzy subset of X= {a, b, c}.

1.2 Definition: Let G be a group. A bipolar-valued fuzzy subset A of G is said to be a bipolar-valued fuzzy subgroup of G (BVFSG) if the following conditions are satisfied,

(i) $A^+(xy) \geq \min\{ A^+(x), A^+(y) \}$,
(ii) $A^+(x^{-1}) \geq A^+(x)$,
(iii) $A^-(xy) \leq \max\{ A^-(x), A^-(y) \}$,
(iv) $A^-(x^{-1}) \leq A^-(x)$, for all x and y in G.

*Corresponding author: K. Arjunan**
Department of Mathematics, H. H. The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

1.2 Example: Let G = {1, −1, i, −i} be a group with respect to the ordinary multiplication. Then A = {< 1, 0.5, − 0.6 >, < −1, 0.4, − 0.5 >, < i, 0.2, −0.4 >, < −i, 0.2, −0.4 >} is a bipolar-valued fuzzy subgroup of G.

1.3 Definition: Let (G, .) be a group. A bipolar-valued fuzzy subgroup A of G is said to be a bipolar-valued fuzzy normal subgroup (BVFNSG) of G if $A^+(xy) = A^+(yx)$ and $A^-(xy) = A^-(yx)$, for all x and y in G.

1.4 Definition: Let G and G^I be any two groups. Then the function f: G→ G^I is said to be an antihomomorphism if f(xy) = f(y)f(x), for all x and y in G.

1.5 Definition: Let X and X^I be any two sets. Let f : X→ X^I be any function and let A be a bipolar-valued fuzzy subset in X, V be a bipolar-valued fuzzy subset in f(X) = X^I, defined by $V^+(y) = \sup_{x\in f^{-1}(y)} A^+(x)$ and $V^-(y) = \inf_{x\in f^{-1}(y)} A^-(x)$, for all x in X and y in X^I. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2. SOME PROPERTIES

2.1 Theorem: Let (G, .) and (G^I, .) be any two groups. The homomorphic image of a bipolar-valued fuzzy subgroup of G is a bipolar-valued fuzzy subgroup of G^I.

Proof: Let (G, .) and (G^I, .) be any two groups. Let f: G → G^I be a homomorphism. Then f(xy) = f(x)f(y), for all x and y in G. Let V = f(A), where A is a bipolar-valued fuzzy subgroup of G. We have to prove that V is a bipolar-valued fuzzy subgroup of G^I. Now, for f(x), f(y) in G^I, $V^+(f(x)f(y)) = V^+(f(xy)) \geq A^+(xy) \geq \min\{A^+(x), A^+(y)\} = \min \{V^+(f(x)), V^+(f(y))\}$ which implies that $V^+(f(x)f(y)) \geq \min\{ V^+(f(x)), V^+(f(y))\}$. For f(x) in G^I, $V^+([f(x)]^{-1}) = V^+(f(x^{-1})) \geq A^+(x^{-1}) \geq A^+(x) = V^+(f(x))$ which implies that $V^+([f(x)]^{-1}) \geq V^+(f(x))$. And $V^-(f(x)f(y)) = V^-(f(xy)) \leq A^-(xy) \leq \max\{A^-(x), A^-(y)\} = \max\{V^-(f(x)), V^-(f(y))\}$ which implies that $V^-(f(x)f(y)) \leq \max V^-(f(x)), V^-(f(y))\}$. Also $V^-([f(x)]^{-1}) = V^-(f(x^{-1})) \leq A^-(x^{-1}) \leq A^-(x) = V^-(f(x))$ which implies that $V^-([f(x)]^{-1}) \leq V^-(f(x))$. Hence V is a bipolar-valued fuzzy subgroup of G^I.

2.2 Theorem: Let (G, .) and (G^I, .) be any two groups. The homomorphic preimage of a bipolar-valued fuzzy subgroup of G^I is a bipolar-valued fuzzy subgroup of G.

Proof: Let (G, .) and (G^I, .) be any two groups. Let f: G → G^I be a homomorphism. Then f(xy) = f(x)f(y), for all x and y in G. Let V = f(A), where V is a bipolar-valued fuzzy subgroup of G^I. We have to prove that A is a bipolar-valued fuzzy subgroup of G. Let x and y in G. Now, $A^+(xy) = V^+(f(xy)) = V^+(f(x)f(y)) \geq \min\{V^+(f(x)), V^+(f(y))\} = \min\{ A^+(x), A^+(y)\}$ which implies that $A^+(xy) \geq \min\{ A^+(x), A^+(y)\}$. Also $A^+(x^{-1}) = V^+(f(x^{-1})) = V^+([f(x)]^{-1}) \geq V^+(f(x)) = A^+(x)$ which implies that $A^+(x^{-1}) \geq A^+(x)$. And $A^-(xy)=V^-(f(xy))= V^-(f(x)f(y)) \leq \max\{V^-(f(x)), V^-(f(y))\}= \max\{A^-(x), A^-(y)\}$ which implies that $A^-(xy) \leq \max\{A^-(x), A^-(y)\}$. Also $A^-(x^{-1}) = V^-(f(x^{-1})) = V^-([f(x)]^{-1}) \leq V^-(f(x)) = A^-(x)$ which implies that $A^-(x^{-1}) \leq A^-(x)$. Hence A is a bipolar-valued fuzzy subgroup of G.

2.3 Theorem: Let (G, .) and (G^I, .) be any two groups. The antihomomorphic image of a bipolar-valued fuzzy subgroup of G is a bipolar-valued fuzzy subgroup of G^I.

Proof: Let (G,.) and (G^I, .) be any two groups. Let f: G → G^I be an antihomomorphism. Then f(xy) = f(y)f(x), for all x and y in G. Let V = f(A), where A is a bipolar-valued fuzzy subgroup of G. We have to prove that V is a bipolar-valued fuzzy subgroup of G^I. Now, for f(x), f(y) in G^I, $V^+(f(x)f(y)) =V^+(f(yx)) \geq A^+(yx) \geq \min \{A^+(x), A^+(y)\}= \min \{V^+(f(x)), V^+(f(y))\}$ which implies that $V^+(f(x)f(y)) \geq \min \{V^+(f(x)), V^+(f(y))\}$. For f(x) in G^I, $V^+([f(x)]^{-1}) = V^+(f(x^{-1})) \geq A^+(x^{-1}) \geq A^+(x) = V^+(f(x))$ which implies that $V^+([f(x)]^{-1}) \geq V^+(f(x))$. And $V^-(f(x)f(y))=V^-(f(yx)) \leq A^-(yx) \leq \max\{A^-(x), A^-(y)\} = \max\{V^-(f(x)), V^-(f(y))\}$ which implies that $V^-(f(x)f(y)) \leq \max \{V^-(f(x)), V^-(f(y))\}$. Also $V^-([f(x)]^{-1}) = V^-(f(x^{-1})) \leq A^-(x^{-1}) \leq A^-(x) = V^-(f(x))$ which implies that $V^-([f(x)]^{-1}) \leq V^-(f(x))$. Hence V is a bipolar-valued fuzzy subgroup of G^I.

2.4 Theorem: Let (G,.) and (G^I,.) be any two groups. The antihomomorphic preimage of a bipolar-valued fuzzy subgroup of G^I is a bipolar-valued fuzzy subgroup of G.

Proof: Let (G,.) and (G^I, .) be any two groups. Let f: G → G^I be an antihomomorphism. Then f(xy) = f(y)f(x), for all x and y in G. Let V = f(A), where V is a bipolar-valued fuzzy subgroup of G^I. We have to prove that A is a bipolar-valued fuzzy subgroup of G. Let x and y in G. Now, $A^+(xy) = V^+(f(xy)) = V^+(f(y)f(x)) \geq \min \{V^+(f(x)), V^+(f(y))\}= \min\{A^+(x), A^+(y)\}$ which implies that $A^+(xy) \geq \min\{A^+(x), A^+(y)\}$. Also $A^+(x^{-1}) = V^+(f(x^{-1})) = V^+([f(x)]^{-1}) \geq V^+(f(x)) = A^+(x)$ which implies that $A^+(x^{-1}) \geq A^+(x)$. And $A^-(xy) = V^-(f(xy)) = V^-(f(y)f(x)) \leq \max\{V^-(f(x)), V^-(f(y))\}= \max\{A^-(x), A^-(y)\}$ which implies that $A^-(xy) \leq \max\{A^-(x), A^-(y)\}$. Also $A^-(x^{-1}) = V^-(f(x^{-1})) = V^-([f(x)]^{-1}) \leq V^-(f(x)) = A^-(x)$ which implies that $A^-(x^{-1}) \leq A^-(x)$. Hence A is a bipolar-valued fuzzy subgroup of G.

2.5 Theorem: Let (G, .) and (G^l, .) be any two groups. The homomorphic image of a bipolar-valued fuzzy normal subgroup of G is a bipolar-valued fuzzy normal subgroup of G^l.

Proof: Let (G, .) and (G^l, .) be any two groups. Let f: $G \to G^l$ be a homomorphism. Then f(xy) = f(x)f(y), for all x and y in G. Let V = f(A), where A is a bipolar-valued fuzzy normal subgroup of G. We have to prove that V is a bipolar-valued fuzzy normal subgroup of G^l.

Now, for f(x), f(y) in G^l, $V^+(f(x)f(y)) = V^+(f(xy)) \geq A^+(xy) = A^+(yx) \leq V^+(f(yx)) = V^+(f(y)f(x))$ which implies that $V^+(f(x)f(y)) = V^+(f(y)f(x))$. And $V^-(f(x)f(y)) = V^-(f(xy)) \geq A^-(xy) = A^-(yx) \leq V^-(f(yx)) = V^-(f(y)f(x)$ which implies that $V^-(f(x)f(y)) = V^-(f(y)f(x))$. Hence V is a bipolar-valued fuzzy normal subgroup of G^l.

2.6 Theorem: Let (G, .) and (G^l, .) be any two groups. The homomorphic preimage of a bipolar-valued fuzzy normal subgroup of G^l is a bipolar-valued fuzzy normal subgroup of G.

Proof: Let (G, .) and (G^l, .) be any two groups. Let f: $G \to G^l$ be a homomorphism. Then f(xy) = f(x)f(y), for all x and y in G. Let V = f(A), where V is a bipolar-valued fuzzy normal subgroup of G^l. We have to prove that A is a bipolar-valued fuzzy normal subgroup of G. Let x and y in G. Now, $A^+(xy)=V^+(f(xy)) = V^+(f(x)f(y)) = V^+(f(y)f(x)) = V^+(f(yx))=A^+(yx)$ which implies that $A^+(xy) = A^+(yx)$. And $A^-(xy) =V^-(f(xy)) = V^-(f(x)f(y)) = V^-(f(y)f(x)) = V^-(f(yx)) = A^-(yx)$ which implies that $A^-(xy) = A^-(yx)$. Hence A is a bipolar-valued fuzzy normal subgroup of G.

2.7 Theorem: Let (G, .) and (G^l, .) be any two groups. The antihomomorphic image of a bipolar-valued fuzzy normal subgroup of G is a bipolar-valued fuzzy normal subgroup of G^l.

Proof: Let (G, .) and (G^l, .) be any two groups. Let f: $G \to G^l$ be an antihomomorphism. Then f(xy) = f(y)f(x), for all x and y in G. Let V = f(A), where A is a bipolar-valued fuzzy normal subgroup of G. We have to prove that V is a bipolar-valued fuzzy normal subgroup of G^l. Now, for f(x), f(y) in G^l, $V^+(f(x)f(y)) = V^+(f(yx)) \geq A^+(yx) = A^+(xy) \leq V^+(f(xy)) = V^+(f(y)f(x))$ which implies that $V^+(f(x)f(y)) = V^+(f(y)f(x))$. And $V^-(f(x)f(y)) = V^-(f(yx)) \leq A^-(yx) = A^-(xy) \geq V^-(f(xy)) = V^-(f(y)f(x))$ which implies that $V^-(f(x)f(y)) = V^-(f(y)f(x))$. Hence V is a bipolar-valued fuzzy normal subgroup of G^l.

2.8 Theorem: Let (G,.) and (G^l, .) be any two groups. The antihomomorphic preimage of a bipolar-valued fuzzy normal subgroup of G^l is a bipolar-valued fuzzy normal subgroup of G.

Proof: Let (G,.) and (G^l, .) be any two groups. Let f: $G \to G^l$ be an antihomomorphism. Then f(xy) = f(y)f(x), for all x and y in G. Let V = f(A), where V is a bipolar-valued fuzzy normal subgroup of G^l. We have to prove that A is a bipolar-valued fuzzy normal subgroup of G. Let x and y in G. Now, $A^+(xy) = V^+(f(xy)) = V^+(f(y)f(x)) = V^+(f(x)f(y)) = V^+(f(yx))=A^+(yx)$ which implies that $A^+(xy) = A^+(yx)$. And $A^-(xy)=V^-(f(xy)) = V^-(f(y)f(x)) = V^-(f(x)f(y)) = V^-(f(yx)) = A^-(yx)$ which implies that $A^-(xy) = A^-(yx)$. Hence A is a bipolar-valued fuzzy normal subgroup of G.

REFERENCES

1. Anitha.M.S, Muruganantha Prasad.K.L., & Arjunan.K, Notes on Bipolar Valued Fuzzy Subgroups of a Group, Bulletin of Society for Mathematical Services and Standards, Vol. 2 No. 3, 52-59(2013).
2. Anthony.J.M. and Sherwood.H, Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69,124 -130 (1979).
3. Arsham Borumand Saeid, bipolar-valued fuzzy BCK/BCI-algebras, World Applied Sciences Journal 7 (11): 1404-1411(2009).
4. Azriel Rosenfeld, Fuzzy groups, Journal of mathematical analysis and applications 35, 512-517 (1971).
5. Kyoung Ja Lee, bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull. Malays. Math. Sci. Soc. (2) 32(3), 361–373 (2009).
6. Lee, K.M., Bipolar-valued fuzzy sets and their operations. Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, pp: 307-312(2000).
7. Lee, K.M., Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolar valued fuzzy sets. J. Fuzzy Logic Intelligent Systems, 14 (2): 125-129(2004).
8. Samit Kumar Majumder, Bipolar Valued Fuzzy Sets in Γ-Semigroups, Mathematica Aeterna, Vol. 2, no. 3, 203 – 213(2012).
9. Young Bae Jun and Seok Zun Song, Subalgebras and closed ideals of BCH-algebras based on bipolar-valued fuzzy sets, Scientiae Mathematicae Japonicae Online, 427-437 (2008).
10. Zadeh, L.A., Fuzzy sets, Inform. And Control, 8: 338-353(1965).

Source of support: UGC-MANF, Conflict of interest: None Declared