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THE INFLUENCE OF THERMAL RADIATION ON THE CONVECTIVE FLOW PAST A VERTICAL RADIALLY STRETCHING SHEET IN THE PRESENCE OF A HEAT SOURCE

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ABSTRACT

A boundary layer flow analysis is performed to study the influence of thermal radiation and heat source on the flow of an incompressible viscous fluid over a vertical radially stretching sheet. The governing system of partial differential equations is first transformed into an ordinary differential equations using similarity transformations and then the transformed boundary layer equations are solved numerically. The present results are compared with the available published results and it is found that the results are in excellent agreement. The salient features of the flow, heat transfer characteristics for different values of thermal radiation, thermal conductivity and intensity of heat generation are discussed. The influence of the physical parameters on the coefficients skin friction, local Nusselt number are calculated and discussed. It is observed that increase in values of the radiation parameter increase the skin friction and Nusselt number, while the increase in the Prandtl number decreases the skin friction and Nusselt number.

Key words: *Boundary layer flow, Radially stretching sheet, Heat generation, Radiation effect.*

1. INTRODUCTION

The study of heat and momentum transfer in a laminar boundary layer flow over a stretching sheet attracted the attention of several researchers due to its applications in polymer engineering and metallurgy. The analysis of the flow due to a stretching sheet is very essential in the extrusion of sheet materials so as to ensure the finished product to possesses required quality specifications for vertical sheets.

Sakiadis (1961) is the first researcher to study the boundary layer flow over a continuous surface with a constant speed. Crane (1970) investigated the boundary layer flow of a Newtonian fluid owing to the stretching of an elastic sheet moving in its own plane linearly. He studied the flow and heat transfer aspects and presented both analytical and experimental results. Subsequently following the above studies several researchers studied the fluid flow problems pertaining to stretching sheet. Gupta and Gupta (1977) have studied heat and mass transfer in hydrodynamic fluid flow over an isothermal stretching sheet with suction/blowing effects. Chakrabarti and Gupta (1979) examined the flow and heat transfer of an electrically conducting incompressible fluid over a stretching sheet. Vajravelu and Hadjinicalaou (1997) analyzed the MHD convective heat transfer from a stretching surface with a uniform free stream in the presence of a heat source/sink. Chamkha (1999) studied the steady, laminar, free convection flow over a vertical porous surface in the presence of a magnetic field and heat source or sink. Emad and Aziz (2004) investigated the steady, laminar, hydromagnetic heat transfer by mixed convection over a continuously stretching surface with power law variation in the surface temperature or heat flux in the presence of heat generation/absoption. Anjali Devi and Thiyagarajan (2006) investigated the steady hydromagnetic flow of an incompressible, viscous and electrically conducting fluid with heat transfer over a surface of variable temperature stretching with a power – law velocity in the presence of a variable transverse magnetic field. Subhas Abel and Mahesha (2008) studied the magnetohydrodynamic boundary layer flow and heat transfer characterstics of a non-Newtonian viscoelastic fluid over a flat sheet with a linear velocity in the presence of thermal radiation and non-uniform heat source. Elbashbeshy and Aldawody (2010) investigated the heat transfer over an unsteady stretching surface with variable heat flux in the presence of heat generation/absorption. Ibrahim and Shanker (2012) studied the MHD boundary layer flow and heat transfer in an incompressible fluid with heat source.

In all the above studies radial stretching of the sheet is not considered. Wang (2007) analyzed the natural convection flow of a viscous fluid over a vertical radially stretching sheet and discussed the asymptotic properties and uniqueness of the solution. In the present analysis an effort is made to study the effect of radiation and heat generation on the boundary layer flow of a viscous incompressible fluid over a vertical radially stretching sheet.

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2. FORMULATION OF THE PROBLEM

We consider the laminar boundary layer flow and convective heat transfer of a incompressible viscous fluid in the presence of heat source over a vertical radially stretching sheet. The temperature of the sheet T_w is assumed to be constant. It is also assumed that a temperature heat source is present. Using Boussinesq approximation the governing equations for the convective flow and heat transfer of the viscous fluid are

$$u_x + v_y + w_z = 0 \tag{1}$$

$$uu_x + vu_y + wu_z = \nu\nabla^2 u + g\beta(T - T_\infty) \tag{2}$$

$$uv_x + vv_y + wv_z = \nu\nabla^2 v \tag{3}$$

$$uw_x + vw_y + ww_z = \frac{-p_z}{\rho} + \nu\nabla^2 w \tag{4}$$

$$uT_x + vT_y + ww_z = \frac{k}{\rho_{c_p}}\nabla^2 T - \frac{1}{\rho_{c_p}}\frac{\partial q_r}{\partial z} + \frac{Q}{\rho_{c_p}} \tag{5}$$

Subject to the boundary conditions

$$u = ax, \quad v = ay, \quad w = 0, \quad T = T_w \qquad at \;\; z = 0 \tag{6}$$

and $\quad u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad p \to p_\infty \quad as \;\; z \to \infty \qquad (7)$

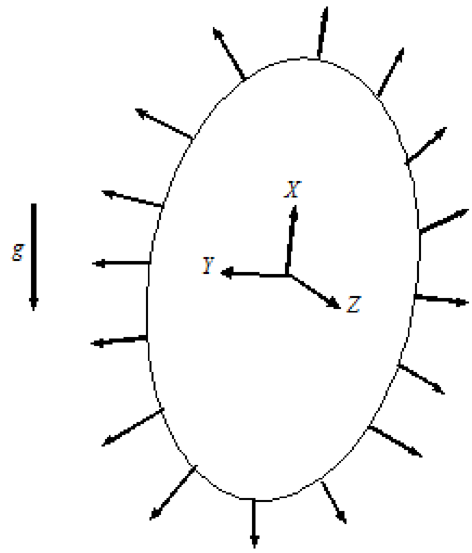

Fig. 1: The radially stretching surface

where (u, v, w) are the velocity components in the (x, y, z) directions respectively, ν is the kinematic viscosity, g is the acceleration due to gravity, β is the coefficient of thermal expansion, T is the fluid temperature, $T_\infty$is the ambient temperature, p is the pressure, ρ is the bulk density and k is the thermal diffusivity, Q is the strength of the heat source a is a constant having dimension t^{-1}. Using Roseland approximation the radiative heat flux can be taken as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{8}$$

Here, σ^* and k^* are respectively, the Stephan-Boltzmann and the mean absorption coefficient. It is assumed that the temperature difference within the flow is such that the term T^4 can be expressed as a linear function of temperature. Giving a Taylor series expansion about T_∞ and neglecting high order terms T^4 can be written as

$$T^4 \approx 4T_\infty^3 T - T_\infty^4$$

3. METHOD OF SOLUTION

We introduce the similarity variables as

$$u = axf'(\eta) + \frac{g\beta(T_w - T_\infty)h(\eta)}{a}, \quad v = ayf'(\eta), \qquad w = -2\sqrt{av}\; f(\eta)$$

$$T = T_\infty + (T_w - T_\infty)\theta(\eta) \tag{9}$$

where $\eta = \sqrt{\frac{a}{\nu}}\, z$

Substituting the similarity variables the equation of motion and heat transfer equations (2), (3), (5) – (7) reduced to the following equations

$$f''' + 2\,f\,f'' - f'^2 = 0 \tag{10}$$

$$h'' + \theta - h\,f' + 2\,h'\,f = 0 \tag{11}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + 2\,\Pr f\,\theta' + \alpha\,\Pr = 0 \tag{12}$$

Where $R = \frac{4\,\sigma^*\,T_\infty^3}{k\,k^*}$ is the Radiation parameter

$\Pr = \frac{\mu_{c_p}}{k}$ is the Prandtl number

$\alpha = \frac{Q}{a\rho_{c_p}(T_w - T_\infty)}$ is the heat source parameter

The corresponding boundary conditions are

$$f'(\eta) = 1, \quad f(\eta) = 0, \quad h(\eta) = 0, \quad \theta(\eta) = 1 \quad \text{at } \eta = 0 \tag{13}$$

$$f'(\eta) \to 0, \quad h(\eta) \to 0, \quad \theta(\eta) \to 0 \quad as\ \eta \to \infty \tag{14}$$

The governing equations (10)-(12) along with the boundary conditions have been solved numerically.

4. RESULTS AND DISCUSSION

The objective of the present investigation is to understand the effect of radiation, heat generation on velocity and temperature. Since the governing equations of the flow are non-linear coupled equations, they are solved using Runge - Kutta fourth order method along with shooting method. The accuracy of the method is ensured by comparing the values of the flow variables with those of Wang (2007) in the absence of radiation and heat generation (Table 1) which are found to be in excellent agreement.

Table - 1

Pr	Wang(2007) $\theta'(0)$	Present study $\theta'(0)$	Wang(2007) h′(0)	Present study h′(0)
2	-1.32391	-1.32393	0.38407	0.38402
7	-2.72297	-2.72288	0.20331	0.20325
20	-4.78914	-4.78884	0.12164	0.12163
70	-9.18731	-9.18736	0.06593	0.06592
200	-15.7061	-15.7060	0.03933	0.03932

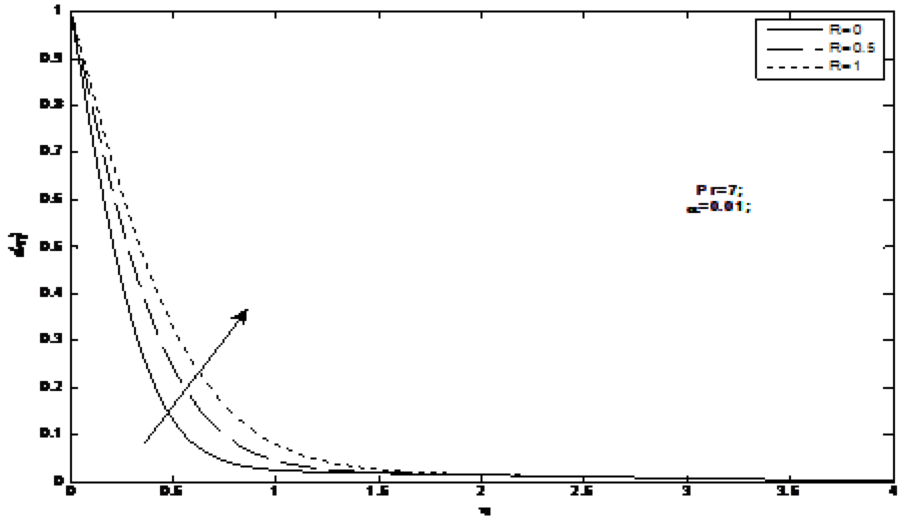

Fig. 2: Effect of radiation parameter on temperature profiles with Pr = 7, α = 0.01.

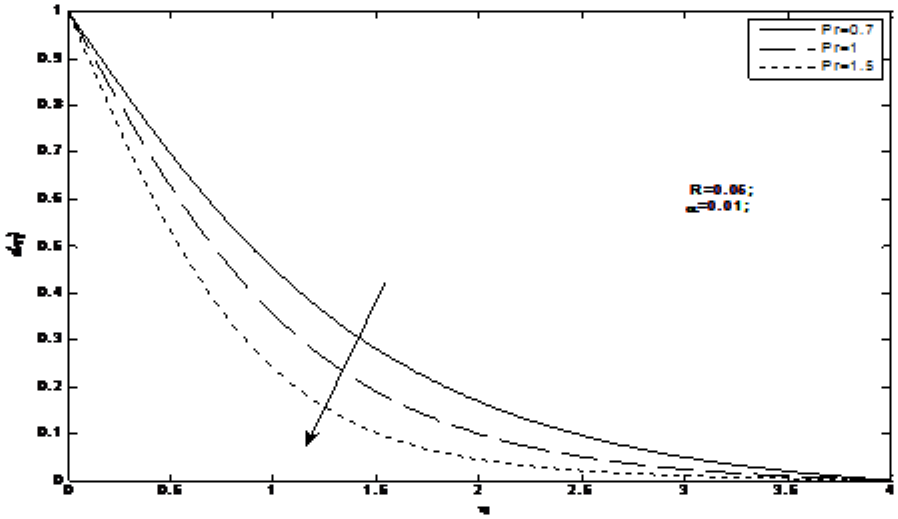

Fig. 3: Effect of Prandtl number on temperature profiles with R = 0.05, α = 0.01.

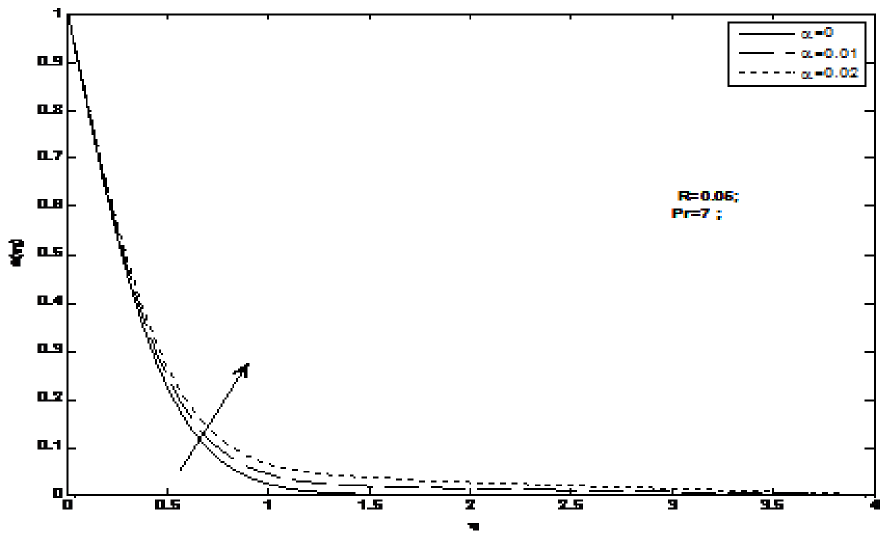

Fig. 4: Effect of heat source parameter on temperature profiles with R = 0.05, Pr = 7.

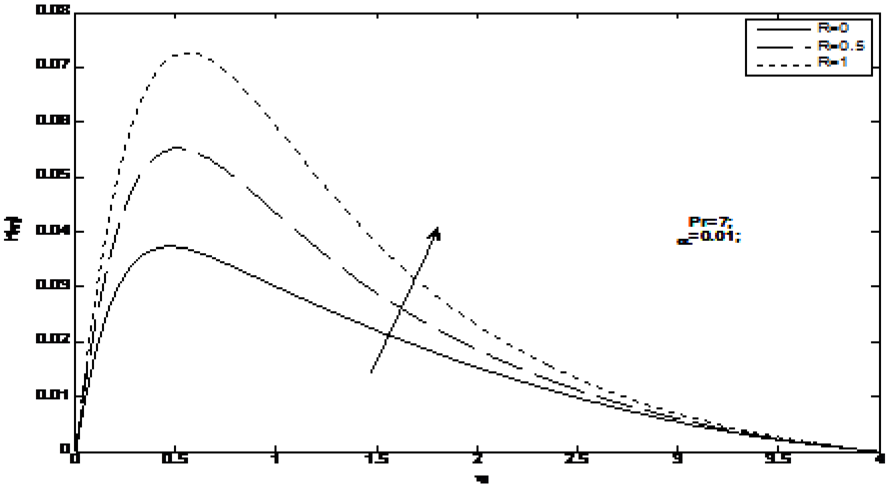

Fig. 5: Effect of radiation parameter on natural convection velocity profiles with Pr = 7, α = 0.001

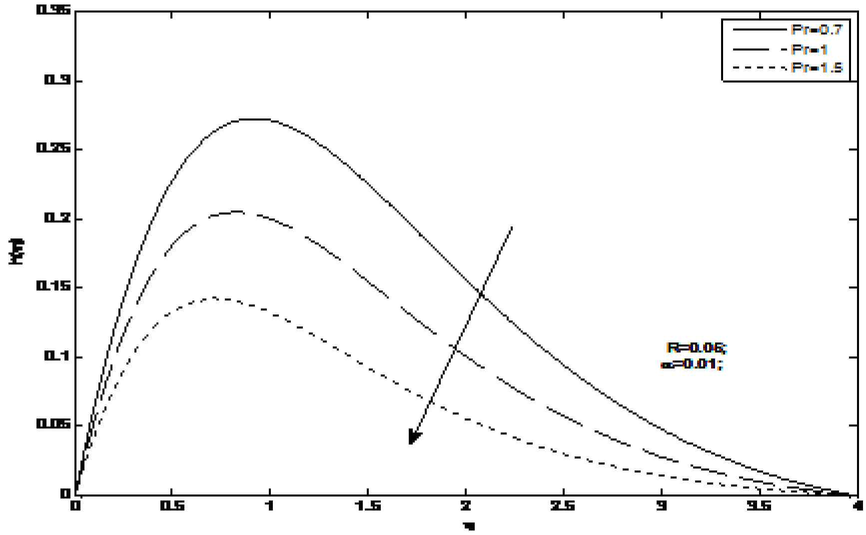

Fig. 6: Effect of Prandtl number on natural convection velocity profiles with R = 0.05, α = 0.01.

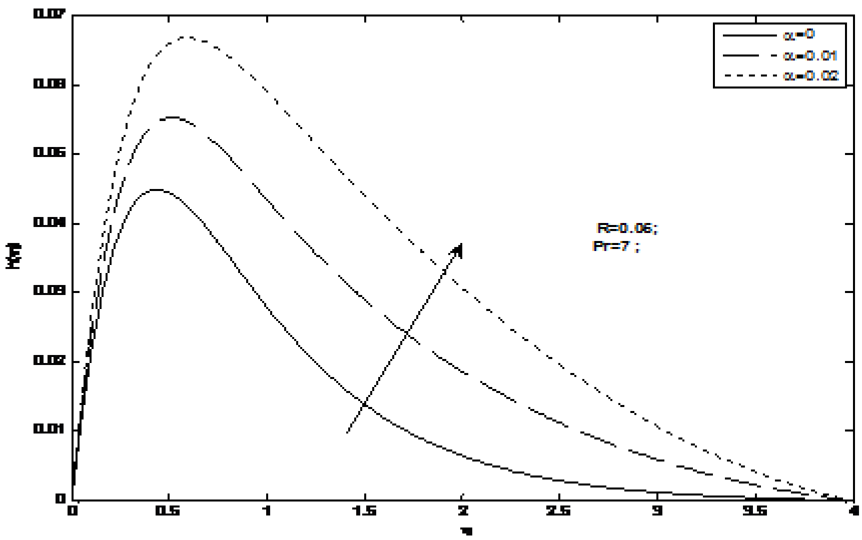

Fig. 7: Effect of heat source parameter on natural s convection velocity profiles with R = 0.05, Pr = 7.

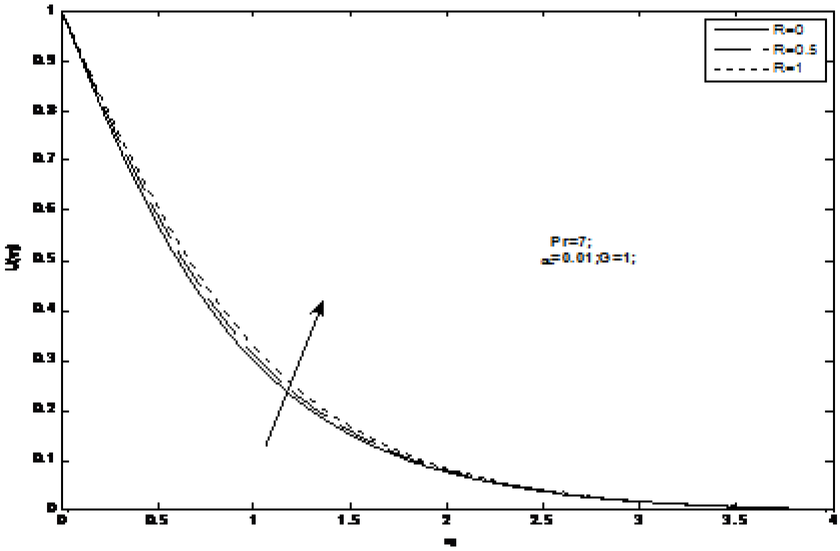

Fig. 8: Effect of radiation parameter on velocity profiles with Pr = 7, α = 0.01, G = 1.

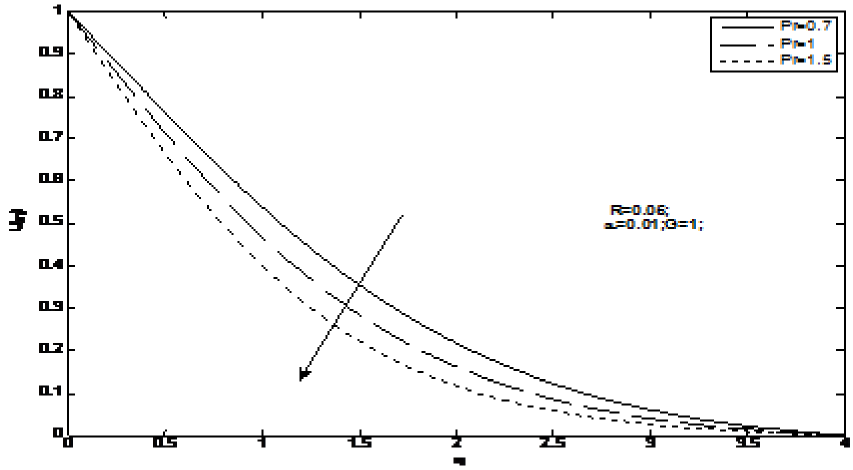

Fig. 9: Effect of Prandtl number on velocity profiles with R = 0.05, α = 0.01, G = 1.

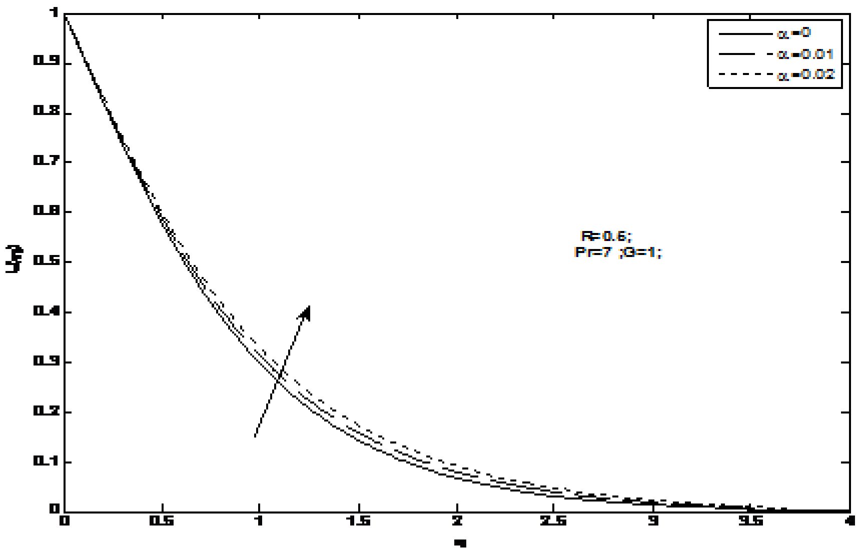

Fig. 10: Effect of heat source parameter on velocity profiles with R = 0.5, Pr = 7, G = 1.

Fig (2) presents the variation of temperature for different values of radiation parameter. The influence of radiation parameter R is to increase the temperature distribution in the flow region. The temperature profiles indicate that the thickness of the thermal boundary layer of the fluid increases with the radiation parameter.

Fig (3) shows the effect of Prandtl number on temperature. It is observed that increasing values of Prandtl number reduces the temperature. This might be due to fact that the reduction in the thermal conductivity and hence the decrease in the thermal boundary layer thickness. When Pr = 1.5 the thickness of the thermal boundary layer is half of that when Pr = 0.7.

Fig (4) indicates that the presence of heat generation increases the temperature distribution. The thickness of thermal boundary layer further increases with increase in the intensity of heat generation.

The velocity (h) induced due to free convection is plotted in Fig (5). The presence of the radiation parameter is to enhance the convective velocity. Increased values of R enhance the magnitude of h. When R = 1, the peak value of h is twice that of the corresponding value when radiation is absent.

The effect of Prandtl number is illustrated in Fig (6). Lesser the thermal conduction, lesser is the convective velocity. When Pr = 1.5 there is a twofold reduction in the peak value of the convective velocity to that when Pr = 0.7.

The variation of heat source parameter is illustrated in Fig (7). The presence of heat source is to increase h. Increasing values of the heat source parameter results in significant enhancement in the magnitude of h throughout the flow region. The total non-dimensional velocity U from the similarity variables defined can be written as

$$U = f'(\eta) + G\ h(\eta)$$

where $G = g\,\beta\,(T_w - T_\infty)/a^2 x$ is the local Grashoff number. The total velocity U is plotted in Fig (8) varying the radiation parameter. The increased values of Grashoff number increase the total velocity. However, its impact on U is not as significant as on h. Hence, the momentum boundary layer thickness does not vary much. The variation of Prandtl number on U is shown in Fig (9). U also decreases for increased values of Pr. The influence of heat source on U is plotted in Fig (10). The total velocity enhances with increase in the heat source parameter.

The skin friction and the Nusselt number on the stretching sheet are tabulated in table (2). The impact of all the physical parameters on the numerical values of skin friction and the wall temperature gradient may be analyzed from this table. It is observed that the wall shear stress and the Nusselt number increase with increase in the radiation parameter. The influence of the Prandtl number is to reduce the Nusselt number and wall shear stress. An increase in the heat source parameter leads to an increase in the Nusselt number as well as skin friction (corresponding to an increase in the thickness of thermal and momentum boundary layers)

Table: 2

R	Pr	α	τ	Nur
0	7	0.01	-0.945251	-2.631474
0.5	7	0.01	-0.888348	-1.981119
1	7	0.01	-0.841116	-1.633921
0.05	0.7	0.01	-0.480061	-0.638689
0.05	1	0.01	-0.584097	-0.799502
0.05	1.5	0.01	-0.692633	-1.034559
0.5	7	0	-0.913014	-2.046089
0.5	7	0.01	-0.888348	-1.981119
0.5	7	0.02	-0.863681	-1.916148

5. CONCLUSIONS

The present investigation brings out some interesting results on the natural convective flow of a viscous fluid over a radially stretching sheet in the presence of a constant heat source. It is observed that increase in the Prandtl number leads to a reduction in the thickness of the boundary layers of the hydrodynamic momentum, convective momentum as well as the mixed convective momentum. The thermal boundary layer thickness is also decreased with decrease in the thermal conductivity. A reversal behavior is observed for an increase in the radiation parameter i. e, increasing values of the radiation parameter resulted in enhancement of the thickness of all momentum boundary layers and the thermal boundary layer. The heat source parameter increased the values of the velocity of base flow, free convective flow, mixed convective flow and temperature and thus the thickness of the corresponding boundary layers are observed to grow.

6. REFERENCES

[1]. Ali J. Chamkha, Hydromagnetic three-dimensional free convection on a vertical stretching surface with heat generation or absorption, International Journal of Heat and Fluid Flow 20 (1999) 84-92.

[2]. Anjali Devi S.P, Thiyagarajan M, (2006), steady nonlinear hydromagnetic flow and heat transfer over a stretching surface of variable temperature, Heat Mass Transfer 42: 671 – 677.

[3]. Chakrabarti A, Gupta AS, (1979), Hydromagnetic flow and heat transfer over a stretching sheet. Quart Appl Math 37: 73 – 78.

[4]. Crane, L.J.: Flow past a stretching plate. ZAMP 21, 645-647(1970).

[5]. Elbashbeshy E. M. A. and Aldawody D.A., Heat transfer over an unsteady stretching surface with variable heat flux in the presence of heat source or sink, Computers and Mathematics with applications 60 pp.2806-2811 (2010).

[6]. Emad M. Abo-Eldahab, Mohamed A. El. Aziz, Blowing/Suction effect of hydrodynamic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, International Journal of Thermal Sciences 43 , 709-719(2004).

[7]. Gupta P. S, Gupta A. S, Heat and Mass Transfer on a stretching sheet with suction or blowing, Canada. J. Chem, J. Engng. 55 (1977) 744 – 746.

[8]. Ibrahim W and Shanker B, Unsteady MHD Boundary-Layer flow and Heat Transfer due to stretching sheet in the presence of Heat Source or Sink by Quasi-Linearization Technique, Int. J. of Appl. Math. 8(7): 18-30(2012).

[9]. Sakiadis, B.C., Boundary layer behaviour on a continuous solid surface: II. The boundary layer on a continuous flat surface. AICHE J.7, 221-225(1961).

[10] .Subhas Abel M. and Mahesha N, Heat transfer in MHD viscoelastic fluid over a stretching sheet with variable thermal conductivity, non - uniform heat source and radiation, Applied Mathematical Modelling 32 (2008) 1965 – 1983.

[11]. Vajravelu K, Hadjinicolaou A, convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream, Int. J. Eng. Sci. 35 (1997) 1237-1244.

[12]. Wang C. Y., Natural convection on a vertical radially stretching sheet, J.Math.Anal.Appl.332 (2007) 877-883.

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